INDIAN SCIENTIFIC HERITAGE

DR. N. GOPALAKRISHNAN

FOREWORD BY DR. R.A. MASHELKAR, F.R.S.

Indian Institute of Scientific Heritage
Thiruvananthapuram

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He has also received four awards for the literary works and has 41 books to his credit, both in scientific and cultural subjects and many popular articles on Indian Scientific Heritage.

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ananthaayaa api samyoga: thasya ananthaanaam api kalpyamaanasya yogasyaaddhyaavayavina: parasparamacchedaad ekonacchedaa mamsa saadhyam sarvathraapi samaanam eva...

Thus the sum of an infinite series, whose later terms (after the first) are got by diminishing the preceding or by the same divisor, is always equal to the first term divided by one less than the common mutual divisor.

When all the facts on the development of mathematics as a branch of knowledge, both in theoretical and applied field, put together one can understand that there are many novel and entirely original contributions of ancient Indians, just like a few of which are presented here. Many more are yet to see the light. These are the basis of **Indian mathematical heritage,** not merely the discovery of Zero and Pythagorus Theorem!

CHAPTER VI : INDIAN CONTRIBUTIONS TO ASTRONOMY

Navagrahas are part of Indian astrology which have a strong base of mathematics and astronomy. Many of the fundamental principles and calculations of astrology follows the astronomical concepts precisely. However the prediction part of astrology has nothing to do with the astronomical sciences. There is a misunderstanding that astrology and astronomy (the subject in its modern and ancient sense) are not related. Calculations of the positions, movements, velocity, etc., of planets used in astrology have a direct bearing on astronomy. Important points relating astrology and astronomy i.e the positions of planets are specifically calculated on the basis of astronomical knowledge. Both astrology and astronomy are known as Jyothisha in Sanskrit, which is the fifth Vedanga of the vedic related literature. For some people, Jyothisha means only 'astroloy'. Thus formed the

misunderstanding that there is no science in Jyothisha. Modern concepts of astronomy can only be searched in the Jyothisha (in its sanskrit meaning) books. Each and every astronomical calculation given in these books is based on mathematics and they have a perfect and sound scientific background too. The ancient Indian scholar Suryayajvan has defined astronomy thus:

"Calculation of time, planets, spherical bodies, celestial spheres, rotation and revolution of planets, position of earth, etc, are explained in astronomical mathematics". In short astronomy means these subjects.

Thus, as it is said in modern science, astronomy is nothing but applied mathematical studies of celestial bodies, so has been said in ancient Indian astronomical books too; for a perfect understanding of celestial bodies, knowledge of mathematics is essential: says Jishnu Nandana:

"Only that mathematician who is an expert in spherics knows the path of planets, i.e astronomy. If one knows other than this, how can he understand the movement of planets" (Jishnu Nandana referred in Neelakanta's Aryabhateeya bhashyam - 33p)

गणितज्ञो गोलज्ञो गोलज्ञो ग्रहगतिं विजानाति
यो गणित गोलबाह्यो जानाति ग्रहगतिं स कथम् ?

Ganithajno golajno golajno grahagathim vijaanaathi
yo ganitha gola baahyo jaanaathi grahagathim sa katham

An astronomer's qualification has been given 1500 years ago in India by Varahamihira. He clearly gives the Indian approach to astronomy as a branch of applied mathematics:

सममण्डले (खा) संप्रवेशवेला: करोति योऽर्कस्य तत्प्रत्ययं च
जनयति जानाति स भास्कर: सम्यक्

Samamandale (khaa) sampravestavelaa: karothi yoarkasya
thathprathyayam cha janyathi jaanaathi sa bhaskara: samyak

One is fit to be called an expert astronomer only if he knows the problem dealing with the Sun crossing the prime vertical and prove his method mathematically and graphically. (Panchasiddhantika 4-36)

Quality of this definition is in no way inferior to that given by modern science. In astrology the Sun, Moon, Venus, Mercury, Mars, Jupiter, Saturn, Rahu and Ketu were considered as Grahas, which is wrongly translated as planets. Perhaps the use of the word graha might have done with a meaning 'that is holding or influencing'. Real planets are separately classified in ancient Indian astronomy. One chapter of the Panchasiddhantika and Sishyadhi vruddhi Tantra, exclusively deals with the real planets which are called Tara grahadhyaya. In this class Sun, Moon, Rahu and Ketu are not included as planets. In all astronomical books - both theoretical and practical explanations - real planets are mentioned as Mars, Mercury, Jupiter, Saturn, Venus and Earth. In Aryabhateeya, Tara graha word is used only for the real planets. It is important to mention here the status of Rahu and Ketu. Lallacharya's famous book Sishyadhi vryddhi Tantra, written 1200 years ago says about the superstitions on 'Rahu' and its 'role' in eclipse. This mention is made in the 20th chapter namely 'chapter of superstition' (Sishyadhi vruddhi Tantra 20-21)

असुरो यदि मायया युतो नियतोऽतिग्रसतीति ते मतम् ।
गणितेन कथं स लभ्यते ग्रहकृतो विना कथञ्चन ।

Asuro yadi maayaya yutho niyathoathi grasatheethi the matham
ganithena katham sa labhyathe grahakrutho vinaa kathanjana

If Rahu the artificial demon is always the cause of an eclipse by swallowing (The Sun or the Moon), then how is it that an eclipse can be determined by means of calculations. Moreover

why is it that there is not an eclipse on a day other than the day of new moon or full moon.

Thus the Rahu concept as a planet, had nothing to do with ancient Indian astronomy. There is no mention of Rahu in Aryabhateeya. In Pancha siddhantika and Bhaskaracharya's work, 'Rahu' is used to refer to the earth's shadow. In these books the parameters related to Rahu have been determined using mathematical calculations. Few astrologers attribute the qualities of Rahu and Ketu to Neptune and Pluto, which is neither astrology nor astronomy.

There is another impression that in India, all the planets are given divine zeal, as it is described in puranas and they are worshipped. It is true! Even the earth has been personified as goddess and given the status of mother, so are the celestial bodies given divine status. But no astronomical calculations have a bearing on this divinity while looking for the scientific measurements. Only in the first stanza, of the astronomical books, the authors bow the gods and the celestial bodies, before they are explained scientifically. Similar approach was common in all general books written in ancient India and towards all branch of knowledge. Hence an attempt to reject the excellent scientific heritage in Indian astronomy on the above grounds, will be tantamount to ignoring the great Indian scientific contributions. As for the spiritual approach towards the celestial bodies Aryabhatta says thus:

ब्रह्म कु शशि बुध भृगु रवि कुज गुरु कोण भगणान्
नमस्कृत्य आर्यभटस्त्विह निगदति कुसुमपुरेऽभ्यर्चितम् ज्ञानम्

Brahma ku sasi budha bhrugu ravi kuja guru kona bhaganaan
namaskruthya aaryabhatasthviha nigadhathi kusumapure
bhyarchitham jnaanam

Having bowed with reference to Brahma, Earth, Moon, Mercury, Venus, Sun, Mars, Jupiter and Saturn and the asterisms, Aryabhatta sets forth the knowledge honoured in Kusumapura. (Aryabhateeya 1-1)

This is a perfect example of the adoration of the divinity on all the celestial bodies. Those who have studied this book can never say that it contains spiritual explanations on planets. The scientific definition and character of the planets are given by Nilakanta in his commentary to Aryabhateeya (4-5):

ज्ञात भोगग्रहं वृत्तं सर्वत्र प्रतिमण्डलं कक्ष्यावृत्तं च
तत्तुल्यं ज्ञेयं भोगप्रदेशगम्

Jnaatha bhogagraham vruttham sarvathra prathimandalam kakhhyaarvuthham cha thathhulyam jneyam bhogapradesagam

Know that all planets are spherical. And also know that it revolves in the eccentric (pratimandalam) orbits which are equal to circular orbit of planets.

पञ्चमहाभूतमयस्तारागण पञ्जरे महीगोल: ।
खेयस्कान्तस्थो लोह इवावस्थितो वृत्त: ।।

Panchamahaabhootha mayasthaaraagana pancjare mahigola:
kheayaskaanthastho yohaivaa vasthitho vruthha:

In the group of great celestial bodies, all planets made of Panchabhootas, exist in the space, like a magnet attracts a piece of iron, from all sides. (Panchasiddhantika 13-1).

Panchabhootas are solid, liquid, gas, ionic state and probably wave nature of particles(?). These are two among a variety of explanations given by the Indian scientists on planets and celestial bodies. They could also distinguish clearly the stars and planets.

तेजसां गोलक: सूर्येा ग्रहर्क्षाण्यम्बुगोलका: ।
प्रभावन्तो हि दृश्यन्ते सूर्यरश्मिविदीपिता: ।।

Thejasaam golaka: sooryo graharkshaanyambugolakaa:
prabhaavantho hi drusyanthe sooryarasmivideepithaa:

Due to the powerful light from the Sun, the planets also appear as shining, in the space. Due to this light (reflected from the planet) we see the planets. (Neelakanta bhasyam to Aryabhateeya 4-37).

There is a misunderstanding that ancient Indians have said that Venus and Mercury are stars. It is wrong! Clear explanations are given that they are considered as inferior planets, as in modern astronomy. Because their orbits fall within the orbits of earth around the Sun. About superior/inferior planets Aryabhatta says:

शनिगुरुकुजेषु मन्दादर्धमृणं धनं भवति पूर्वे ।।
Sanigurukujeshu mandaadardhamrunam dhanam bhavathi poorve

In the case of superior planets, Saturn, Jupiter and Mars, first apply the mandaphala negatively or positively (Aryabhateeyam 3-22)

Superior planets are those which have orbits outside the orbits of earth, around the Sun. This line informs us that the positions of the above planets around the Sun and also correction factors to be applied were known while determining different parameters on their revolutions around the Sun. Similarly scentific approach in giving correction factors for the inferior planets (i.e. Mercury and Venus) was also existed, even from the period of Aryabhatta I.

Some Scientific definitions and graphical figures in Astronomy:

In modern astronomy, definitions and pictorial representation of the zodiac are given with many scientifc terms. Similar representation were given in the ancient Indian astronomy too. A few examples are cited here to show that the modern

astronomy and Indian astronomy approached the celestial studies in similar pattern.

Day radius according to modern astronomy is the small circle parallel to the celestial equator. A small circle parallel to the celestial equator is known as diurnal circle. The Sun's diurnal circle is the small circle parallel to the equator when the Sun is described in the course of a day. Earthsine is R sine of the arc of a diurnal circle intercepted between the local horizon and 6 o' clock circle. The 6 o' clock circle is a great circle of the celestial sphere which passes through east and west points of the celestial horizon and the poles of celestial equator. Equinoctial horizon is the horizon of a place on the equator. Ascensional difference is defined by the arc of celelestial equator lying between the equatorial horizon and the secondary to equator passing through the intersections of the diurnal circle and eastern or western horizon.

Exactly the same explanation, methods and definitions were followed in the ancient Indian astronomy too. Determination of these parameters is also described with perfect clarity. Eight types of circles known to modern astronomy have been described with definitions by Indian astronomers. Vateswara in the gola part of the Vateswara siddhanta, described them in detail. Earlier to Vateswara, Aryabhatta I has also given definitions and explanations to some of these astronomical terms and parameters. Vateswara's approach was highly systematic and further commentary is not required, for understanding.

Vateswara's definition for the celestial and terrestrial circles:

From the earliest siddhantas to the latest ones, many astronomical parameters have been described and mathematically calculated for understanding the celestial phenomena. One chapter is fully devoted to such astronomical definitions by Acharya Vateswara. Those definitions and their modern

equivalents are quoted. The definitions on construction of the Armillary Spheres known as Gola are given in the third chapter. Chapters 1-4 of Vateswara siddhanta deal with definitions of celestial circles:

ऊर्ध्वमधो/परपूर्वमिहाद्यम् प्राहुरिदं सममण्डलमन्यत् ।
तद्वदिहोत्तरदक्षिणदिक्स्थम् वृत्तयुगम् विदिशोरपि तद्वत् ।।

Urdhvamadho apara poorvamihaadyam praahuridam
samamandalamanyath thadvadihotthara dakshinadikstham
vrutthayugam vidisorapi thadvath

Vertical circle passing through the west and east cardinal points is the first circle: this is called the samamandala. (This circle is the prime vertical. Another similar vertical circle (called the yaamyottara-vrutta) which passes through the north and south cardinal points is called the meridian.

Two vertical circles (called drg vrutta), similarly pass through the intermediate cardinal points (i.e north east and south west, north west and south east points)

आवेष्टमानमथ तानि दलप्रवृत्या यद्वृत्तमत्र हरिजं क्षितिजं तदाहुः
यस्मिन् भवेत् समुदयो/स्तमयो/खिलानां प्राच्यां क्रमादपरदिश्युडुखेचराणां

Aaveshtamaanamatha thaani dalapravruthyaa
yadvrutthamathra harijam kshithijam thadaahu:
yasmin bhaveth samudayasthamayo akhilaanaam
praachyaam kramaadaparadisyudu khecharaanaam

The great circle which goes round them, dividing each of them into two equal parts, is called harija or kshitija. This in modern astronomy is horizon. This is the circle on which rising and setting of stars and planets take place towards east and west respectively.

पुर्वापरक्षितिजसङ्गमयोर्गतञ्च याम्यादधः पललवैः क्षितिजाद्विलग्नम्
सौम्यादथोपरि समध्रुवमार्ग संस्थमुन्मण्डलं दिननिशोः क्षयवृद्धिकृतात्

Poorvaaparakshithija sangamayorgathamcha yaamyaadadha:
palalavai: kshithijaadvi lagnam
soumyaadathopari samadruvamarga samstha munmandalam
dinaniso: kshayavruddhikruthaath.

Passing through the two points of intersection of prime vertical and horizon, lying below the south cardinal point by the degrees of local latitude, fasterned to the horizon, and lying above the north cardinal point, passing through the north celestial pole, is the Unmandala, the cause of decrease and increase of the day and night. This in modern astronomy is known as the 6'o clock circle.

ऊर्ध्वाधरं ग्रहसमाभिमुखं च यत्तद् दृङ्मण्डलं त्रिग्रहहीनविलग्नलग्नम् ।
दृक्षेपवृत्तमपि तद्वदमूनि चाष्टौ भांशङ्कितानि वलयानि खगोलबन्धे ।।

Urdhvaadharam grahasamabhimukham cha yatthath
drungmandalam thrigraha heena vilagnalagnam
drukshepavrutthamapi thadvadamooni chaashtow
bhaamankithaani valayaani khagola bandhe.

The vertical circle which goes through the planet is the drunmandala. (in modern terms this is known as the planet's vertical circle). Vertical circle that passes through the central ecliptic point which lies three signs behind the vilagna is Drikshepavrutta. (It is the rising point of the ecliptic)

According to Vateswara the above eight circles which are graduated by the divisions of signs and degrees on the Khagola-known as sphere of the sky.

खस्वस्तिकाद् दक्षिणतोऽक्षभागौ पाता(ल)संज्ञाच्च तथोत्तरेण ।
नाड्यङ्कितम् वैषुवतम् तदुक्तम् वृत्तम् भगोलस्य खगोलमध्ये ।।

Khasvasthikaad dakshinathoakshabhaagow paathaa(la)
samjnachha thathothharena naadyankitham vaishuvatham
thaduktham vrutham bhagolasya bhagolamaddhye

The sphere of the asterisms lie within the sphere of the sky. Great circle of the sphere of asterisms which lies towards the south of the zenith by an amount equal to the degrees of local latitude and towards the north of nadir by the same amount and which is graduated with the division of nadis is the vishuvadvrutta. This circle is called the equator.

याम्योत्तरं वृत्तमिहान्यदेतद्विवेष्टमानम् कुजवत् समन्तात् ।
तद्दक्षिणोदग्ध्रुवयोर्निबद्धा यष्टिर्ध्रुवा भूश्च भगोलमध्ये ।।

Yaamyottharam vrutthamihaanyadetha dviveshtamaanam
kujavath samanthaath thaddakshinodagdhruvayornibaddhaa
yashtirdhruvaa bhooscha bhagolamadhye

Surrounding it on all sides like the horizon is another great circle of this sphere called the meridian. Fastened to north and south pole, equator is polar axis which is fixed in postions. In the centre of sphere of asterisms lies the earth.

नाड्याह्ववृत्ते/जतुलादिलग्नं जिनांशकैर्दक्षिणतो मृगादौ ।
सौम्ये शीतमन्दिरादावपक्रमाख्यं तदुशन्ति वृत्तम् ।।

Naaddyaahvavruttha f ajathulaadilagnam
jinaamsakairdakshinatho mrugaadow soumye seetha
mandiraadaavapakramaakhyam thadusanthi vruttham

Fastened to the so called nadivrutta or the equator at the points of Aries and Libra and lying 24 degrees of the south (of equator) at the first point of Capricon and 24 degrees to the north (of equator) at the first point of Cancer, there is a great circle called the apakrama vrutta (now known as the ecliptic)

पाताग्रहाणां च विलोमगत्या विमण्डले स्वे मृगलाञ्छनाद्या: ।।

Paathaagrahaanam cha vilomagathyaa
vimandale sve mrugalaancchanaadyaa:

The Moon, the planets and their nodes, move on their own

orbits which is called the vimandala. It is known as the orbits of the planets and celestial bodies.

कक्ष्यावृत्तम् नाडिकावृत्तगत्या देयं तद्वद्याम्यसौम्यं तथाऽन्यत् ।
भूजस्थित्या क्रान्तिवृत्तम् च तद्वत् तस्मिन्केन्द्रे स्वोच्चनीचे यथोक्त्या ।।

Kakshyaavruttham nadikaavruttha gathyaa
deyam thadvadyaamya soumyam thathaanyath
bhoojasthithyaa kraanthivruttham cha thadvath
thasminkendre svocchaneeche yathokthyaa

In the plane of equator fix a circle equal to the planets orbit. This is the equator in the sphere of a planet. Similiary fix the meridian and also another circle in the plane of the horizon (each equal to planet's orbit). Similarly fix the ecliptic. Here the planet's orbit is known as kakshyavrutta.

उन्मण्डले लग्नमपक्रमाग्रे पूर्वापरे दक्षिणसौम्यवृत्ते ।
नतांशकैस्तद् भ्रम वृत्तमुक्तम् व्योमौकसां व्यस्तमत: प्रभा स्यात् ।।

Unmandale lagnamapakramaagre poorvapare dakshina soumya vrutthe
nathaamsakaisthad bhrama vrutthamuktham
vyomoukasaam vyasthamatha: prabhaa syaath

The small circle which is fastened to the eastern and western halves of six o' clock circle at the distance of the planet's declination, from the equator and to the meridian at the distance of degrees of the planet's meridian zenith (distance) from the zenith is called the circle of the planet's diurnal motion.

हरिजे परपूर्वमण्डलद्युज्यावृत्तविशेषशिञ्जिनी ।
उदयाग्रगुणो द्युमण्डले भूज्योद्वृत्तकुजान्तरांशजीवा: ।।

Harije parapoorva mandala dyujyaavruthha visesha sinjinee
udayaagraguno dyumandale bhoojyothavruttha kujaantharaam sajeevaa:

R sine of the arc of the horizon lying between the prime vertical and the diurnal circle of the planet is the R sine of agra

(now known as the rising point of the planet) and the R sine of the degrees of diurnal circle lying between six o' clock circle and the horizon is bhoojya (bhujya) which is termed as Earthsine.

व्यासार्धवृत्तेऽन्तरमेतयोः स्याच्चरार्धजीवा परपूर्वयोस्तत् ।
अग्राग्रयोर्यद् हरिजे निबद्धम् सूत्रम् ग्रहाणामुदयास्त संज्ञम् ।।

Vyaasaardhavruttheantharamethayo: syaaccharaardha jeevaa parapoorvayosthath agraagrayoryad harije nibaddham soothram grahaanaamudayaastha samjnam

The arcual distance between the six o' clock circle and the horizon measure, along the R circle trijyavrutta known as great circle of the celestial sphere, supposed to be of radius 3438′ (minute of angle) is the charardhajya. It is called the R sine of the Ascensional difference. A thread tied to the extremities of the agra on the eastern and western halves of the horizon is called the udayaastasutra. (In modern astronomy it is known as the rising - setting line of planets).

प्राच्यांकुजापक्रमवृत्तसङ्ग प्राग्लग्नमाहुः (परितोऽस्त लग्नम्) ।
(लग्नाद्भवेत्) स (प्त) म(रा) शि(र) स्त तस्या (स्त)
कालोऽभ्युदयोऽस्य भूयात् ।।

Praachyaamkujaapakrama vrutthasanga praaglagnamaahu (parithoasthlagnam) (lagnaadbhaveth) sa(pta) ma (raa) si (ra) stha thasyaa stha kaalo abhyudayosya bhooyaath

Point of intersection of horizon and the ecliptic in the eastern half of the celestial sphere is called praglagna. I.e. the rising point of ecliptic; the same in the western half is called astalagna, known as setting point of ecliptic.

दिग्वृत्ते समदिग्विदिग्जवलये दृक्षेपवृत्ते तथा याम्योदग्वलये
खमध्यग खगक्षोणीजयोरन्तरम् ।
भागा उन्नतसंज्ञिता नतलवाः तत्खाङ्क भागान्तरं सर्वत्रैव नराः

समुन्नत गुणा दृग्ज्या नतज्याः क्रमात् ।।

Digvrutthe samadigvidigjaavalaye drukshepa vrutthe thathaa yamyodayvalaye khamadhyaga khaga kshoneejayoranthram bhaagaa unnathasamjnithaa nathalavaa: thathkhangka bhaagaantharam sarvathraiva naraa: samunnatha gunaa drugjyaa nathajyaa: kramaath

Whether the heavenly body be on the prime vertical or on the intermediate vertical on the drkshepa vrutta or on the meridian or on any veritical circle, the distance in degree between the body in the sky and the horizon gives the degrees of altitude; and 90 minus this degree measurement gives the degrees of the zenith distance. In all positions of the heavenly body, they are called nara i.e Rsine of the altitude and drugjya which is R sine of zenith distance.

There are many scientific definitions given in the ancient Indian astronomical books which are exactly the same in the corresponding terms of modern science. Some more of the ancient Indian astronomical definitions, terms and their modern astronomical equivalents are given below as glossaries:

Akshajya	Rsine of latitude	A
akshakarna	equinoctial midday shadow	B
akshakoti	colatitude	B
akshachapa	arc of latitude	B
akshamsa	lattitude of the place	B
akshonnati	inclination of the earth's axis	B
asita	measure of moon's unilluminated part	V
astarka	Sun's position at the time of star's heliacal setting	V
agra (1)	amplitude of the rising or Rsine of that	A

agra (2)	the arc of celestial horizon lying between the east point and the point where a heavenly body rises, or between the west point and the point where a heavenly body sets	B
apakrama	greatest declination	A
apamandala	ecliptic	A
adhva	distance of place from meridian	B
ardhajya	R sine	A
astamayodayasutra	rising and setting line	A
ativakra	midst of retrograde motion	B
ahoratrardhavishkambha	day radius	A
avanama	zenith distance	B
astalagna	setting point of the ecliptic	B
ayanachalana	precision of the equinoxes	B
udayajya	the rising point of planet's orbit	B
utkramana	R verse of sine	A
uttarayana	Sun's northward journey from winter solistics to summer solistics	A
unnatabhaga	degrees of latitude	A
unmandala	equatorial horizon	A
kakshyamandala	mean orbit of planet	A
kalardhajya	Rsine difference in terms of angle minutes	A
kuavarta	rotation of earth	A
kshitija	horizon	A
kshitijya	earthsine. The distance between the rising setting line and the line joining the point of the intersection of the diurnal circle and six o' clock circle	A

kshitijaa	earthsine	A
kshepa	celestial latitude	A
konamandala	intermediary vertical circle	L
krantimandala	ecliptic	L
ghanabhoomadhya	centre of earth	A
chandroccha	apogee of moon	A
chara	Ascensional difference, it is defined by the arc of the celestial equator lying between the six 'o clock circle and hour circle of a heavenly body at rising	B
chayabhramana	the path of the last point of the shadow	B
Jeevabhukti	true daily motion derived mathematically	B
tamovishkambha	diameter of shadow	A
taragraha	real planets	A
akshavala	deflection due to latitude	A
dyudalasanku	Rsine altitude at midday	L
drikshepa	ecliptic zenith distance	A
drukshepa	shortest arcual distance of the planet's orbit from the zenith. It is also used for the Rsine of distance	A
desantara	the longitude of the place. It is also the distance of local place from the prime meridian or the difference between the local and standard times	B
desantra karma	correction of the longitude for the place	B
drkshepamandala	vertical circle through central ecliptic point	A

drkchaya	parallax	A
drggati	arc of ecliptic between the Sun or the moon and central ecliptic point	A
drggola	visible celestial sphere	A
drngmandala	visible vertical circle	A
dhyujyamandala	diurnal circle	L
natajya	Rsine of the zenith distance	A
nakshatra divas	sideral day	A
neecha	perigee	A
palabha	equinoctial shadow	L
purvaparasutra	east-west line	L
paramapakrama	greatest declination, obliquity of ecliptic	A
paramasanku	Rsine of greatest altitude/ meridian altitude	A
parilekha	graphical representation	L
pratiloma	retrogression	A
praglagna	rising point of ecliptic	A
pragjyakashta	right ascension	A
bhabhrama	locus of shadow	V
bhakakshya	orbit of asterisms	V
bhavrutta	shadow circle	V
bhoogola	sphere of earth	A
bhoodivas	terrestrial day	A
bhramavrutta	diurnal circle or day circle	V
madhyagraha	mean planet	A
madhyajya	Rsine of the zenith distance of the meridian ecliptic point	A
mandakendra	longitude of the planet minus longitude of the planets apogee	B
madhyamakranti	declination of place of planet on ecliptic	L

mandagatiphala	correction to mean motion of planet	L
nati	meridian zenith distance. Celestial latitude as corrected for the parallax in latitude	B
natakala	hour angle	L
natakalakotijya	Rcosine of the hour angle	L
natakalajya	Rsine of the hour angle	L
nadivalaya	celestial equator	L
neechochharekha	line of apses	L
valana	deflection related to an eclipsed body. It is the angle subtended at the body by the arc joining the north pole of the ecliptic. The akshavalana is the angle subtended at the body by the arc joining the north point of the celestial horizon and the north pole of the equator. Ayanavalana angle subtended at the body by the arc joining north pole equator and ecliptic	B
vikshepa	celestial latitude	A
vishuvat	equator	A
samamandala	prime vertical	L
sankvarga	distance of planet's projection on the plane of horizon from rising setting line	A
suddhi	mean heliocentric position	L
sphutakranti	declination of planet's centre	L
sphutavikshepa	celestial latitude corrected by parallax	L
sphutamadhya	true mean planet	A
sphuta	true planet	A

A: Aryabhateeya of Aryabhatta I, B: Mahabhaskareeya of Bhaskaracharya I, L: Sishyadhi vruddhi Tantra of Lallacharya, V: Vateswara siddhanta of Vateswara.

These parameters were determined with the sound support of mathematics and experimental proofs. Exciting inference on the observational skills can be obtained when searched through the astronomical instruments used by Indians. More than a hundred instruments used by them are discussed in Sanskrit books. Using these instruments various parameters were calculated as given below.

Day radius: Vateswara gives the following calculation in Vateswara siddhanta (3(4) - 1) for getting the day radius:

क्रान्तिज्यावर्गोनात् त्रिज्यावर्गात्पदं द्युजीवा स्यात्।
त्रिज्याक्रान्तिज्यान्तरसमासघातस्य मूलं वा।।

Kraanthijyaa vargonaath thrijyaavargaath padam dyujeevaa syaath
thrijyaakraanthijyaanthara samaasaghaathasya moolam vaa

Day radius is equal to the square root of the difference obtained by subtracting the squares of R sine of the declination from the square of the radius or the square root of the product of the difference and the sum of the radius and the Rsine of the declination. This explanation can be mathematically summarised:

If θ be the declination then day radius = $R \cos \theta$

Where $R = 3437' 44''$ and $R \cos \theta = \sqrt{(R^2 - (R \sin \theta)^2}$

Here radius is taken in radians of the angle in minutes which is the same as the modern value obtained from $360 \times 60 \div 3.14$.

Day diameter and Colatitude : By the diurnal rotation of the earth on its axis, the Sun apparently moves round the earth every day in a circular pathway. This circle is called the diurnal circle or day circle and its diameter is the day diameter.

विषुवज्ज्याff्यामर्धवर्गविश्लेषमूलमवलम्बकः ।
क्रान्तित्रिज्याकृत्योरन्तरपदं द्विगुणं दिनव्यासः ।।

Vishuvajyaa ayaa mardha varga vislesha moolamavalambaka:
kranthithrijyaakruthyo rantharapadam dvigunam dinavyaasa

Square the sine of latitude and deduct from the square of the radius. Its square root is the sine of the co-latitude (its arc being the co-latitude). Square the sine of the declination deduct from the square of the radius and find its root. Twice the result is the day diameter. (Panchasiddhantika 4-23)

Relations between day radius and earthsine:

क्रान्तित्रिभान्तरज्या द्युज्या वा चरदल जीवया हृता
त्रिज्याक्षितिजीवघ्ना स्वाहोरात्रार्धजीवा वा ।।

Kraanthi thribhaantharajyaa dyujyaa vaa charadalajeevayaa
hruthaa thrijyaa kshithi jeevaghnaa svaahoraathraardhajeevaa vaa

Rsine of the difference between the three signs and the declination is also equal to the day radius. Day radius multiplied by earthsine and divided by the R sine of the Ascensional difference gives the day radius. (Vateswara siddhanta 3(4)-3).

Day radius = Rsin (3 signs - θ = R x earthsine/R (Ascensional difference), where 1 sign is 30 degree and θ is the angle of declination.

Division of zodiac: Method of dividing the zodiac into 360 degrees was well known to Indian scholars. Even the measurement of degree minute and seconds for angular dimensions have been taken from Indian system.

विकलानाम् कला षष्ट्या तत्षष्ट्या भाग उच्यते
तत्त्रिंशता भवेद्राशिः भगणो द्वादशैव ते ।।

Vikalaanaam kalaa shashtyaa thathshashtya bhaaga ucchyathe
thathrimsathaa bhavedraasi: bhagano dvaadasaiva the

Sixty vikala is one kala, and 60 of that is one bhaga, 30 of that is one rasi and 12 of that is the Bhagana, which is zodiac (Suryasiddhanta 1.28).

The angular equvalent: Vikala is second, kala is minute, bhaga is degree, rasi is sign and bhagana is zodiac in modern astronomical terms.

Sun's prime vertical : Vateswara's definition for prime vertical is the same as the modern definition:

ऊर्ध्वमधोऽपरपूर्वमिहाद्यं प्राहुरिदं सममण्डलमन्यत्।
तद्वदिहोत्तरदक्षिणदिक्स्थं वृत्तयुगं विदिशोरपि तद्वत्।।

Urdhvamadho'parapoorvamihaadyam praahuridam samamandalamanyath thadvathihottharadakshina dikstham vrutthayugam vidisorapi thadvath.

Vertical circle passing through the west and east cardinal points is the first circle called samamandala or the prime vertical (Vateswara siddhanta Gola. 3-1,2).

In Mahabhaskareeya (3-37, 38) the method for determining the Sun's prime vertical altitude and zenith distance are given as follows:

स्फुटरविभुजनिघ्नम् यां परां क्रान्तिजीवां हरतु समवलम्बज्याकलापेन भूयः ।
सफुटदिवसकराग्रा सा यदाक्षांशहीना रविरपि यदि गोले चोत्तरे लम्बकघ्नाम् ।।
अक्षज्यया हरेद् भूयः शङ्कुः स्यात् सममण्डले।
तद्वर्गव्यासकृत्योर्यद् विश्लेषं तत्पदं प्रभा ।।

Sphutaravi bhujanighnam yaam paraam kraanthi jeevaam harathu samavalambajyaa kalaapena bhooya: sphutadivasakaraagraa saa yadaakshamsaheenaa ravirapi yadi gole chotthare lambakaghnaamakshajyayaa hared bhooya: sanku: syaath samamandale thadvargavyaasakruthyoryad visleesham thathpadam prabhaa.

Multiply R sine of the Sun's greatest declination by the R sine of the Sun's true longitude, then divide by the R sine of colatitude. Result is the agra of true Sun. When that is less than the latitude and when the Sun is in the northern hemisphere, multiply the Sun's agra by R sine of the colatitude and then divide the product by the R sine of latitude. The result is R sine of Sun's prime vertical zenith distance.

The Sun's agra is the amplitude of the Sun at rising and setting lines. R sine of the Sun's agra is equal to the distance between the east-west and rising lines. Mahabhaskareeya was written in 528 AD during when many modern astronomical terms were used for calculating a mathematical solution of complicated porblems. Vateswara gives the following method for determining the prime vertical:

रविभुजजिनमौर्व्योर्वधात्पलज्याहृतात् समः शङ्कुः
उत्तरगोले स्वाक्षादत्पक्रान्त्यामयं (ग्राह्यः)

Ravibhujajina mourvyorvadhaalpalajyaahruthaath sama: sanku:
uttaragole svaakshaadalpakraanthyaamayam (graahya:)

From the product of Rsine of the Sun's bhuja and the R sine 24° divided by the Rsine of the local latitude, then is obtained the Rsine of Sun's prime vertical altitude. This exists when the Sun is in the northern hemisphere and the Sun's declination is less than the local latitude. (Vateswara siddhanta 3(11)-8)

I.e. if θ, is the Sun's tropitcal longitude, Ap, the Sun's prime vertical altitude and $, latitude of the local place, then

Rsin Ap = R sin θ x Rsin 24°/ R sin $. R sine of the declination multiplied by the radius and divided by the Rsine of the local latitude gives the Rsine of the prime vertical altitude.

Earthsine: Definition of earthsine has been given before. The method of determining earthsine according to Vateswara siddhanta (3(5)-I)is:

क्रान्तिज्या/क्षज्याघ्ना लम्बकजीवा विभांजिता कुज्या ।
विषुवत्छायागुणिता क्रान्तिज्या/कोंद्धृता वा स्यात् ।।

Kraanthijyaa f akshajyaaghnaa lambakajeevaa vibhaanjithaa kujyaa
vishuvathcchaayaagunithaa kraanthijyaa f arkoddrutha vaa syaath

R sine of declination multiplied by R sine of the latitude and divided by R sine of colatitude gives the earthsine. R sine of declination multiplied by equinoctial midday shadows and divided by 12, too yields the same.

Earthsine = R sin θ x R sin $/R cos $; where θ declination and $ is the local latitude.

Bhaskaracharya I also has correleated these parameters.

इष्टक्रान्तिक्षितिजावर्गसमासस्य मूलमर्काग्रा क्षितिजा व्यासार्धहृता पलस्य गुण:

Ishtakraanthikshithijaavarga samaasasya
moolamarkaagraa kshithijaa vyaasaardhahruthaa palasya guna:

Square root of the sum of the square of R sine of the Sun's declination and earthsine for the desired time is the R sine of the Sun's agra for that time.

I.e.R sine of the Sun's agra is equal to the square root of the product of squares of R sin θ and Earthsine where θ is the Sun's declination. Similarly the product of radius and earthsine divided by the product of Radius and the Sun's agra gives R sine $ where $ is the latitude of the place. (Mahabhaskareeya 3-53)

Bhaskaracharya has given a complex method for the calculation of various astronomical parameters in different ways. In following lines he said thus:

इष्टज्यां मुनिरन्ध्रपुष्करशशिक्षुण्णां सदा संहरेद्
व्यासार्धेन भवेदपक्रमगुणस्तात्कालिकस्तत्कृतिम्
विष्कम्भार्धकृतेर्विशोध्य च पदं द्युव्यासखण्डं विदु:
स्वेष्टक्रान्तिहतं पलं प्रविभजेल्लम्बेन जीवा क्षिते:

व्यासखण्डगुणितं क्षितेर्गुणं संहरेद् दिवसजीवया पुनः ।
काष्ठितं च यदवाप्तमत्र तु प्रोच्यते चरदलं सतां वरैः ।।

Ishtajyaam munirandrapushkarasasikshunnaam sadaa samhared vyaasaardhena bhavedapakramagunasthaalkaalika sthathkruthim vishkambhaardha krithervisodhya cha padam dyuvyaasakhandam vidu sveshtakraanthi hatham palam pravibhajellambene jeevaa kshithe: vyaasakhandagunitham kshipergunam samhared divasajeevayaa puna: kaashtitham cha yadavaapthamathra thu prochyathe charadalam sathaam varai:

Mutiply R sine of the given longitude by 1397 and divide by radius; the result is the R sine of the declination of that time. Subtract that from the squares of radius and then take the square root of that difference. Result is called Day radius. Multiply the R sine of latitude by R sine of the given declination and divide by the R sine of colatitude; the result is the earthsine. Multiply earthsine by radius and then divide the product by the day radius, then reduce the resulting R sine to arc. Whatever is thus obtained is termed the ascensional difference by the best among the good astronomers. This can be mathematically represented thus: (Mahabhaskareeya 3-6,7)

R sine θ	=	1397 x R sin $/R
Day radius	=	Square root of $(R^2 - R\sin\theta)^2$
Earthsine	=	Rsine # x R sin θ/Rsin (90° - #)
Rsine of ascensional difference	=	Earthsine x radius/day radius

Where R is the radius, # is the latitude of the place, $ and θ are sayana longitude and declination respectively.

Rsine of moon's meridian and zenith distance: Vateswara has defined the meridian as the circle passing the north and south cardinal points. (Vateswara siddhanta Gola 3-1,2)

Bhaskaracharya gives the explanation to determine the

meridian zenith distance by a process in which the moon is on the meridian which literally means that the moon's longitude is equal to that of the meridian ecliptic point:

मध्यलग्नसमश्चन्द्रो जायतेऽनेन कर्मणा ।
तत्क्षिप्त्यपक्रमाक्षैस्तु मध्यच्छाया प्रसाध्यते ।।

Madhyalagna samaschandro jaayatheff anena karmanaa
thathkshipthyapakramaakshaisthu madhyacchayaa prasadhyathe

By this process the meridian zenith distance of the moon is obtained when she is on the meridian. From the moon's celestial latitude, declination and local latitude, the R sine of her meridian zenith distance is determined. (Then he described the mathematical equation)

Moon's meridian zenith distance = local latitude +/- true declination.

Here + or - sign is taken depending upon the position of the moon in south or north of equator. (Mahabhaskareeya 6-41)

First, one has to calculate the true declination of the moon and then apply the above formula for determination of R sine of the moon's meridian zenith distance.

Oblique ascension:

विंशतिरष्टि: सार्धा: 'पादोना: सप्त' चाजपूर्वाणाम्
विषुवच्छायागुणिता: क्रमोत्क्रमाच्च रविनाड्योर्धे ।।

Vimsathirashti: saardhaa: paadonaa: saptha chaajapoo rvaanaam
vishuvacchayaagunithaa:kramothkramaachha ravinaaddyo fardhe

Multiply the constants 20, 16.5 and 6.75 with equinoctial shadow. The results are the oblique ascensional differences in vinadis, the first in the given order, then in the reverse order of the first 6 months and again the given and the reverse orders for the second half of the ecliptic and the second 6 months (Panchasiddhantika 3-10).

For the months starting from Mesha the order of multiplying the equinoctial shadow is 20, 16.5, 6.75 (then reverse) 6.75, 16.5, 20 upto the month of Mina.

Winter summer solistics: The movement of the Sun, viewed from the earth, to the north and the south of the equator has been examined thoroughly by ancient Indians. Commencement of the northward/southward movement has also been identified specifically. The information mentioned here, belong to the period of Varahamihira i.e 505 AD.

उदगयनं मकरादावृतव: शिशिरादयश्च सूर्यवशात् ।
द्विभवनकालसमानं दक्षिणमयनं च कर्कटकात् ।।

Udagayanam makaraadaavruthava: sisiraadayascha sooryavasaath
dvibhavanakaalasamaanam dakshina mayanam cha karkatakaath

The Sun's turning northward is when it reaches the 0 point of Makara (Capricon) at winter solistic and its turning southward is at 0 point of Karkitaka (Cancer). (Panchasiddhanitka 3-25)

Celestial sphere, longitude and latitude: Information on the celestial sphere and its longitude and latitude, was not confined to merely theoretical level but went deep into the application level.

हरिजमिति गगनमवनौ प्रसक्तमिव यत् प्रदृश्यतेऽन्तेषु ।
सममितिपूर्वापरतो ह्येवमेव दक्षिणोत्तरत: ।।
ध्रुवहरिज विवरमक्षोऽक्षनवति विवरं च लम्बकोऽभिहित: ।
(लम्बो) नमति ख (मध्याद्) द्युव्यासोऽस्तोदयाख्यस्य ।।

Harijamithi gaganamavanow prasakthamiva yathpradrsyathe antheshu
samamithipoorvaaparatho hyevameva dakshinoththaratha:
dhruvaharija vivaramaksho akshanavathi vivaram cha lambakoabhihitha:
(lambo) navathi kha (madhyaad) dyuvyaaso asthodayaakhyasya

Where the sky appears to meet the earth at their skirt it is called the horizon. The vertical circle which runs from the east

to west is called the prime vertical. Similarly, the circle which runs north to south is called the meridian. Actual distance between the north pole and the horiozon is called latitude. The difference between 90° and the latitude is called the colatitude. The day daimeter is the diameter of the so called diurnal circle (Panchasiddhantika 14-17,18)

Ascensional difference: Vateswara has given the following method for the determination of Ascensional difference.

कुज्या त्रिज्यागुणिता द्युज्याभक्ता चरार्धजीवा स्यात् ।
अन्त्याहता कुजीवा धृतिभक्ता वा चरार्धज्या ।।

Kujyaa thrijyaagunitha dyujyaabhakthaa charaardhajeevaa syaath
anthyaahathaa kujeevaa dhruthibhakthaa vaa charaardhajyaa

Earthsine multiplied by the radius and divided by the day radius is the R sine of the ascensional difference. Earthsine multiplied by antya and divided by dhruti is also Rsine of ascensional difference (Vateswara siddhanta 3(7)-1).

R sine of ascensional difference = Earthsine x R/day radius.

The Sun's druti is the distance of the Sun from its rising and setting line.

Rising of the planets: Determining the position of rising of planets in the horizon needs a lot of observational skill. It needs the knowledge on planets, their revolution pathway, angular velocity, division of zodiac, declination, etc. Without the use of a telescope or other instruments, the skill was shown by the Indian scholars to calculate the position (in angular values of zodiac), of rising of the planets. For understanding the content of modern concepts in this achievement, one should know about the first point of Ariesa. The point of Aries is where the equatorial plane and the ecliptic meet. This is fixed as the point of measuring the position of planets in degree. In Sanskrit this point is called

the Mesha sankranti. It is the first point of Zodiacal constellation, which is taken as 0 degree on the path of revolutions of planets. Positions of planets are referred from this point, as followed in modern astronomy.

The rising and setting of the planets have definitions in modern astronomy as follows: In the course of the diurnal motion of the position of a celestial body in the horizon, where it enters the visible hemisphere from the invisible one is called its rising. This has been explained in Aryabhateeya with details.

बुध-भृगु-कुज-गुरू-शनि न-व रा-ष-ह-गत्वांशकान् प्रथम पाना:

Budha-bhrugu-kuja-guru-sani-va-raa-shaa-ha-gathvaamsakaan prathama paathaa:

The ascending nodes of the Mercury, Venus, Mars, Jupiter and Saturn are 20°, 60°, 40°, 80°, from the first point of Sign Aries (Aryabhateeyam 1-9a).

This means that Mercury rises at 20° from the Aries point (Mesha Sankranti point) Venus 60° and so on for other planets. These values are remarkably correct and in agreement with the modern values. Other information connected with the rising and the setting of planets has been described by ancient Indian scholar. Lallacharya:

वसुयमै: शरपूर्णविलोचनैर्मनुभिरग्निभुजङ्गनिशाकरै: ।
खनयनैरुदयो दिशि वज्रिणो भवति यश्चलकेन्द्रभवैर्लवै:।।
गगनषट्कहुताशपरिच्युतै: ककुभि पाशभृतोऽस्तमयो भवेत् ।
शशिशरै स्त्रियमैर्बुधशुक्रयोर्जलपतेरुदयो दिशि जायते ।।
निगदितै: पतितैश्च भमण्डलाद्दिशि सहस्रदृशोऽस्तमयो भवेत् ।
यमगुणैर्दिवसै: शशिभूधरैर्भवति चाभ्युदयोऽस्तमयोस्तयो: ।
व्योमार्का: क्षितिपा नभोहुतभुजो नागास्तथा षड्गुणा: ।

Vasuyamai: sarapoornavilochanairmanubhi ragnibhujanga nisaakarai: bhanayanairudayo disi vajrino bhavathi yaschala kendrabhavairlavai: gaganashatkahuthaasaparichyuthai:kakubhi paasabhrutho asthamayo bhaveth sasisarai sthriyamaibhudha sukrayorjala patherudayo disi jaayathe nigadithai: pathithaischa bhamandalaaddisi sahasradriso asthamayo bhaveth yamagunairdivasai: sasi bhoo dharairbhavathi chaabhyu dayo fsthamayosthayo: vyomaarkaa: kshithipaa nabhohutha bhujo naagaasthathaa shadgunaa:

When the sighrakendras of Mars, Mercury, Jupiter, Venus and Saturn are respectively 28°, 205°,14°, 183°, and 20° they rise in the east. When their sighrakendras are 360° minus these values respectively they set in the west. When their sighrakendras are respectively 51° and 23°, Mercury and Venus (when direct motion) rise in the west. When they are 360°, minus these values respectively they set in the east. Mercury and Venus once set, rise again 32 and 71 days respectively after setting. For 120, 16, 30, 8 and 36 days respectively Mars, etc., (in that order) remain invisible in the west. They set 660, 37, 372, 251 and 342 days respectively after rising (Sishyadhi vrudhi Tantra 3-22-25)

Complex scientific information is elucidated by continuous observations and calculations are described in the above stanza. They are examples to show the careful observations undertaken for calculating the rising and setting of the planets. In this example also inferior planets, Mercury and Venus are separetely dealt.

ऊन: प्रागुदयं प्रयाति सवितु: प्रागेव चास्तं ग्रह: ।
पश्चादभ्यधिकोऽर्क सन्निधिवशान्नित्यं प्रवाहेण च ।।

Una: praagudayam prayaathi savithu: praageva chaastham graha: paschaadabhyadhikoarka sannidhivasaannithyam pravaahena cha

The planets which have a motion slower than the Sun (earth) always rise helically in the east and set in the west. The planets which have faster motion rise in the west and set in the

east. So, Mars, Jupiter and Saturn which have slower motion than the Sun (earth) always rise in the east and set in the west. Mercury and Venus when retrograde do the same. But when they have direct motion-which is faster than that of Sun (earth) - they rise helically in the west and set in the east and so does the moon. (Sishyadhi vruddhi Tantra 8-1)

Here, the speed of Sun is directly attributed to the motion of earth by Lallacharya. Unlike Aryabhatta, the author of Sishyadhi vruddhi Tantra, Lallacharya believed that the Sun is revolving around the earth. Data given for the Sun can be extrapolated to earth, which are comparable with modern astronomical values.

Position of planets: According to modern astronomy position of planets at any time and also the rising/setting are calculated based on the celestial longitude of that planet. The celestial longitude of planets is the distance of the foot of the secondary through the body to the ecliptic measures from the first point of Aries along the direction of the Sun's annual motion, it lies between 0° - 360°. The same way the ancient Indians calculated the position of the planets. The position of planets given for March 21, 499 by Aryabhatta and corresponding (back calculated) modern astronomical values are given for comparison. Given here is the mean position of the celestial bodies on Sunday, March 21, 499 at mean noon in the prime meridian at Ujjaini.

Celestial body	Aryabhateeya	Modern calculations
Sun	360° 00′ 00″	359° 42′ 05″
Moon	289° 48′ 00″	280° 24′ 52″
Moon's apogee	035° 42′ 00″	035° 24′ 38″
Moon's ascending node	352° 12′ 00″	352° 02′ 26″
Mars	007° 12′ 00″	006° 52′ 45″

Mercury	186° 00′ 00″	183° 09′ 51″
Jupiter	187° 12′ 00″	187° 10′ 47″
Venus	356° 24′ 00″	356° 07′ 51″
Saturn	049° 12′ 00″	048° 21′ 13″

Generally, positions of planets are calculated after giving due correction to the year upto which previous corrections were given. The corrected year is termed as the epoch era. Modern astronomy also gives corrections to time in seconds or minutes. 522 AD was the epoch era taken and it was followed by astronomers for two centuries. Based on this, corrections were given for the planetary positions. Manjulacharya has given the position of planets for epoch era as Saka year 854. Here the given position was also accurate, even upto the angular value in minutes.

कृतशरवसुमितशाके चैत्रादौ सौरिवारमध्याह्ने
राश्यादिराजनृपार्का रविरिन्दुर्भवधृति द्वियमाः
सूर्यान्मन्दोच्चंशा वसुतुरगाः पर्वतास्तु सत्र्यंशाः
स्वररवयः खाकृतयो द्विनागभुवोऽशितिरद्रिजिनाः

Kruthasaravasumitha saake chaithradow sourivaaramadhyaahne
raasyaadi raajanrupaarkaa ravirindurbhava dhruthi dviyamaa:
sooryanmandochhamsaa: vasathuragaa: parvathaasthu sathryamsaa
svararavaya: khaakruthayo dvinaagabhuvo fasithiradrijinaa:

On Saturday noon, the beginning of Chaitra, Saka year 854, position of the Sun 11s 16° 12′ (346° 12′), moon 11s 18° 22′ (348° 22′), apogee of the Sun 2s 18° (78°), that of Moon 0s 7° 20′ (7° 20′) that of Mars 4s 7° (127°), Mercury 220°, Jupiter 5s 22°(172°), Venus 2s 20° (80°), Saturn 8s 7° (247°). (Laghumanasa 1-12).

These values agree very much with the modern values on the position of planets. Puthumana Somayaji has given the position of the planets for the Kali day 1772786 at sunrise.

गानात् सुमेरौ न सुमन्त्रनीति: Sun 6s 2° 57′ 2″ 57‴ 3⁗
gaanaath sumerow na sumanthraneethi:

लोकेकृशांग्या निजकामपुञ्ज Moon 8s 15° 18′ 1″ 51‴ 13⁗
lokekrusaangyaa nijakaamapunja

अनंगमानं कृतनामरेण Mars 5s 25° 6′ 10″ 53‴ 0⁗
anangamaanam kruthanaamarena

निर्भासभानो: कथनेक्षिपाल: Mercury 3s 16° 7′ 10″ 47‴ 40⁗
nirbhaasabhano: kathanekshipala:

लीनाविभावे भवतद्धनारी Jupiter 2s 9° 34′ 44″ 44‴ 3⁗
leenaavibhave bhavathaddhanaaree

तयाप्रियोसौ रसभिद्वनस्थ: Venus 7s 4° 47′ 27″ 12‴ 16⁗
thayaapriyosow rasabhidvanastha:

हेमाद्रिनेता गिरिशोनिनादै: Saturn 8s 0° 52′ 36″ 2‴ 58⁗
hemaadrinethaa girisoninaadai:

Signs, degrees, minutes, seconds, one sixtieth of a second and one sixtieth of that value, are given here. (Karanapaddhathi 1-23 with the commentary)

Mathematical values for angular positions given above are according to Ka ta pa yadi number system. It is interesting to note that positions are given in angular measurements, beyond minutes and seconds, which in Sanskrit are called liptam and praliptam (respectively 1/60 of a second and 1/60 of that value). Celestial ascending nodes of planets are given by Manjulacharya.

संक्रान्तितिथिध्रुवका: शुक्रवसुनवरसेषवो राहो: ।
कृतयमवसुरसदशका दशाहता: शेषपातांशा: ।।

Sankraanthi thithi dhruvakaa: sukravasunavaraseshavo raaho:
krutha yamavasu rasadasakaa dasaahathaa: seshapaathamsaa:

At the end of the completion of the Sun's revolution in Saka year 854, the ascending nodes were, for Moon 8s 9° 56′, Mars 40°, Mercurcy 20°, Jupiter 80°, Venus 60°, and Saturn 100° (Laghumanasa 1-4).

Precision of equinoxes : As mentioned earlier the above angular values for the position of planets are given based on the fixed point of Mesha sankranti. The two imaginary equinoctial points are first degrees of Aries and Libra. During the revolution of the total solar system, it has been observed that these points also move slowly. I.e the precision of the equinoxes change, resulting in the change of the Mesha sankranti day/time. It can be seen that Manjulacharya has given the rate of change of precision of equinoxes about 1200 years ago.

अयनाचलनाष्षडंशा: पंचाशल्लिप्तिकास्तथैकैका: ।
प्रत्यब्दम् तत्सहितो रविरुत्तरविषुवदादि: ।।

Ayanaachalanaashaddamsaa panchaasalliptikaa sthathaikaikaa:
prathyabhdam thathsahitho raviruttharavishuvadaadi:

Precision of equinoxes is (Ayana chalana) 6° 50′ and the rate of change of ayanachalana is 1′ (minute of angle) per year. The longitude of the Sun, increased by the ayanachalana is to be reckoned from the vernal equinox (Laghumanasa 1-5).

This value has great scientific significance. Given here is the total change of precision of equinoxes from 522 AD to the period of Manjulacharya. This was calculated by the author himself as 6° 50′, at the rate of 1′ per year. Important application of this knowledge can be seen in the following observations. In 499 AD, Aryabhatta I has mentioned that the Mesha sankaranti was on March 21. Now Meshasankrati is celebrated on April 14th. This change is due to the ayanachalana i.e change in the precision of equinoxes for the last 1500 years, at the rate of 1′ per year, resulting in the change of the position is 1500÷60, nearly

25° . If earth takes 1 day for revolving about 1° (exactly 59′ 8″) around the Sun, the number of days the earth had to move more to reach the Aries point in 1999, compared to 499 AD is nearly 25 days. Thus Mesha sanskranti is changed from March 21st to April 14. The angular value calculated by Indian scholars for precision equinoxes was 1′, whereas the modern value is 50″ 26‴. Even though other Indian astronomers have not mentioned specifically about this phenomenon it appears that they have included this correction factor in all their astronomical calculations. In another book Brhmamanasa, the same author Manjulacharya has given the number of revolution of vernal equinoctial point as 199669 for a period of 4320000000 years!. This number is also very near to the correct modern value.

Explanation for ecliptic and equinoxes: The path of apparent annual motion of the Sun is called ecliptic. Plane of the ecliptic inclined to an angle of 23° 27′ to the plane of equator, intersects, at first points of Aries and Libra. Between the two equinoctial points the first half plane is upward and the second downwards. This is the modern explanation for the Aries and Libra points. Same definition had also been followed one and a half millennia ago by Aryabhatta I and others.

मेषादेः कन्यान्तं सममुदगपमण्डलार्धमपयातम् ।
तौल्यादेर्मीनान्तं शेषार्धम् दक्षिणेनैव ।

Meshaade: kanyaantham samamudaagapamandalaardhamapayaatham
thoulyaadermeenaantham seshaardham dakshinenaiva

One half of the ecliptic running from the beginning of the sign Aries to the end of the Virgo, lies obliquely inclined to the equator northwards. Remaining half of ecliptic running from the beginning of sign Libra to the end of sign Pisces lies equally inclined to the equator to southwards (Aryabhateeyam 4-1).

This inclination is defined as apakrama (meaning: not in

order but inclined). In Sankaranarayana's commentary for Laghubhaskareeya, he had clarified this declination for moon while interpreting Aryabhatta's above stanza, as follows:

अपक्रममण्डलाद्विक्षेपमण्डलस्यार्धपञ्चमांशप्रमाणविक्षेपत्वादित्यर्थ:

Apakramamandalaadvikshepamandalasyaa rdhapanchamaamsa pramaana vikshepathvaadithyartha:

From apakrama mandala to the path of revolution (vikshepa mandala), it is 4° 30′ for the Moon, that is the meaning..... (Sankaranarayana's commentary for Laghubhaskareeyam 5-4). The corresponding modern value for the declination of the Moon is 5° 8′ and it ranges between 4° 57′ and 5° 20′.

Visiblity of planets: Planets are revolving around the Sun in an elliptical path way. They may be at different distance at different times. At one instance, each can be at the farthest and that position is called apogee and about 180° of apogee, planets are at the nearest point which is called perigee. This phenomenon is due to the elliptical orbit of the revolution of planets and the Sun is at one corner of this ellipse. This farthest and nearest positions were well explained by Indian scholars. Apogee was known as the neechocha and perigee as sighrocha.

While viewing the planets, it is obvious that when they are near, they appear bigger and when they are farther they appear smaller. In fact in modern astronomy the angular radius of the planet θ is said to be inversely proportional to the distance of the body.

I.e θ = radius of the body/distance. This has been clearly mentioned by Lallacharya:

अत एव द्युचराणामणुना भवति क्रमान्महत्वं वा ।
सवितुश्चासन्नानां दूरगतानां क्रमेणैव ।।

Athaeva dyucharaanaamanunaa bhavathi kramaanmahathvam vaa savithuschaasannanaam dooragathaanaam kramenaiva

The planets appear small or large accordingly. In the same manner, they appear large or small as they are near the Sun or far from it (Sishyadhi vruddhi Tantra 14-11)

This is a well known fact. Based on this, mathematical equations have been formulated for calculating angular diameter of planets the Sun and the Moon.

Visibility of planets. Minimum angular distance from the Sun: Due to the brightness of the Sun, planets cannot been when they approach the same line of the Sun and the observer, when viewed from the earth. Depending upon the brightness of the Sun and the planets, minimum angular distance is to be maintained, in time equivalents, to be waited for getting vision of the planets. This information was documented here.

चन्द्रोंऽशैर्द्वादशभिरविक्षिप्तोऽर्कान्तरस्थितो दृश्यः
नवभिर्भृगुर्भृगोस्तैर्द्वयाधिकैर्द्वयधिकैर्यथाश्लक्ष्णाः

Chandroamsairdvaadasaabhiravi kshiptho f arkaantharasthitho drusya: navabhirbhrugurbhrugosthairdvaryadhi - kairdvaryadhikairyathaa slakshnaa:

When the moon has no latitude it is visible when situated at a distance of 12 degrees of time from the Sun. Venus is visible when 9° of time distance from the Sun. The other planets taken in the order of decreasing size (as per Aryabhatta). Jupiter, Mercury, Saturn, and Mars are visible when they are 9°, increased by 2° (i.e 11°,13°,15° and 17°) distance from the Sun (Aryabhateeyam 4-4)

One degree is equivalent to 4 minutes in time duration. Hence when 12° is taken the time equivalent is 48 minutes. If the arc of the ecliptic joining the Sun and the Moon takes at least 48

minutes in rising above horizon, moon will be visible. But if moon has some latitude this minimum degree varies. This fact had been supported by Bhaskaracharya I. The same modern astronomical facts has also been supported by Lallacharya. Even if the required visibility correction is given, the planets are visible only by maintaining the above angular distance: Says Lallacharya:

कालांशै रविभि: शशि महिसुतोऽत्यष्ट्याथ नन्दै: सित:
शाशाङ्किगुणभूमिभिरक्षाशराशिभि: सौरि: शिवैरङ्गिरा: ।
वक्रस्थौ बुधभार्गवौ रवि गजै: सूर्यान्तरैदृश्यतां यान्ति
प्रत्यगिनो भषट्कसहित: कार्यस्तथा दृग्ग्रह: ।।

Kaalamsai ravibhi sasi mahisutho fathyashtyaatha nandai:
sitha: sasaankaguna bhoomibhi rakshaa saraasibhi: souri :
sivairangiraa: vakrasthow budhabhargavow ravi gajai:
sooryaantharairdrusyathaam yaanthi prathyagino
bhashatka sahitha: kaaryasthathaa druggraha:

Corrected by the two visibility corrections, the Moon, Mars, Jupiter and Saturn are helically visible when separated from the Sun by (a degree or time equivalent to that) respectively 12°, 17°, 11° and 15°. Mercury and Venus are helically visible when in direct motion separated from the Sun by the kalamsa 13° and 9°, when retrograde by kalamsa 12° and 8° respectively (Sishyadhi vruddhi Tantra 8-5)

Thus the result of a series of observation is given by different scholars for getting the vision of the planets from earth.

Visibility correction of planets: Visibility of the planets are affected by various factors and many of them have been known to ancient Indians. The depth down to which they had gone for adopting these mathematical corrections can be made clear from the quotations taken from Lallacharya, Bhaskaracharya I, Brahmagupta, Varahamihira and many others .

राशित्रयान्वितखगोत्क्रमजातमौर्वी क्षेपेण षष्टिलववर्जितभूरसैश्च ।
हत्वा हरेत् खखशराक्षियमैः फलं स्याल्लिप्तादि दृष्टिभवकर्मणि खेचरस्य ।।

Raasithrayaanvitha khagothkramajaatha mourvee kshepena shashti lavavarjitha bhoorasaischa hathvaa hareth khakhasaraakshiyamai: phalam syallipthaadi drushti bhavakarmani khecharasya

Multiply the Reversed sine of the true longitude of a planet increased by 90° by its latitude and by 60 and 50/60, and divide the product by 22500. The result in minutes is the dark phase or visibility correction of the planet due to the obliquity of ecliptic (Sishyadhi vruddhi Tantra 13-14)

Lallacharya has further mentioned the correction given in minutes of angle as "Or multiply the difference of the radius and the R sine of the true longitude of a planet by its latitude and divide by 370. The result is the visibility correction" (in minutes), (Sishyadhi vruddhi Tantra 13-15)

This is exactly the view on the subject. More refined values have been incorporated in modern astronomy so as to get higher level of accuracy.

Parallax on visibility: Parallax of a body is the angle subtended at the body by any two positions of observations. Therefore this is the angle between the two directions of observations. According to modern astronomy there are two types of parallax affecting the calculation of positions of the celestial bodies. They are geocentric parallax and horizontal parallax. In geocentric parallax the centre of the earth is taken as the standard observer. The centre of the earth observes the celestial body which is taken as the standard direction. Direction in which the observer on the surface of the earth sees is the apparent direction. The angle between these two is geocentric parallax. This is mathematically defined as follows:

Geocentric parallax = (Radius of the earth x 180 x 60 x 60 x Sin z)÷ ∏ x distance from earth. Geocentric parallax is directly related to the spherical shape of the body and the spherical shape of the earth.

Horizontal parallax is the parallax observed when the same body is observed from two different places having same meridian. This is also related to the geocentric parallax. If Pi and Pii are the geocentric parallaxes when the same body is observed in two places on the same meridian, then horizontal parallax is:

Pi=g x sin zi and Pii =g x sin zii; where g is the geocentric parallax and zi and zii, the zenith angle distance of the two places. In both cases the parallax is directly related to the radius of the earth. In the geocentric parallax when distance remains the same the parallax is the same, as radius of the earth multiplied by a factor. The Parallax affects the visibility of celestial bodies. It is because the observer is standing on the earth surface and also because of the parallax, the observer sees the visible and non visible part of the planet as big or small. According to modern science the horizontal parallax for Moon is 57′ and that of the Sun is 8.8″′. In both geocentric and horizontal parallaxes (as mentioned above) the radius of the earth and the distance from the body are the two important factors which influence. This information has been given by Aryabhatta I:

....कुवशात् । क्षितिजे स्वा दृक्छाया भूव्यासार्धं नभोमध्यात्

..... *Kuvasaath / kshithije svaa drukcchaayaa*
bhoorvyaasaardham nabhomadhyaath

Because of the spherical shape of the earth, the parallax equal to a maximum level of the earth's radius is possible for observing the celestial bodies, from the earth (Aryabhateeyam 4-34)

Drkchhaya is parallax. According to the commentator, parallax in longitude is a product of radius of earth and druggatijya and the result will be in yojanas. Lallacharya has connected the parallax with the hour angle and meridian ecliptic, and parallax was given in time unit.

तिथेर्नतस्य क्रमशिञ्जिनी हता खमध्यलग्नप्रभवेन शङ्कुना ।
क्षमाषडङ्काब्धिशराङ्कनेत्रहृद् बिलम्बने स्याद् घटिकादि वा फलं ।।

Thithernathasya kramasinjanee hathaa khamadhyalagnaprabhavena
sankunaa kshamaashadangkaabdhi saraankanethrahrud
vilambane syaad ghatikaadi vaa phalam

Rsine of the hour angle at the amavasya multiplied by R sine of the altitude of the meridian ecliptic point and divided by 2954961 gives the parallax in ghatikas at mid eclipse (Sishyadhi vruddhi Tantra 6-8)

The time equivalent of parallax is the time required for the disappearance of the body equivalent to that which is observed by the parallax. When the Sun and the Moon are in different hemispheres, the method followed for calculating the parallax is discussed in detail by Lallacharya.

हृताथवा दृष्टिगतिः खषड्गजैर्विलम्बनं तत् स्वमृणं क्रमाद् भवेद्
तिथेर्विरामे परपूर्वभागयोर्मुहुस्तदुत्थांशकलाः शशीनयोः

Hruthaathavaa drushtigathi: khashadgajaivilambanam
thath svamrunam kramaath bhaveth thitherviraame
parapoorvabhagayormuhu sthaduthhaamsakalaa: saseenayo:

Driggatijya divided by 860 gives the parallax in ghatikas. It should be added to the calculated time when the amavasya ends, if the Sun is in the western hemisphere. The longitude of the Sun and the Moon must be found for the correct time by adding or subtracting the minutes resulting from the motion according to as the parallax is additive or subtractive, (Sishyadhi vruddhi Tantra 13 -9).

Parallax in longitude: Separate chapters have been set apart in many ancient astronomy books for discussing the parallax in longitude and that for latitude. Lallacharya's method for calculating parallax in longitude is as follows:

त्रिराशिजीवा वलनज्यका हृता शिलीमुखैरङ्गुलतां व्रजन्ति ता:
द्विसङ्गुणा दृष्टिगति: शराचलैर्विभाजिता लम्बननाडिका फलम्

Thriraasijeevaa valanajyakaa hruthaa sileemukhai rankulathaam
vrajanthi thaa: dvisankunaa drushtigathi: saraachalai-
rvibhaajithaa lambana naadikaa phalam

Radius and the valanajya when divided by 5, are converted into angulas. The R sine of driggati multiplied by 2 and divided by 75 gives ghatika of the parallax in longitude. (Sishyadhi vruddhi Tantra 13-11)

Parallax in longitude is known as Nati in Sanskrit which is discussed in detail under the title, longitude in this text.

Parallax in latitude: Parallax in latitude has also been discussed in detail in ancient astronomy books. Thus says Lallacharya for parallax in latitude.

नतक्रमज्याम्बरशङ्कुनिघ्ना स्याल्लम्बनं तत्त्वरसेषुहृद्वा
दृक्क्षेपभुक्त्यन्तरयोश्च घात: खबाणयुग्माक्षिहृतो नति: स्यात्

Nathakramajyaambara sankunighnaa syaallambanam
thathvaraseshubhrudvaa drukshepabhukthyanthara yoscha ghaatha:
khabaanayugmaakshihrutho nathi: syaath

R sine of the hour angle multiplied by Rsine of altitude of the merdian ecliptic point and divided by 5625 gives parallax in longitude. The Difference of true motions of the Sun and the moon multiplied by the Rsine of drikshepa and divided by 2250 gives the parallax in latitude. (Sishyadhi vruddhi Tantra 13-12)

Thus for every planet for calcualating the vision point of

celestial phenomena like rising, setting, eclipse, conjunction, etc., the parallax on both latitude and longitude are to be added.

Apogees of the planets: While explaining positions of planets, their apogees have also been described by Indian astronomers. Aryabhatta has given apogees for planets as follows:

बुध-भृगु-कुज-गुरु-शनि न-व-रा-ष-ह गत्वांशकान् प्रथमपाता: ।
सवितुरमीषां च तथा द्वा-ऽखि-सा-ह्दा-ह्लय-खिच्य मन्दोच्चम् ।।

Budha-bhrugu-kuja-guru-sani-na-va-raa-sha-ka- gathvaamsakaan prathama paathaa: savithurameeshaamcha thatha dvaa-njakhi-saa-hdaa-hlaya-khichya mandochham

Apogees of the Sun, Mercury, Venus, Mars, Jupiter and Saturn are respectively 78°, 210°, 90°, 118°, 180°, 236° (Aryabhateeyam 1-9)

Aryabhateeya number system is followed to give the above numerical values. When the first point, Aries is taken, the above planets and the Sun come at the farthest distance in their respective apogees. Apogee of the Sun from the earth is given here as 78°, which means that after travelling 78° from the Mesha sankranti point, the distance between the Sun and the earth is maximum. Here, a comparison of the modern values of apogee to that given by Aryabhatta is cited:

Planet	Aryabhatta	Modern values
Earth (Sun)	78°	77° 15′
Mars	118°	128° 28′
Mercury	210°	234° 11′
Jupiter	180°	170° 22′
Venus	90°	290° 04′
Saturn	236°	243° 40′

Except for Venus the other values have a degree of accuracy.

Aryabhatta's values were recorded 1500 years ago! Regarding apogee, the same line has been mentioned in bhootha sankya system by Lallacharya.

वस्वीशा दशबाहवोऽम्बरधृतिः खाङ्का रसत्र्यश्विनो ।
..... क्षितिजज्ञजीवभृगुजच्छायासुतानां क्रमात् ।।

Vasveesaa dasabahavo fambaradruthi: khankaa rasathryasvino
..... kshithijajnajeeva bhrugujacchaayaa suthaanaam kramaath

Apogees of Mars, Mercury, Jupiter, Venus and Saturn are respectively, 118°, 210°, 180°, 90°, 236°, (Sishyadhi vruddhi Tantra 3-1)

An interesting scientific conclusion that can be arrived at from the above observation is that the first point of Mesha sankranti (also called vishu) falls on April 14th. It was well documented that, every day the earth revolves around the Sun at an angular velocity 59′ 8″ (*ekona shashti lipta: ashtow vilipta:* one less than 60 minutes and eight seconds of angular velocity, when viewed from earth, the Sun moves). When the earth revolves at this rate around the Sun for reaching the point of apogee at 78°, it takes about 79 days. The number of days from April 14 to July 3rd is 79 (including the former). July 3rd is the day on which the Sun and the Earth come farthest according to modern astronomy. This ancient discovery agrees absolutely with the modern observation. Going one more step ahead on this wonderful mathematical astronomy, one can also see that the exact time during when the distance between the earth and the Sun attains the farthest distance, can be calculated from the correct Mesha sankaranti time to the time required to reach the apogee 78°.

Perigee is the nearest point which is opposite of the apogee. If the opposite point of apogee is the nearest point between the Sun and earth, then Sun should be located in one corner of the ellipse,and they should be separated by 180°. This information has been clearly mentioned.

स्वोच्चात् षड्भागाध्यधिको यदा तदा भवति स्वनीचस्थः ।
दूरेणोच्चग उर्व्याः कर्णवशान्नोच्चगो निकटे ।।

Svochhaath shadbhaagaadhyadhiko yadaa thadaa bhavathi svaneechastha: doorenochhaga urvyaa: karnavasaannochhago nikate

When a planet is at a distance of 6 signs from its apogee, it is said to be at the perigee or neecha. When a planet is at the apogee, it is farthest from the earth when at the perigee, it is nearest to the earth. This is so because of the length of the hypotenuse in each case (Sishyadhi vruddhi Tantra 14-10)

One sign is 30°, then 6 signs are 180°. I.e the Sun comes closest to the earth when the angular distance (from mesha sankranti) is 180°+78°. So the modern astronomical conclusion that the Sun is at a corner of an elliptical orbit of the earth (around the Sun) was known to Indians. For revolving 180° around the Sun from the point of apogee, the earth takes 183 days according to Indian observation (at the rate 59′ 8″ per day). When the date of apogee (i.e July 3rd) is added with 183 days, we get January 3 as the date on which the Sun and earth come closest. (I.e. when the earth travels 180° in 183 days after July 3rd). According to modern science on this date the earth is in perigee.

Varahamihira in Panchasiddhantika has given the special method for calculating the apogee of the inferior planet, Mercury, using the period calculation based on ahargana. Ahargana is the number of years (or days) elapsed after the (arbitrarily corrected) fixed date of epoch. Various scholars take epoch as Mesha sankranti in 522 AD as mentioned earlier and some other give appropriate corrections for days and fix the year of thier choice to start counting the ahargana. (Number of years elapsed since then, is ahargana).

शतगुणिते बुधशीघ्रम् स्वरनवसप्ताष्ट भाजिते क्रमशः ।
अत्रार्ध पञ्चमास्तत्पराश्च भगणाह(ताः) क्षेपः ।।

Sathagunithe bhudha seeghram swara nava sapthaashta bhaajithe kramasa:
athraardha pancha masthathparaasche bhaganaaha (thaa:) kshepa:

For finding the apogee of Mercury multiply the ahargana with 100 and divide by 8797. The revolution numbers are obtained. Add 4½″ per revolution. Add 4 sign 28° 17′ 0″, the sighra at epoch. (Panchasiddhantika 16-7)

From the apogee, the perigee can be calculated by the addition of 180°. In the next stanza Varahamihira has given the method of adding 10.5″ per revolution to Venus to get the position of apogee. These correction factors are also very much in agreement with modern astronomy.

Revolutions of apogee: The total solar system is also revolving/ moving at some directions, and also various other types of motions of the planets take place. Due to this these imaginary points like apogee, perigee, first point of Aries, in the celestial orbit, also revolve around the Sun. In one of the above calculations, it has been pointed out that, the first point of Mesha sankranti, has moved about 25° from 499 AD to 1999 AD. I.e. during the last 1500 years. In the same way the rate of revolution of apogee is also described while discussing the revolutions of the celestial bodies. Ancient values and the modern astronomical values have also been given. Aryabhatta says on the revolution of apogee as follows:

चन्द्रोच्च र्जुषिखिध, बुध सुगुशिथृन, भृगु जषबिखुछृ, शेषार्का:

Chandrochaa rjushikhidha, bhudha sugusithruna,
brugu jashabikhrucchru seshaarkaa:

In a Yuga the revolutions of moon's apogee is 488219, that of Mercury 17937020, Venus 7022388, for other planets the same as that Sun/ earth. The counting revolution commences at the beginning of the sign Aries on Wednesday at Sunrise at Lanka (Aryabhateeyam 1-4)

These values are correct according to the present day knowledge. Here the revolutions of the imaginary point of apogees around the Sun are given for one Mahayuga (4320000 years). Manjulacharya has also given the method for calculating the apogee of the moon and also the rate of motin of the apogee.

द्युगणो द्विगुणाब्दोनाश्चन्द्रोच्चांशा नवोद्धृता: ।
खवेदघ्नाब्दसंयुक्तारसाष्टंशाब्दकलोनिता: ।।

dyugano dvigunaabdo naaschandrochhamsaa navoddhruthaa:
khavedaghnaabda samyukthaarasaashtamsaabda kalonithaa:

Subtract 2 times the years elapsed from the Dyugana and divide that by 9. To this add 40 times the years elapsed. These are the degree measure of the moon's apogee. Subtract minutes equal to (1+1/8) times the years elapsed. This is the mean motion of moon's apogee, since the epoch. (Laghumanasa 1-5).

This was obtained by a series of mathematical calculations, in which various astronomical parameters are used. It is described in the Laghumanasa, separately.

For calculating the apogee of Mercury a method has been suggested by Lallacharya. This is by adding correction factors to the final position of the planet and the apogee of the previous years (which is called here as druva).

((D - 2Y)/9 + 40Y) degrees - (1+1/8) min. Where D; Dyugana, Y; number of years elapsed since epoch. The method of calculating Dyugana has been discussed in detail by Manjulacharya (Laghumanasa 2.1)

दिने दिने भागचतुष्टयं क्षिपेच्छरर्तुभि: षट् च बुधध्रुवे लवम् ।
शरघ्नमब्ध्यक्षगुणांश वर्जितं कलादि जीवध्रुवके दिवागणम् ।।

Dine dine bhaagachathushtayam kshipeccharartubhi:
shat cha budhadhruve lavam saraghnamabdhyakshyagunaamsa
varjitham kalaadi jeeva dhruvake divaaganam

To obtain the apogee of Mercury add 4 + 6/65 degrees daily to its druva (position at the end of the previous year) (Sishyadhi vruddhi Tantra 1-36).

This is the method adopted for finding out apogee positions of other planets also i.e by giving correction factors to the previous positions.

मन्दतुङ्गभगणा: कजीविते भूवियद्गजशराष्टयोरवे: ।
लोहितस्य शरषट्छिवोरगा-धीकृताङ्कदहनेंदवो गुरो: ।।
कृतसप्तनगर्तव: शने: क्षितिगोदोर्मुनिभूभृदब्धय: ।
शशिजस्य सुरारिमन्त्रिणो द्विकृताष्ट द्विकपञ्चभूमय: ।।
नवकुनगाष्टकुवेदरसेषु श्रुतिहरिणाङ्कभधीमतिनन्दा: ।
शरबिलधीरस रामरसाभ्रद्विपकृतिभेन्दुरसाङ्कशशाङ्का: ।।
जलधिगजर्तुनखा-यमशून्य द्विघनगुणा-द्विकृतेषुभुव-श्च ।
बुध-सित-भूज-सुरेज्य-शनीनां कमलभवायुषि पातभसङ्घा: ।।

Mandathunka bhaganaa: kajeevithe bhooviyadgaja saraashtayorave:
lohithasya sarashatcchivoragaa dheekruthaanka dahanendavo guro:
kruthasapta nagarthava: sane: kshithi godormuni bhoo bhrudabdhaya:
sasijasya suraarimanthrino dvikruthaashta dvikapancha bhoomaya:
navakunagaashta kuvedaraseshusruthi harinaanka bhadheemathinandaa:
sarabiladheeraraamarasaa bhradvipa kruthi bhendu rasaanka sasaankaa:
jaladhigajarthunakhaa yamasoonya dvighanagunaa dvikrutheshu bhuva-scha
budha-sitha-bhooja-surejya saneenaam kamala bhavaa yushi paatha bhasanghaa:

Vateswara has given here accurately by mathematical calculations the number of revolutions of the apogee in 14 Manwantharas i.e one Brahmakalpa, which is a well defined period comprising of 4320000 x 72 x 14 years. Vateswara gives the number of revolution of apogees:

During the life of Brahma (Brahmakalpa) the revolutions performed by the apogee of rhe Sun are 165801, that of Mars, 81165, of Jupiter 13948, of Mercury 477291 and Venus 152842.

And the revolution of their ascending nodes are 988271456418719 for Mercury, 196127640636895 for Venus, 20684 for Mars, 3802 for Jupiter, 1542 for Saturn. (Vateswara siddhanta 1-16-19).

In modern astronomy the rate of annual motion of the apogee of the Sun 11.63″, Mars 15.99″, Mercury 5.73″, Jupiter 7.72″, Venus 0.38″ and Saturn 22.25″. That of ascending nodes from Mars, in the order -22.524″, 7.606″, -13.888″, -17.87″ and -18.85″. Dividing 360° by the above rate of motion the number of years required for one complete revolution of the apogee will be obtained. From that value, the number of revolutions possible in a Brahmakalpa can be calculated, i.e for a period of 14 x 72 x 4320000 years.

Revolutions of planets: It is well known that the planets revolve around the Sun. In ancient times some scholars believed that, all the planets revolve around the earth. Number of revolutions of the planets given by Indian scholars agree marvellously with the modern values and it is given by almost all the astronomers. They followed the basis, for calculation of revolutions, as one Mahayuga, which is 4320000 years, as defined by Aryabhatta I and also others.

दन्ताब्धयोऽयुतहता युगवत्सराःस्युः

Danthaabdhayo ayutha hathaa yugavathsaraa: syu:

In one Mahayuga, total number of years is the product of 432 and ten thousand (Mallikarjuna Suri 1-3) i.e 4320000 years

Suri has also given the number of revolutions of the planets in a Mahayuga which is the same number as mentioned by Aryabhatta I and other astronomers of ancient India. Aryabhatta gives the values as follows:

युगरविभगणाः ख्युघृ, शशि चयगियिङुशुछ्लृ, कु ङिशिबुण्लृष्वृ प्राक् ।
शनि ढुङ्ढ्वघ्व, गुरु रिखिच्युभ, कुज भदिलिझ्नुखृ, भृगुबुधसौराः ।।

Yugaravi bhaganaa: khyughru, sasi chayagiyingusucchru lru kungisibunlru shkhru praak sani ddungivaghva, guru rikhichyubha kuja bhadilijnukhru bhrugu budhasouraa:

In a yuga the eastward revolutions, of Sun is 4320000, of the Moon 57753336, of earth 1582237500, of Saturn 146564, of Jupiter 364224, of Mars 2296824, of Mercury and Venus the same as that of the Sun (Aryabhateeya 1-3).

Here the number of revolutions of planets and number of rotation of the earth are given. In fact the revolution of the Sun given here is equal to the number of revolution of earth around the Sun. And that given for the earth is its self rotation for the period of 4320000 years. (Numbers are given in Aryabhateeya system)

Bhaskaracharya I, in Lakhubhaskareeyam, has also given the same values as those given by Aryabhatta I. But the number is given in Bhoothasankya. All the values are just repetition of Aryabhatta's data. However Bhaskaracharya I has not included the rotation number of earth. This may due to the fact that Bhaskara was not supporting Aryabhatta's theroy of the earth's rotation.

Bhaskara has given the following data: Revolutions of the Sun 4320000, that of the Moon 57753335, Mars 2296824, Jupiter 364224, Saturn 146564, for Venus and Mercury, equal to that of the Sun (Laghubhaskareeyam 1-9-11)

In both the cases the values given for Venus and Mercury are not correct whereas those for other planets are correct.

Revolutions of the planets differ when viewed from earth. Nilakanta, on his commentary for Aryabhateeya, has mentioned this clearly:

ग्रहनक्षत्रभ्रमणं न समं सर्वत्र भवति भूस्थानाम् ।
तद्विज्ञानं गोलाद् यतस्ततो गोलमभिधास्ये ।।

grahanakshathra bhramanam na samam sarvathra bhavathi bhoo sthaanaam thadvijnaanam golaad yathasthatho golamabhidhasye

Revolutions of the stars and the planets are never equal everywhere, when viewed from earth. That knowledge is explained in the astronomy known as spherical bodies (golam) (Neelakanta's Aryabhateeyabhashya 4-10)

The number of sidereal days equivalent required for each planet (or its apogee or ascending node) to complete one full revolution around the Sun is compared with modern data.

Planet	Revolution	Period in sidereal days for one revolution	
		Aryabhatta I	Modern values
Sun (earth)	4320000	365.25868	365.256636
Moon	57753336	27.32167	27.32166
Mars	2296824	686.99974	686.9797
Jupiter	364224	4332.27217	4332.5887
Saturn	146564	10766.06665	10759.201
Moon's apogee	488219	3231.98708	3232.37543
Mercury's apogee	17937020	87.96988	87.9693
Venus' apogee	7022388	224.69814	224.7008
Moon's asc.node	23226	6794.951	6793.39108

The number of revolutions given by Aryabhatta I is accurate. He has also given the number of earth's rotation as 1582237500, it is for rotation and not revolution of earth. When the number is divided by 4320000, we get the number of days in one year i.e 366.25..... which is the number of rotation of earth in one year. Rotation of earth is separtely discussed elsewhere in the text.

According to Siddhanta darpana the number of revolution of the planets in a Kalpa is as follows: Commencing from the Sunrise at Lanka the number of eastward revolution in a Kalpa, of the Sun is of 43200000000, of the Moon 57753332321, of the Earth 1582237839500, of Mars 2296862137 of Mercury 17937120175 of Jupiter 364160611, Venus 7022270552 and of Saturn 146571016 (Siddhantadarpana 2-5). One Kalpa is 14 x 72 x 4320000 years!

Again more refinement on the scientific information took place in the course of time in ancient Indian astronomy. Nilakanta Somayaji, in Siddhanta darpana has given above value, on the basis of calculations. Here, Nilakanta gives the number for rotation of earth in Kalpa: i.e number of rotation of earth in 435450000 years is 1582237839500 - as given in bhoothasankhya:

खखेषुगोगुणाष्टाश्वाग्न्याश्विद्वयाष्टशरेन्दव:

Khakhashugogunaashtaasvaagnyaasvidvayaashta sarendava:

What stands unique is that the information on the apogees and the revolutions of the planets was a subject matter of in-depth study for Indians, more than one millennium before Kelper, Newton, Galileo and Copernicus. In modern astronomy these discoveries are attributed to these scientists.

Inclination of planet's orbit of revolution: All planets revolving around the Sun are not in the same plane. There are inclinations in the planet orbits which has been documented in modern astronomy. Inclination of planet orbits given by ancient Indian astronomers and modern values are compared here.

Planets	Suryasiddhanta	Aryabhatteeya & Sishy.vruddhi	Siddh. Sekhara	Modern value
Mars	90′	90′	110′	111′
Mercury	133	120	152	421 11
Jupiter	60	60	76	78 21
Venus	123	120	136	203 39
Saturn	126	120	130	149 25

There are variations for Mercury and Venus which has been attributed to the fact that they are inferior planets and also due to the approach taken by Indians from the geocentric vision, wheras the above given are heliocentric values. (Vateswara siddhanta commentary 6-4 page 533)

Various types of motions of planets: Vateswara says that the celestial bodies are made in such a way that they have to move continuously.

आदौ ससर्ज भगणं झषमेषसन्धि संस्थग्रहै: सह ग्रहस्फुरदंशुजालम्
ब्रह्मा प्रतिक्षणगमर्कजसो(म) कक्ष्यावक्त्रध्रुवप्रति निबद्धमिनेन्दुवश्यम्

Aadov sasarja bhaganam jyashameshasandhi samsthagrahai:
saha grahasphuradamsujaalam brahmaa prathi kshanagamarkajaso (ma)
kakshyaa vakthradhruvaprathi nibaddhaminendu vasyam

In the beginning, Brahma created the ever revolving circle of asterisms a net of twinkling stars, fastened to the pole star lying in front of the orbit or planets ranging from Saturn to Moon. (Vateswara siddhanta 1: 1-6).

Planets undergo different types of motions. The ancient Indian scholars have described eight types of motions. In Suryasiddhanta these motions have been well defined. Perhaps it is the oldest book which mentions about varieties of planetary motions. It is important to note that various types of motions were analysed viewing from the earth. Since the earth is also revolving and rotating, observations on motions vary. This is one of the reasons for some types of motion, of other planets.

Suryasiddhanta (2-12) explains that there are eight types of motions as:

वक्रानुवक्रा विकला मन्दा मन्दतरा समा ।
तथा शीघ्रतरा शीघ्रा ग्रहाणामष्टधा गति: ।।
तत्रातिशीघ्रा शीघ्राख्या मन्दा मन्दतरा समा ।।

ऋज्वीति पंचधा ज्ञेया यान्या वक्राधिका मता ।।

Vakraanuvakraa vikalaa mandaa mandatharaa samaa
thathaa sighratharaa seeghraa grahaanaamashtadhaa gathi:
thathraathi seeghraa seeghraakhyaa mandaa mandatharaa samaa
rujveethi panchadhaa jneyaa yaanyaa vakraadhikaa mathaa

Vakra; beginning of regression, ativakra; maximum regression, kutila; end of regression and beginning of direct motion, manda; (slow), mandathara (slower), sama, (mean) sighra; (fast), and sighrathara; (faster). The first three are different types of retrograde motion and the other five are various forms of direct motions. For both set of inferior and superior planets, the observer sees from earth that they move steadily in different ways.... forward, retrograde, mean motions, faster or slower. Also sometime attain a drastic change in their speed. Some of these changes are due to the movement of the earth in its elliptical orbit.

अनुलोमगतिर्वृत्ते मन्दगतिर्यो ग्रहो भ्रमति
अनुलोमगानि मन्दात् प्रतिलोमगानि वृत्तानि ।।

Anulomagathir vruthhe mandagathiryo graho bhramathi
anulomagaani mandaath prathilomagaani vrutthaani

In the process of the revolution of the planet in a circular path, from a manda speed, the planet moves clockwise and from sighra it moves anticlockwise (Nilakanta's commentary for Aryabhateeya - page 67).

He has used the word Apamandala which can probably be the elliptical orbit. Since the apogee and perigee concept was cleraly known during his period, one can interpret his Sanskrit commertary in this way also:

कक्ष्याया: प्रतिमण्डलं कक्ष्याप्रतिमण्डलं तेनमार्गेण गच्छन्तीति
कक्ष्याप्रतिमण्डला: ..... भ्रमन्ति ..... वृत्ताकारेण पर्यटन्ति.....

Kakshyaayaa: prathimandalam kakshyaaprathi mandalam thena margena gacchantheethi kakshyaa prathi mandalaa .....bhramanthi vrutthaakaarena paryatanthi

The Orbit which is eccentric/elliptic is known as elliptical orbit, movement through that orbit is the elliptical movement (Nilakanta's commentary - page 42)

Lallacharya has used the terms equivalent to those used in modern astronomy for defining the motion of planets. The mean circular orbit, mean motion etc. were correctly used here.

मध्यमकक्ष्यावृत्ते मध्यमया गच्छति ग्रहोगत्या ।
उपरिष्टात् तल्लघ्व्या तदधिकगत्या वधस्थ: स्यात् ।।

Maddhyamakakshyaavrutthey gacchathi grahogathyaa uparishtaath thallaghvyaa thadadhikagathyaa vadhastha:syaath

A planet moves along its mean circular orbit at the rate of its mean motion. When it is above its mean orbit, it moves at slower rate and when below, at a higher rate (Sishyadhi vruddhi Tantra 19-37)

The directions of movement given by him for all planets are in agreement with that of modern astronomy both for direct and retrograde motions.

वक्री यात्यपराशां निसर्गतो गच्छति ग्रह: प्राचीम् ।
क्रान्त्या याम्योदीच्योर्ग्रहगतिरेवं भवेत् षोढा ।।

Vakree gaathyaparaasaam nisargatho gacchathi graha: praacheem kranthyaa yaamyodeechyorgrahas gathirevam bhaveth shoddaa

A planet naturally moves to the east, but when retrograde, to the west, it is drawn north or south by its declination. Thus the motion of planets are of six kind (Sishyadhi vruddhi Tantra 19-38)

True planet and mean planet: Concept of true planet and mean planet is frequently used in modern astronomy which has also

been mentioned in Indian books.

कक्ष्यायां ग्रहवेगो यो भवति स मन्दनीचोच्चे ।

Kakshyaayaam grahavego yo bhavathi sa mandaneechochhe

The velocity of a true planet moving on the sigra epicycle is the same as the velocity of the true mean planet moving in its orbit. (Aryabhateeyam 3-25b)

Aryabhatta has correlated motion of the true planets with other parameters like:

कक्ष्याप्रतिमण्डलगा भ्रमन्ति सर्वे ग्रहा: स्वचारेण ।
मन्दोच्चादनुलोमम् प्रतिलोमम् च शीघ्रोच्चात् ।।

Kakshyaa prathi mandalagaa bhramanthi sarve grahaa: svacharena mandochhadanulomam prathilomam cha seeghrochhaath

All the mean planets move in their own orbits and the true planets on their eccentric circles, (pratimandalam), in shape. All the planets whether they move in their own orbits or eccentric circles move with their own mean motion anticlockwise from their apogee and clockwise from their sighroccha (Aryabhateeyam 3-17)

The third chapter in Laghumanasa, describes with details and complex mathematical formula, on a variety of calculations on true motion of planets and in the second chapter the mean motion of planets.

Motion of planets in angular and linear values: It is known that when a celestial body revolves around the Sun during one complete revolution it moves through 360°. This angle is accommodated in 12 signs, each comprising of 30°. When the number of signs multiplied with 30, degrees in angular measurements will be obtained, for converting it into minutes, it is further multiplied with 60 and when multiplied once again

with 60, the value in seconds will be obtained. For the moon, during the revolutions each angular dimention in seconds is said to be equal to 10 yojanas. (one yojana is approximately equal to 12.11 Kilometers).

There is no proof for the last part of the above statement that 1″ angular distance is equal to 10 yojana equivalent for moon, in modern astronomy. But it gives an indication of the capability to calculate the circumference of the circular orbit of moon and come to a conclusion on angular and linear dimensions. Thus says Aryabhatta I.

शशिराशयठ चक्रं तैं/शकलायोजनानि य-व-ञ-गुणा: ।।

Sasiraasayata chakram theamsakalayojanaani ya-va-nja-gunaa:

Reduce the moon's revolutions in a Yuga to sign (Rasi) multiplying with 12, this when multiplied respectively with 30, 60 and 10 gives the total degrees, minutes and yojanas through which the moon has moved, will be obtained. (Aryabhateeyam 1-6a)

Rate of angular motion of planets during their revolution: The rate of angular motion(i.e revolution) of planets and their apogees around the Sun has been given by Lallacharya upto the finest possible angular values.

From the number of days taken for one complete revolution, astronomers calculated the rate of the motion per day. These data are given in angular minutes and seconds. The angular data given for the Sun is meant for the earth in modern astronomy for obvious reason.

दिवाकरादेर्गतिलिप्तिका: क्रमान्नवेषव: खाङ्कनगा: क्षमाग्नय:
शिलीमुखाम्भो बाहव: निधि: शरास्तत: षडङ्का द्वितयं रसा गुणा: ।
क्रमात् विलिप्ता वसव:शरानला: षडश्विनो/युग्मगुणा नभो गजा:
नभ: कुवेदा: कुभुवो दिवात्यये गतेर्ग्रहा: स्यु: स्वगतेर्दलोनिता: ।।

Divaakaraadergathiliptikaa: kramaannaveshava: khankanagaa: kshamagnaya: sileemukhambho bahava: nidhi: saraasthatha: shaddankaa dvithayam rasaa gunaa: kramath vilipthaa vasava: saraanalaa: shadasvino fayugmagunaa nabho gajaa:nabha: kuvedaa: kubhuvo divaathyaye gathergrahaa: syu: svagatherdalonithaa:

Dauly mean motion of Sun etc. (Moon, Mars, Sighroccha of Mercury, Jupiter, Sighroccha of Venus, Saturn and moon's apogee and node) are respectively 59′8″, 790′35″, 31′26″, 245′26″, 5′0″, 96′8″, 2′0″, 6′41″, 3′11″ (Sishyadhi Vruddhi Tantra 1-40, 41)

There value agree with modern values. Puthumana Somayaji has given the following angular Velocity for planets in Karanapaddhathi

गोपाज्ञया दिनधाम (gopaajnayaa dinadhaama)	Sun	59′	8″	10‴	13⁗
चण्डिकेशो भर्ग स्निग्धोसौ (Chandikeso bharga snigdhosow)	Moon	790′	34″	51‴	36⁗
प्रभुर्धराचक्रपाल: (Prabhurdharaachakra paala)	Mars	31′	26″	29‴	42⁗
रागीतुम्बुरुर्गजेश्वर (Rageethumbururgajeswara)	Mercury	245′	32″	36‴	32⁗
प्रज्ञासन्नो धर्मवान् (Prajnaasanoo dharmavaan)	Jupiter	4′	59″	7‴	2⁗
काशी साम्बसन्न चोल: (Kasi saambasanna chola:)	Venus	96′	7″	37‴	51⁗
प्रभलप्राज्ञो नर: (Prabhalapraajno nara:)	Saturn	2′	0″	23‴	32⁗

It is for the first time in any ancient astronomical book

angular velocity upto to the level of fractions of seconds are described in lipta and pralipta units which is one sixtieth of a second and one sixtieth of that value. The modern values of angular motions are Mercury 245.7′, Venus 96.13′, Earth/Sun 59.14′, Mars 31.45′, Jupiter 4.99′ and Saturn 2′.

Corrections for the motions: In the modern astronomy correction of time, degree of motion, etc., are incorporated due to the influence of various factors on the planet's motion. This approach of giving correction were very common in the ancient books dealing with this subject. An example of giving the correction to the daily motion of the moon is given below. The verses are directly quoted here.

इन्दूच्चोनार्ककोटिघ्ना गत्यंश विभवा विधो:
गुणो व्यर्केन्दुदो: कोट्यो रूपपञ्चाप्तयो क्रमात् ।
फले शशाङ्कतद्गत्योर्लिप्ताद्ये स्वर्णयोर्बधे ऋणं
चन्द्रे धनम् भुक्तौ स्वर्णसाम्यबधेऽन्यता ।।

Indoochhonaarkakotighnaa gathyamsa vibhavaa vidho:
guno vyarkendudo kotyo roopa panchapthayo kramaath
phale rasaankathadgathyorlipthaadye swarnayorbadhe runam
chandre dhanam bhukthow swarnasaamyabadheranyathaa

Multiply the degrees of the moon's true daily motion after reducing 11 from it, with R cosine of the true longitude of the Sun minus the longitude of the moon's apogee. This is the multiplier of the Rsine and Rcosine of the true longitude of the moon diminished by that of the Sun, divided by 1 and 5 respectively. This will give the correction, in terms of minutes of arc, for the moon and its true daily motion, respectively. If, in the above product one factor is positive and the other negative, the correction for the moon is subtractive and that for its true daily motion additive. If both are of like signs, both positive or both negative, corrections are to be applied contrarily (Laghumanasa 4-1,2)

I.e 8°8′ cos (S - U) (Moon's true daily motion - 11) x 8°8′ sin (M - S). Where S,M,U denote the true longitude of the Sun, Moon and moon's apogee. Here correction for moon and method applied for getting its true daily motion are given. Application of true longitudes of the Sun, the moon and the moon's apogee have been brought in the calculation for arriving at the correction factor. Application of sine and cosine of the angle have been done very systematically 1300 years ago. Correction of motion for planets has also been given by Haridatta (Grahacharanibandhana (I-17,18) as:

Moon	=	5s	10°	07′	40″	48‴
Mars	=	2s	27°	33′	07″	12‴
Mercury	=	7s	06°	57′	31″	00‴
Jupiter	=	6s	22°	45′	07″	12‴
Venus	=	3s	24°	37′	26″	24‴
Saturn	=	6s	15°	04′	19″	12‴

During a fixed period (this period appears to be one Mahayuga) the correction that should be applied for each planet is given above. S is for sign which is 30 degrees. Application of correction for the revolution of the planets, to calculate the position of the planet is as followed in modern astronomy.

Retrograde motion of planets: Every planet moves in space from west to east i.e. in the anticlockwise direction. This motion of the planet is called the direct motion. As the earth also moves simultaneously in space, in some regions of the planet's orbit, it appears to the viewer that the planet moves in the opposite direction. I.e. from east to west. This apparent motion of the planet is called the retrograde motion. At some point, the planets appear as stationary when viewed from earth. This is known as stationary position. A planet attains this position when it changes from retrograde to direct motion and also in the opposite. Mathematical calculations follow a different formula for getting

retrograde parameters for inferior and superior planets. This type of motion observed from the earth, is due to the rotation and revolution of the earth. Almost all the Indian astronomers have described this subject. Lallacharya has given the following observations:

गुणनृपतिभिर्बाणाब्ध्यकै: शराक्षिनिशाकरै:
शररसकुभिर्विश्वक्ष्माभिर्लवैश्चलकेन्द्रजै: ।
भवति नियतं वक्रारम्भ: कुजादिनभ:
सदां पुनरपि भवेद्वक्रत्यागश्च्युतैस्तु समण्डलात् ।।

Gunanrupathibhirbaanaabdhyakai: saraakshanisaakarai:
sararasa kubhirviswakshmaabhirlavaischala kendrajai:
bhavathi niyatham vakraarambha: kujaadinabha: sadaam
punarapi bhavedvakrathyaagaschyuthaisthu samandalaath

Retrograde motion of the planets beginning with Mars (Mars, Mercury, Jupiter, Venus, Saturn in that order) commences when the Sighrochakendras (apogees) are respectively 163°, 145°, 125°, 165° and 113°. When sighrocchas are respectively 360° minus each of these values their retrograde motion ceases. (Sidhyadhi vruddhi Tantra 3-20)

Lallacharya has also given the number of days of retrograde motion of the planets. Bhaskara I and Varahamihira have made such observations. Lallacharya's quotation is given here:

रसरसा: क्रमश शशिबाहवो यमनिशाकरशीतमरीचय: ।
यमशरा युगपावकभूमयोऽनृजुगतेर्दिवसा: कथिता: कुजात् ।।

Rasarasaa: kramasa sasibaahavo yamanisaakaraseetha mareechaya:
yamasaraa yugapaavaka bhoomayoamrijugatherdivasaa: kathithaa: kujaath

Retrograde motion of the planets Mars, Mercury, Jupiter, Venus and Saturn lasts for 66, 21, 112, 52 and 134 days respectively (Sishyadhi vruddhi Tantra 3-21)

These numbers can be obtained only if the astronomers have a deep scientific observation skill in planet watching. Period of retrograde motion of planets in number of civil days are given by Vateswara:

पञ्चर्तव: कुदस्रा बाहुशिकाद्विषवो द्विगुणचन्द्रा:
वक्रदिनान्युर्वीजानि निरंशदिनशोधिताण्यृजूनि स्यु: ।
खाष्टनगा रसरुद्रा नवनवरामा: पयोधिधीपवना:
वसुशैलगुणा: क्रमशो भौमादीनां निरंशनिशा: ।।

Pancharthava: kudasraa baahusikaadvishavo dvigunachandraa:
vakradinaanyurveejaani niramsadina sodhithanyrujuni syu:
khaashtanagaa rasarudraa navanavaraamaa: payodhi deepavanaa:
vasu salagunaa: kramaso bhoumaadeenaam niramsanisaa:

The number of civil days of retrograde motion for the planets (Mars, Mercury, Jupiter, Venus and Saturn) are 65, 21, 112, 52 and 132. These numbers are subtracted from the days of their synodic period and days, of their direct motion. 780, 116, 399, 584 and 378 are in days the synodic period of planets (in the above order) (Vateswara siddhanta 2 (4) 10,11)

Vateswara and Lallacharya differ only in the cases of Mars and Saturn.

Studies on tracing planetary motion: Observations on the motion of planets have been undertaken with great scientific accuracy. Rate of motion, number of days during when a direction is followed, etc. were systematically studied by Varahamihira in Panchasiddhantika. He has given details for every planet. For Mars he describes thus:

षड्विषयै स्ति(थ्यु)न: दृष्टो वसुधृति(भि) रंशका(ष)ष्टि:
अष्टशतेन (च) षष्टि: सप्तत्या(त्र्य)धिकया नवति: ।
षष्ट्याष्टयुक्तया(श) तदलं च खाश्वि द्विकै: स्वराद्रि(घ्ना:) ।
अस्तमितोऽत: सप्ताष्टकेन तिथयो निरंशाग (गति:) ।।

Shadvishayai sthi (thyu:) na: drushto vasudhruthi (bhi) ramsaka (sha) shti: ashta sathena (cha) shashti: saptha thyaa thryadhikayaa navathi: shashtyaashta yukthayaa (sa) thadalam cha khaasvi dvikai: svaraadrighmaa: asthamitho atha: sapthaashtakena thithayo niramsaga (gathi)

Mars is seen in its longitude in more than 15°, then within 188 days it moves through an angle of 60°, during next 108 days 60°, next 72 days 90°, next 68 days 50°, next 240 days 70° and then sets. (total days 768 and total degrees equal to 360°) (Panchasiddhantika 16-68, 69)

Mean motion of Mars per day is given by Bhaskaracharya I:

द्विकनिघ्ने ग्रहदेहे स्वविंशभागरहिते तु लिप्ताद्या: ।
पञ्चाशदंश विकला: क्षोण्या भैमो रवेरर्धे ।।

Dvikanighne grahadehe svavimsabhaagarahithe thu lipthaadyaa: panchaasadamsa vikalaa: kshonyaa bhaumo raverardhe

Multiply the grahathanu by two and subtract 1/20 th of itself from that,. These are minutes etc., Then divide the grahatnu by 50. These are seconds. Add these to half the Sun's longitude that is the mean longitude of Mars.

Grahatanu is defined as the number of years elapsed since Kaliyuga x 360 (Mahabhaskareeya 1-38) From this mean longitude the motion and period can be concluded.

प्रागुदये षट्(चत्वार्येक)मष्टादश(ग) स्ततो वक्रम् ।
अध्यर्धं च शतं शीघ्रां (स्ततो/स्तमितो द्यूनां षष्ट्या।)
समतीत्या दशत्रियु(तं) निरंशगतो (/तास्त्रिंशतं) व्यतीत्य कुज: ।
उदयमुपयाति वक्ष्ये गतिचारदि(न) क्रमे चात: ।।

Praagudaye shatchathvaaryeka mashtaadasaga sthatho vakram adhyardham cha satham sighram sthathoasthamitho dyoonam shashtyaa samatheethyaa dasathriyutham niramsagatho athaathrimsatham vyatheethya kuja: udayamupayaathi vakshye gathi chaaradina krame chaatha:

After rising in the east, Mars moves 146° in quick motion and then 18° slow motion, retrograde and after that 150° of quick motion, then setting, it reaches conjuction, in 60 days moving 13 plus 30 degrees (43°). Then it rises moving same degrees in the same number of days. (Panchasiddhantika 18-25,26)

दशभिर्द्वादशहीनः प्रागुदितो मनुभिरून नन्दांशाः ।
धृतिभिः सनवोऽस्तमितः त्रिंशाद्भिरुदेति सशराश्विः ।
अष्टादशभिः सनवः षोडशभिश्चार्क वर्जितोऽस्तमितः
पश्चाद्वसुभिर्नववर्जितो निरंशं बुधो याति ।।

Dasabhirdvaadasaheena: praagudithо manubhiroona nandaamsaa: dhruthibhi: sanavo asthamitha: thrimsaad bhirudethi sarasaasvi: ashtaadasabhi sanava: shodasabhischaarka varjitho asthamitha: paschaadvasubhir navaarjitho niramsam budho yaathi.

When Mercury is behind the Sun by 12°, within 10 days it rises in the east. During 14 days it travels 10°, then for 18 days 9°, and it sets, for 30 days it travels for 13° and rises in the west, then for 18 days 9°, 16 days 8°, it travels and sets in the west. Then for 8 days it retrogrades by 9° and then reaches the starting point, called niramsaka means 0°, the starting point (Pancha siddhantika 18-71, 72)

दिन षष्ट्यंशान् द्वादश खकृतैर्वेदान कृताश्विभिद्वौ च
सप्ताष्टकेन वक्री षड्भागान् षष्टितः षट् च
अनुवक्रोऽशीत्यर्कान् द्वयूनार्धशतेन नव ततोऽस्तमितः
स्थित्वा सैकं मासं स्फुटोदयोऽष्टोत्तरैरङ्गैः

Dina shashtyamsaan dvaadasa khakruthairvedaan kruthaasvi bhidvow cha saptaashtakena vakree shad bhaagan shashtitha: shat chaanuvakro aseerthyaarkaan dvyoonardhasathena nava thatho asthamitha: sthithvaa saikam maasam sphutodayoashtottharai rangai:

By 60 days, 12°, by 40 days 4° and by 24 days 2° becoming

retrograde by 56 days, it moves 6° (-6°) and by 60 days 6° (-6). Following after retrograde it moves 12° in 80 days and 9° in 48 days. Then setting, staying so for a month + one day. Jupiter clearly rises moving at 6°8′. (Panchasiddhantika 18-12,13)

All the above values are accurate except the last value of 6°8′ which according to modern astronomy is 6° 12′. I.e. a difference of about 4 minutes in the angular dimension for the total period. Astounding level of accuracy can be seen!

Lallacharya has given the method for calculating the position of Jupiter in longitudinal angles. This longitude could be calculated only by applying the correction factors for the planet.

शरघ्नमब्ध्यक्षगुणांशवर्जितं कलादि जीवध्रुवके दिवागणम्

Saraghnamabdhyakshagunaamsavarjitham kalaadi jeeva dhruvake divaaganam

Mean longitude of the Jupiter is obtained by adding to its dhruva (at the end of the previous year) in minutes equivalent to 5 times the ahargana diminished by its 5/354 (Sishyadhi vrudhi Tantra 1- 36b)

विषयैर्नवकविहीनः प्रागुदित स्तिथि(भि)रेक(य) महीन ।
वसुकृत्याति(थ्यून)कृताष्टिभिः स(पञ्चकस्त्रिंशत्) ।।
पञ्चाष्टकेन सदशः निरंतो विलोमगः पश्चात् ।
उदेति निरंशकाले प्रयाति चास्तं विलोमगतिः ।।

Vishayairnavaka viheena: praaguditha sthithi bhirekaya maheena vasukruthyaathi thyoonakruthaashtibhi: sapanchaka thrimsath panchaashtakena sadasa: nirantho vilomaga: paschaath udethi niramsakaale prayaathi chaastham vilomagathi:

Fove 5 days, Venus retrogrades and rises in the east, then for 15 days further retrogrades through an angle 21°, further for 208 days 15° retrograding, for 12 days 5° moves forward and

sets. Then it travels for 10° during 48 days and becomes niramsaka (Panchasiddhantika 18-77,78)

Varahamihira gives similar explanation for saturn also

षट्कृत्या त्रीनंशान् मुनिभिर्लिप्ताश्चेषु गुणास्सप्त
षोडशभिश्चाशीति कृतोनषष्ट्या वेदयम पक्षान् ।।
वक्री विभूतषष्ट्या त्रीनंशान् षष्टित: कृतान् सौर: ।
अनुगोऽर्कशतेनाष्टौ षट्कृत्या चास्तगो दहनम् ।।

Shatkruthyaa threenamsaan munibhirliptha scheshu gunaassaptha shodasabhischaaseethi kruthonashashtyaa vedayama pakshaan vakree vibhoothashashtyaa threenamsaan shashtitha: kruthaan soura: anugoarkasathenaashtow shatkruthyaa chaastago dahanam

Saturn moves 3° in 36 days 35′ in 7 days, 80′ in 16 days and 22′ in 56 days. Then becoming retrograde, it moves 3° in 55 days and 4° in 60 days then following direct motion moves 8° in 112 days and setting, it moves 3° in 36 days in the set period and rises in the east. (Panchasiddhantika 18-19, 20).

The examples give a notable support for the fact that practically this knowledge ought to have generated through observational science and skill rather than intuition or the so called divinity!

Going through the general information on the planets, one has to search for the scientific understanding of the planet earth which remained mainly as a bundle of superstition till the 15th century in the West. Hence the ancient Indian knowledge on planet earth is also traced in the next part.

Studies on the planet Earth

In modern and ancient astronony, the earth has been a subject of great interest and importance. Upto Copernicus, Galileo and Kepler, there was a belief that the earth was plane.

The story of Columbus who went in search of new landscape and continents, is really amusing. The travellers, who accompanied him feared that they may fall to the depth, when they reach the edge of the plane earth. Surprisingly it was not a story, but a history, which occured in the 15th century AD. However there was absolutely no such superstition among Indians on the size and shape of the Earth. In Vedas, Itihasas and Puranas, the common use of the word Bhoogola (spherical earth) clarifies the shape of earth as Gola i.e spherical. Aryabhatta in 499 AD has defined the shape and composition of the earth:

मृज्जलशिखिवायुमयो भूगोल: सर्वतो वृत्त:

Mrujjalasikhivayumayobhoogola sarvatho vruttha:

Spherical earth made of soil, water, fire and air is circular (when viewed) from all sides i.e it has spherical shape (Aryabhateeyam 4-6)

This definition is complete and it was given more than 1000 years before Columbus and the above modern scientists, in whose names many discoveries on the nature of earth were attributed to. If earth is spherical then why we see it as plane? Answer has been given here by Indian astronomers to this common man's question!

प्रकृत्या वृत्ताकारैव भू: । तस्या विक्रियमाणत्वाद् यो
विशेषस्तेनापि न वृत्ताकारतया हानि: स्यात् ।
भूपरिमाणापेक्षया निम्नोन्नततापरिमाणस्यात्यल्पत्वादिति ।।

Prakruthyaa vrutthaakaaraiva bhoo: thasyaa vikriyamaanathvaath
yo viseshasthenaapi na vrutthakaarathayaa haani: syaath
bhooparimaanaapekshayaa nimnonnathathaa
parimaanasyaathya lpathvaadithi

Naturally (when viewed from a distance place) earth is spherical throughout. Due to the large area, one does not feel

that it is circular (spherical) because we see only a small part of earth and also ups and downs on the surface of the earth. (Neelkanta's commentary to Aryabhateeyam. Golapada 2-8)

Lallacharya has described the shape of the earth thus:

गगनमरुदाग्निजलमृण्मयो महाभूतगुणयुत: खस्थ: ।
कक्षाभिरावृतोऽयं भपञ्जरान्तश्च भूगोल: ।।

Gaganamarudaagni jalamrunmayo mahaabhootha gunayutha: khastha: kakshaabhiraavruthofayam bhapancharaanthascha bhoogola

Spherical earth, made of ether, fire, air, water and clay (Panchabhoothas) and thus have all the properties of the five elements, surrounded by the orbits and extending upto the sphere of stars, remain in the space (Sishyadhi vruddhi Tantra 17-1)

He has commented on why the earth looks plane and answered, using mathematical approach.

प्रगुण:परिधे: शतांशकोगणितज्ञा: कथयन्ति दृश्यते ।
प्रतिभाति तदा समा मही विषये यत्र तथैव गम्यते ।।

Praguna paridhe: sathaamsako ganithajnaa: kathayanthi drusyathe prathi bhaathi thadaa samaa mahee vishaye yathra thathaiva gamyathe

Mathematicians say that one hundredth of the circumference of the earth appears to be plane. So, that portion of the earth appears to be plane to an observer (Sishyadhi vruddhi Tantra 20-35)

Lallacharya has further explained the reason for the planar appearance of the earth.

परित: क्षितिजे प्रदृश्यते गगनं सङ्गमुपागतं नृभि: ।
तस्यावनिरन्तरे स्थिता सुसमा दर्पणवद् विभाव्यते ।।

Paritha: kshithije pradrusyathe gaganam sangamupaagatham nrubhi: thasyaavaniranthare sthithaa susamaa darpanavad vibhavyathe

To men, the sky appears to meet the earth all round the horizon. The earth thus surrounded appears to be plane like a mirror (Sishyadhi vruddhi Tantra 20-37)

Varahamihira and Aryabhatta have defined the golayantra (now we call as globe) which is a prototype of earth with lines of longitude and latitude on it:

समवृत्तपृष्ठमानं सूक्ष्मं गोलं प्रसाध्य दारुमयं
स्थगितार्कसमाङ्कितकालभोगरेखाद्वये परिधौ ।।

Samavrutthaprushtamaanam sookshmam golam prasaadhya daarumayam
sthagithaarka samaankitha kaala bhogarekaadvaye paridhov

Perfectly circular throughout and spherical, made of wood, marked with degrees and minutes, incorporated with lines both longitude and latitude lines at ends, is the golayantra. (Panchasiddhantika 14 -23)

Aryabhatta I was definitely the first to write on the rotation of the earth. Hence he added this aspect of the rotation of the earth also while defining the globe.

काष्ठमयं समवृत्तं समन्ततः समगुरुं लघुं गोलम् पारदतैलजलैस्तं
भ्रमयेत् स्वधिया च कालसमम् ।।

Kaashtamayam samavruthham samanthatha:
samagurum laghum golam paaradathaila
jalaistham bhramayeth svadhiyaa cha kaalasamam

Made of wood, fully circular, uniform , equally dense throughout and spherical shaped golayantra, which rotates at a fixed rate of time as the earth does by the help of mercury, oil and water, by the application of our intelligent calculation, is the golayantra-Globe (Aryabhateeyam 4- 22)

Aryabhatta and others have not only mentioned the spherical shape of the earth, but also have given the diameter of the earth correctly. They used the unit of measurement, yojana.

Aryabhatta has defined the yojana, using which an equivalent term in modern measurement, can be obtained. However, in puranas, yojana unit was used for a different dimension. Aryabhatta's yojana is well definied as 8000 times the average height of a man (nara = Nr). Using this unit he has given the diameter of earth, (Aryabhateeyam 1-7):

नृषियोजनं, ञिला भूव्यासो .....

Nrushiyojanam, njilaa bhoovyaaso

8000 Nr units is equal to one yojana. The diameter of earth is 1050 yojana.

It has been estimated that one yojana can be approximately 12.11 km. (explanation given in the Aryabhateeya published by Indian National Science Adademy). Thus the diameter of the earth in kilometers is 1050 x 12.11 km. The result gives 12716km which is very near to the correct value of earth's diameter i.e 12756 km. Lallacharya has given equatorial circumference of the earth which can be obtained by multiplying the diameter of earth with 3.14. Ie 3.14 x 1050 yojanas.

खखमरा योजनवेष्टनं भुवो नभः शराभ्रक्षितयोऽस्य विस्तृतिः ।
दिवाकरघ्नम् पलकर्णभाजितं स्फुटं महीगोलकवेष्टनं भवेत् ।।

Khakhamaraa yojanaveshtanam bhuvo nabha:
saraabhrakshithayoasya visthruthi divaakaraghnam
palakarna bhaajitham sphutam maheegola kaveshtanam bhaveth

The circumference of the earth is 3300 yojanas. Its diameter is 1050 yojanas. The mean circumference of the equinoctial shadow (palkarna) of a place gives the corrected circumference at that place (Sishyadhi vruddhi Tantra 1-43).

The modern value for the circumference of the earth is 40090.3 km whereas Lallacharya's value is 39964.5 km. This variation in Aryabhatta's or other estimation may also be due to

the fact that the Yojana is taken as 12.11 km. Any minor variation even in decimal place can contribute to the variation in original value, significantly.

Lallacharya has also given the surface area of the earth. Howerver this value is not correct, because the equation for calculating the surface area of a globe should be 4 x 3.14 x r x r. whereas Lallacharya has calculated the area using √10, for 3.14.

नगशिलीमुखबाणभुजङ्गमज्वलनवह्निरसेषुगजाश्विन: ।
कुवलयस्य बहि: परियोजनान्यथ जगु:खलु कन्दुकजालम् ।।

Nagasileemukha baana bhujangama jvalana vahni raseshugajaasvina: kuvalayasya bahi: pariyojanaanyatha jagu: khalu kandukajaalam

The earth surface is said to have an area, externally which is like a net enclosing a ball of 286338557 sq.yojana (Sishyadhi vruddhi Tantra 17-11).

Lallacharya has refuted the belief that the earth is of infinite size using the logic that a circle has 360° which is 21600′. He argued that how, there can be infinite size.

खखषट्कुयमैर्मिता: कला ग्रहचक्रे वलये व्यवस्थिता: ।
स्वफलस्य कृवृत्तयोजनैरनुपातादमिता कथं मही ।।

Khakhashat kuyamairmithaa: kalaa grahachakre valaye vyavasthithaa: svaphalasya kruvrutthayojanai ranupaathaadamithaa katham mahee

By using the above circumference of earth in yojana, it is found by means of simple proportion that the circumference of the circle of one revolution of a planet consists of 21600 minutes (angle). Then can the earth be of infinite size? (Sishyadhi vruddhi Tantra 20-32)

Cover above the earth: Lallacharya says that there are seven layers of cover above the earth. This explanation resembles well with the several spheres known to astronomy like troposphere, mesosphere, stratosphere, ionosphere, exosphere, etc.

अवह: प्रवह उद्वहस्तथा संवह: सुपरिपूर्वकौ वहौ ।
सप्तमस्तु पवन: परावह: कीर्तित: कुमरुदावहा/परै: ।।

Aavaha: pravaha udvahasthathaa samvaha: suparipoorvakow vahow
sapthamasthu pavana: paraavaha: keerthitha: kumarudaavahaaparai:

Avaha, pravaha, udhwaha, samvaha, suvaha, parivaha, paravaha are the seven layers above the earth. And the atmosphere where the air is blowing is the avaha layer. (Sishyadhi vruddhi Tantra 18-1).

Circumference of avaha layer of atmosphere is also given

शराद्रिरामानलयोजनानि कुवायुकक्ष्या परित: पृथिव्यां
समुद्रशैलाम्बरशीतभासस्तदीयविष्कम्भमुशन्ति सन्त: ।।

Saraadriraamaanalayojanaani kuvaayukakshyaa paritha pruthivyaam
samudrasailaambaraseethabhaasa sthadeeya vishkambhamusanthi santha:

Circumference of the atmosphere which surrounds the earth is 3375 yojanas and the wise maintains that, it has a diameter of 1074 yojana. (Sishyadhi vruddhi Tantra 18-2).

This when converted to kilometers by multiplying with 12.11, it will be come 1073.9 yojanas x 12.11 km. and taken as the radius. We can see that the height of the atmosphere will be about 6502 - 6357 = 145 kilometres from the surface of the earth. It has been proved that air molecules are seen upto this height above the earth and the actual height of the atmosphere is 100 - 120 km. Thus there was also an awareness on the space above the earth, which is at par with modern astronomy.

Rotation of earth: It was believed that the celestial bodies and planets revolve around the earth and the earth stands stationary. This belief was deeprooted, and nobody was permitted to scientifically search beyond this belief in the European countries. Galileo was even punished in the 15th century for telling the

universal truth that the earth revolves around the Sun. Some of the Indian scholars also believed the same. However from the available ancient Indian literature it can be seen that, Aryabhatta I was the first to say that the earth rotates by itself and revolves around the Sun, with details and mathematical data.

प्राणेनैति कलां भूः ..... *Praanenaithi kalaam bhoo:.....*

(The earth rotates through an angle in the orbit) at the rate of 1 minute per 4 seconds (Aryabhateeyam 1-6)

(Unit of time mentioned here is *prana* which is the time taken for one respiration)

Aryabhatta has correctly defined the prana unit of time equivalent to the duration of one respiration. This rate of earth's rotation given by Aryabhatta I is correct. The earth rotates 21600' of the angle in 24 hours (i.e in a day) which means in 4 seconds it rotates at the rate of 1' of an angle. It is important to mention here that some of the ancient Indian scholars did not accept Aryabhatta's theory of earth's rotation. Even though there are different commentaries on Aryabhatteeya they focus this subject with diversified interpretations to explain Aryabhatta's view. Aryabhatta has given explanation and proofs to explain the rotation of the earth. He has also said that days are formed due to the rotation of earth:

.....कु आवर्तश्चापि नाक्षत्राः ।।
Ku aavarthaaschaapi naakshathraa:

The rotation of the earth is the cause of days (Aryabhateeyam 3-5). Aryabhatta has given the numerical values of the number of rotation of the earth which is the same as that mentioned in modern science. This is perhaps the most authentic and solid proof given by Aryabhatta for the rotation of the earth (in one Mahayuga i.e during a period of 4320000 years.)

कु ङि शि बु ण्लृ षृं खृ प्राक् ....

Ku ngi si bu nlru shru khru praak

Eastward rotations of the earth in one Yuga is 1582237500 (Aryabhateeyam 1-3).

If this number is divided by the number of years in a Mahayuga (4320000), we get the duration of one solar year in number of days as 366.2586806 and one less than this (due to the revolution of earth around the Sun) is 365.25 which is the number of days in a civil year. This is the most important and projectable mathematical proof given by Aryabhatta.

Aryabhatta has further explained the reason for the observer seeing the celestial body revolving around the earth. He has also compared this phenomenon with a passenger's observation while travelling in a boat.

अनुलोम गतिर्नैस्थः पश्यत्यचलं विलोमगं यद्वत् ।
अचलानि भानि तद्वत् समपश्चिमगानि लङ्कायाम् ।।

Anulomagathirnoustha: pasyathyachalam vilomagam yadvath achalaani bhaani thadvath samapaschimagaani lankaayaam

Just as a man in a boat moving forward sees the stationary objects as moving backward, so are the stationary stars and celestial bodies seen by the people at equator (Lanka) as moving exactly towards west. (Aryabhateeyam 4-9).

This statement does not need any further explanation. It states correctly that the rotation of the earth from west to east is the cause of the observer seeing the movement of the celestial bodies from east to west.

Many contemporary (to Aryabhatta) Indian astronomers did not accept the earth's rotation theory. Proof of this can also be seen here. Brahmagupta, a scholar of the period of Aryabhatta, questioned, the view of Aryabhatta on earth's rotation.

प्राणेनैति कलां भूर्यदि तर्हि कुतो व्रजेत् कमध्यानम्

आवर्तनमुर्व्याश्चेन्न पतन्ति समुच्छ्रया: कस्मात् ।।

Pranenaithi kalaam bhooryadi tharhi kutho vrajeth
kamadhyaanam aavarthana murvyaa schenna pathanthi
samucchrayaa: kasmath

If earth rotates at a speed of 1′ of an angle in 4 seconds, will not the things on the loft fall? Where does the earth go in this speed? (Brahmasphuta siddhanta 11-17).

This question was put forward by Brahmagupta to refute Aryabhatta's theory of earth's rotation. Bhaskaracharya I in his commentary for Aryabhateeya (Aryabhateeyam 1-3) has not included the number given in the above lines on earth's rotation, but did not omit any values for other planets. This obviously shows that Bhaskaracharya I has also not agreed to this concept put forth by Aryabhatta. Lallacharya also questions the theory of earth's rotation:

यदि च भ्रमति क्षमा तदा स्वकुलायं कथमाप्नुयु: खगा: ।
इषवो/भिनभ: समुज्झितानिपतन्त: स्युरपांपतेर्दिशि ।।

Yadi cha bhramathi kshamaa thadaa svakulaayaam
kathamaapnuyu: khagaa: ishavo fabhi nabha:
samujnjithaani pathantha: syurapaam patherdisi

If earth rotates how could birds come back to their nest. Moreover arrows shot towards the sky would fall towards the west (Sishyadhi vruddhi Tantra 20-42).

These objections from some astronomers arose because, Aryabhatta has specifically described the earth's rotation. These objections are cited here to stress that even before Galileo and other Western astronomers, specific knowledge on the earth's rotation, existed in ancient India. Varahamihiras words are proof of the ancient Indian theory of the earth's rotation:

भ्रमति भ्रमिस्थितेव क्षितिरित्यपरे वदन्ति तेद्धुगणा: ।
यद्येवं श्येनाद्या: न खाप्पुन: स्वनिलयमुपेयु: ।।

Bhramathi bhramisthitheva kshithirithyapare vadanthi theudduganaa:
yadyevam syenaadyaa: na khaappuna: svanilayamupeyu:

The Axis of earth extends right up and right down to the sphere. The stellar sphere, bound by the axis to the east, rotates by the wind system called pravaha. Others say that the earth rotates on its axis, like an object placed at the hub of a wheel, and not the stars. (Panchasiddhantika 13-6).

He, further said.... "then the eagle etc., flying up into the sky will not reach its cage if earth rotates". Varahamihira was a contemporary scholar astronomer to Aryabhatta I.

Hence Aryabhatta I is the first Scientist who discovered that earth rotates, 1000 years before Galileo.

Days and Rotation of the earth: There was a very clear idea on the civil and sidereal days.and their number, duration and corrections were well documented. Aryabhatta has stated that days are formed due to the rotation of earth (Aryabhateeya 3-5), (quoted earlier). The number of rotation of earth in a Mahayuga has also been given by him from which the number of days, can be determined for a year.

व्योमाक्षिपञ्चविषयेषुभुवो/युतघ्ना: प्रोक्ता युगे/र्कदिवसा: खगुणोद्धृतास्ते
मासा: स्युरभ्रवसुखाम्बरखाभ्रवह्निव्योमाङ्गशीतकिरणा दिवसा हिमांशो:
व्योमाभ्रबाणतुरगक्षिति नन्दशैलक्ष्माभृच्छिलीमुखभुव: क्षितिवासरा: स्यु:
पूर्णाभ्रसायक शिलोच्चयरामदस्रद्वीभेषुशीतकिरणा भपरिभ्रमा: स्यु:

Vyomaakshi panchavishayeshu bhuoayuthaghnaa: prokthaa yuge
arkadivasaa: khagunoddhruthaasthe maasaa: syurabhravasukhambara
khabhra vahni vyomaankaseethakaranaadivasaa himamso:
vyomabhra baanathuraga kshithi nanda saila kshama
bhrucchileemukhabhuva: kshithivaasaraa: syu: poornaabhrasayaka
silochhayaraamadasradveebheshu seethakiranaa bhaparibhramaa: syu:

Lallacharya gives the number of solar days in a Yuga as 155520 x 10000, this divided by 30 gives the solar months and

the number of lunar days which is equal to 10603000080. The number of civil days is 1577917500 and the number of revolution of asterism (sidereal days) is 1582237500. (Sidhyadhi vryddhi Tantra 1-7,8).

These values agree with the modern values. From this the number of civil days and sidereal days, in a civil year and a solar year can be calculated. The above numbers are divided by 4320000 (number of years in a Mahayuga). We get 365.2568 and 366.2568 as the number of civil days and sidereal days in the above years.

Sidereal and civil days are clearly defined by Vateswara, in Vateswara siddhanta:

प्राणेन लिप्ता समुदेति पूर्वे भुजेऽपरेऽस्तं व्रजति ग्रहश्च ।
स्वभुक्तिलिप्तायुतचक्रलिप्ताभोगस्समं तेन यतो जवत्वम् ।।

Pranena liptha samudethi poorve bhuje apare astham vrajathi grahascha sivabhukthi lpthaayutha chakralipthaabhogassamam thena yathojaathvam

The circle of asterism rises on the eastern horizon and sets on the western horizon at the rate of one minute of an arc in one respiration. A planet, on the other hand on account of its own motion has its diurnal motion equal to the minutes in circles + minutes of its own motion and so it rises & after so many respirations have passed. A Star rise after every 24 sidereal hrs. The Sun rises after every 24 sidereal hrs + (59 min 8 secs)/15 (Vateswara siddhanta 1 (7)-24).

That is 24 hrs. 3 min. 56 secs. The same value is given by modern astronomy for mean solar day. Definition for the sidereal and civil days are given by Vateswara in numerical values.

सूर्यभुक्तिकलिकासमासुभिः सावनो भदिवसाद्यतोऽधिकः
तेन सूर्य परिवर्तसावना भोदयाः स्युरिनपर्ययैयुताः ।।

Soorya bhukthi kalikaasamasubhi: savano bhadivasaadyatho f adhika: thena soorya parivarthasarvanaa bhodayaa: syurinaparyayaiyuthaa:

Since a civil day exceeds a sidereal day by as many 'asus' as there are minutes in the Sun's daily motion, the number of rising of a star plus the number of revolution of earth (Sun) is equal to the number of civil days i.e the duration of a civil day = duration of a sidereal day + 59 and 8/60 asus.

Civil days in a Yuga = rising of star in a Yuga + revolution of (earth) Sun in a Yuga which is equal to sidereal days + Sun's (earth's) revolutions. (Vateswara siddhanta - Gola 4-1).

Vateswara has given the number of terrestrial civil days in a Yuga:

जलधररसपञ्चक्ष्माभृदग्निद्विपक्षद्विघनशरशशाङ्काभोदया: स्युर्युगे/मी
निजभगणविहीना: खेचरस्योदया: प्राक् दिनकृदुदयराशि: सावनो भूदिनाख्य: ॥

Jaladhara rasapanchakshma bhrudagnidvipaksha
drighanasara sasaankaa bhodayaa:
syuryugeamee nijabhagana viheenaa: khecharasyodayaa:
praak dinakrududayaraasi: saavano bhoodinaakhya:

1582237560 is the rising of the asterism in a Yuga. This diminished by the revolution of a planet gives the number of rising of the east of that planet in a Yuga. Number of rising of Sun is called terrestrial civil days. (Vateswara siddhanta 1(2) - 1)

The number of revolution of asterisms is 1582237500, one year = 366.2586 civil days, this minus the revolution of earth in Mahayuga i.e 4320000 gives the sidereal days or the rising of the Sun: 1577917500 ÷ 4320000 one year = 365.2586.

Achyutha in Sphutanirnaya has given the number of days in a Kalpa in which one Kalpa is 14 Manwantharas. Each Manwantara is 72 mahayugas and each of that comprise of 4320000 years. The number of days in a Kalpa is thus given in bhootha sakhya as :

...भगणा धात्रीदिनौघा नवेन्दूभ्रात्यष्टिशराद्रिचन्द्रनवशैलाद्रीषुशीतांशव: ।।

..... *Bhaganaa dhaathree dinowghaa navendoobhraa thyashtisaraadrichandranawa sailaadreeshu seethaam sava:*

Number of days in a Kalpa is 1577917517019 (Sphutanirnaya Tantra of Achyutha 5-7)

Observations on the earth in the space : It is said that Indians believed the earth is resting on some support. Puranas are quoted to explain this superstition. However the astronomical observation on this subject was different. The astronomical books never said that earth was supported on the tortoise, sea or on mountains. It was well known that on all the surface of the earth people or living beings exist. No surface of the earth was utilised for getting supported on. Thus says Lallacharya:

भूपरिधिदलान्तरगा: के नृच्छाये यथा तथोर्ध्वमध: ।
सर्वेषामुपरि नभस्तदधो भूरुपरि ते भूमे: ।।

Bhooparidhidalaantharagaa: ke nrucchaaye yathaa thathordhvamadha: sarveshaamupari nabhasthadadho bhoorupari the bhoome:

Those who are at a distance, half the earth's circumference from each other, are at antipodes, just as the men are standing on the bank of a river and his reflection on the water, the sky is above and all the globe earth beneath it. Inhabitants are on all the surface of the earth. (Sishyadhi vruddhi Tantra 17-5).

How the earth is suspended in the space has been explained:

भवनभावतुलोपतुलास्वलं स्थितवती च यथा गृहगोधिका ।
समभिधावति नूनमनाकुला कुवलयस्य तथैव जनोऽधि

Bhavanabhaava thulopathulaasvalam sthithavathee cha yathaa gruhagodhikaa samabhidhaavathi noona manaakulaa kuvalasyasya thathaiva janoadha

Just as a house lizard resting on the ceiling of a house, runs

forward without hesitations, so do people on the bottom side of the sphere earth (Sishyadhi vruddhi Tantra 17-7)

This line gives a proof that earth attracts everything and holds them on its surface. It is further that the earth has no surface to fall or to get supported on.

शिखिशिखा गगनं गुरुमेदिनीं व्रजति यद्वदिहास्मदवस्थितौ ।
तलगतामितरेष्वपि तत्तथा न तलमस्ति भुवः क्व पतत्वसौ ।।

Sikhisikhaa gaganam gurumedhineem vrajathi yadvadi haasmadaasthithow thalagathaa mithareshvapi thattha thaa na thalamasthi bhuva: kva pathathvasow

In our daily life we see the flame of fire goes towards the sky and heavy weight falls towards the earth. In the same manner everything that has a surface to reach makes for it. The earth however has no such surface, where can it fall? (Sishyadhi vruddhi Tantra 17-8)

It has been explained clearly that gravitational pull of the celestial bodies keep earth and other planets in their respective positions. Force of attraction of magnet was compared with the gravitational force of planets and other celestial bodies.

मध्येऽयस्कान्तानां तथा स्थितोऽयोगुडः खमध्यस्थः
तद्वदनाधारोऽपि हि सर्वाधारो महीगोलः ।।

Madhye ayaskaanthaanaam thathaasthithoayogudda: khamadhyastha: thadvadanaadhaaroapi hi sarvaadhaaro maheegola:

Just as a ball of iron which when places amidst pieces of magnet remains suspended in space, in the same manner the spherical earth remains in space unsupported (Sishyadhi vruddhi Tantra 17-2)

Lallacharya used specifically magnets and iron ball. This is a clear proof of the knowledge on the gravitational force of

attraction among the planets and other celestial bodies. Similar explanation has been given, on the influence of other celestial bodies, by Vateswara:

लोहकान्तपरिवेष्टितो यथा खस्थितोऽपि न पतत्ययोगुडः
तद्वदेव भखगैर्निवारिता नो पतत्यविधृताऽपि भूरियम् ।।

Lohakaanthapariveshtitho yathaa khasthithoapi na pathathyayogudda:
thadvadeva bhakhagairnivaarithaa no pathathyavidhruthaapi bhooriyam

Just as an iron ball surrounded by pieces of magnet does not fall though standing in the sky in the same way, this earth though supportless does not fall as it is prevented by the stars and planets (Vateswara siddhanta - Gola 5-2).

Similar explanations are also given in Siddhanta sekhara (15-22)by Sripathy and in Siddhanta siromani (II, iii.2 (a-c) by Bhaskaracharya II. Definition of gravitation for attraction of celestial bodies is same as the modern concept. Vateswara questions the validity of the puranas on the explanation of the earth's position in space:

शेषकूर्मनगकुञ्जरादयो धारयन्त्यविधृताः कथं नु ते ।
तेषु शक्तिमवधारयन्कथं भूगतां नु परिवर्जयन्त्यलम् ।।

Seshakoormanagakunjaraadayo dhaarayanthya vidhruthaa:
katham nu the theshu sakthi mavadhaarayan katham
bhoogathaam nu pari varjayanthyalam

If earth is supported by Sesha (a serpent) or tortoise, mountains, or elephants, how do they (these things) stand supportless in the space? (Vateswara siddhanta Gola 5-5)

Gravity and the related phenomena of the earth have been thus attributed by him:

उष्णताऽर्कशीखिनोरपां गतिः शीतता शिशिरगोरमार्धवम्
अश्मनां सुनियतं निसर्गतः खस्थितिः खलु महीवियत्सदाम् ।।

Ushnathaarkasikhinorapaam gathi: seethathaa
sisiraghora maardhavam asmanaam suniyatham
nisargatha: khasthithi: khalu mahee viyathasadaam

Just as warmth is the natural property of the Sun and fire, motion that of water, coolness that of moon, and hardness that of stone, in the same way to remain suspended in space is undoubtedly the natural attribute of the earth. (Vateswara siddhanta - Gola 5-9)

Gravity and Earth : Other than what is mentioned above as the gravitational force, Bhaskaracharya II has specifically defined the gravity. He has also given the reason for the solid bodies falling from the sky to the earth. This description was given about 400 years before Sir Isaac Newton, who according to modern astronomy discovered gravity of earth, while watching the falling of an apple !.

आकृष्टिशक्तिश्च मही तया यत् खस्थं गुरु स्वाभिमुखं स्वशक्त्या ।
आकृष्यते तत्पततीव भाति समे समन्तात् क्व पतत्ययं खे ।।

Aakrushti sakthischa mahee thayaa yath khastham guru
svaabhimukham svasakthyaa aakrushyathe thathpathatheeva
bhaathi same samanthaath kva pathathyayam khe:

This earth attracts whatever solid materials are in the space, by her own force of attraction towards her (earth). All those subjected to this attractional force fall, to the earth. Due to equal force of attraction among the celestial bodies, where can each among them fall? (Siddhanta siromani - Bhuvanakosham 6)

This gives the correct explanation similar to that given by Newton.

Four quadrants of the earth: Ancient Indians have explained the shape of the earth and have commented on other scientific concepts related with the earth. They had in depth knowledge

on the geography of the earth and its various phenomena which are attributed to the earth due to its spherical nature. These extrapolations and observations were also done correctly. The observers standing in four quadrants of the earth see the Sun in different ways and it is explained precisely by Aryabhatta and others:

उदयो यो लङ्कायां सोऽस्तमय: सवितुरेव सिद्धपुरे ।
मध्याह्नो यवकोट्यां रोमकविषयेऽर्धरात्रं स्यात् ।।

Udayo yo lankaayaam soasthamayo: savithureva siddhapure
madhyahno yavakotyaam romaka vishaye ardharaathramsyaath

When it is Sunrise in Lanka, the same Sun sets in Siddhapura. It is noon in Yavakoti and midnight in Romaka (Aryabhateeyam 4-13).

This is the finest example to show the four cities known, have four different time and vision of the Sun. It is said that Siddhapura may be the present Guatemala of South America, which is nearly on the opposite side of Ujjaini. Romaka is modern Rome and Yavakoti can be a place 90 east of Lanka i.e four cities exited in four quadrants of earth. Lallacharya gives similar explanation.

उदयमेति यदा पुरि रक्षसां व्रजति सिद्धपुरेस्तमयं तदा ।
दिनदलं यमकोटिपुरे रवी रजनिमध्यमुपैति च रोमके ।

Udyamethi yadaa puri rakshasaam vrajathi siddhapure asthamayam thadaa
dinadalam yamakoti pure ravee rajani madhya mupaithi cha romake

When the Sun rises in Lanka, the abode of Rakshasa it sets simultaneously in Siddhapura. When it is midday in Yavakoti, it is midnight in Romaka (Sishyadhi vruddhi Tantra17-12)

Lallacharya gives position of above cities on the earth and included in the list the north and south poles. This definition is

yet another example of excellence in astronomy. The word 'transversely' carries a lot of modern geographical knowledge.

एतेजलस्थलथो मेरु: स्थलगोऽसुरालयो जलग: ।
कुपरिधिपादान्तरगा मन्यन्ते तिर्यगन्योन्यं ।।

Etheyalasthalatho meru: sthalago asuraalayo jalaga:
kuparidhi paadaantharagaa: manyanthe thiryaganyonyam

These cities mentioned earlier are on lands. Meru (north pole) is on the land and south pole the abode of demons is surrounded by water. All these six places are believed to be situated transversely at a distance one fourth of the earth's circumference, each from the next (Sishyadhi vruddhi Tanra 17-4) i.e The four cities separated by 90° and south and north poles of earth together makes six locations.

Brahmagupta has repeated Aryabhatta's lines (Brahmasphuta siddhanta 11-12). Varahamihira gives yet another set of cities in Panchasiddhantika. Even though he was contemporary of Aryabhatta, he refused different cities.

When there was no communication facilities similar to what is present now, the position of these cites could be understood by mere extrapolation of the observations. And that was done with perfect scientific approach and accuracy during a period when the people of different country thought that the earth was plane.

North pole and south pole of the earth: Two poles of earth, Arctic and Antarctica circle were also well defined. Unlike other explanations these two places were correlated with a little puranic knowledge also. North pole is called Meru and is the abode of gods and South pole, the Bhadavamukha is abode of demons. However these appear to be synonyms used for convenience and no puranic stories are mixed with geographical information and descriptions.

देवा: पश्यन्ति भगोलार्धमुदङ्मेरुसंस्थिता: सव्यम् ।
अर्धं त्वपसव्यगतं दक्षिणबडवामुखे प्रेता: ।।

Devaa: pasyanthi bhagolaardhamudangmeru samsthithaa: savyam ardhram thvapasavyagatham dakshinabadavaamukhe prethaa:

Gods (those who are living there) living in the north at the Meru mountain (north pole) see one half of the Bhagola as revolving from left to right (clockwise), the demons living in the south at Bhadavamugha - (south pole) on the other hand see the other half as revolving from right to left in the anticlockwise, when they face each other (Aryabhateeya 4-16).

When the people of north and south pole face each other, movement of the celestial bodies can be from right to left and vice versa. One half of an year, the north pole and other half, in south pole very little sun light is available and practically the Sun is out of sight.

रविवर्षार्धम् देवा पश्यन्त्युदितं रविं तथा प्रेता: ।।

Ravivarshaardham devaa pasyanthyuditham ravim thathaa prethaa:

Gods (those who are in the north pole) see the Sun after it has risen for half a solar year, so are done by the demons in the other half of the year (those who are in the south pole). (Aryabhateeyam 4-17a).

These lines are given by Aryabhatta after defining the north and south poles. Further explanations of these parts of earth are given: In the north pole the Sun is seen far lower. East/west directions cannot be designated looking into the Sun. Hence theoretically there is no direction in the poles. So also says the modern science.

मेरोर्तं दिग्विभागो यस्मात् प्राची न भास्करात्तस्मिन् ।
उदयते यावद्दिवं पर्येतीव सुन्दरी तावत् ।।

अणुमात्रदर्शनात् प्राग्विभाग इति चेत् समार्धमित्वा तु ।
तस्मिन्नेवास्तमये किं वा प्राची भवेत् त्वपरा ।।

Merorthadigvibhaago yasmaath praachee na bhaaskaraathbhasmin udayathe yaavaddivam paryetheeva sundaree thaavath anumaathra darsanaath praagvibhaaga ithi cheth samaardhamithvaa thu thasminnevaasthamaye kim vaa praachee bhavethvaparaa

There is no distinction of direction at north pole, because east and west cannot be determined there, with the help of Sunrise and set, for as long as the Sun stays risen, it goes round the sky like a beautiful damesel. It is argued that from the point where the Sun just appeared above the horizon east is determined, as the Sun sets at the same point after half an year. Can this east be west also? (Panchasiddhantika 15-11,12)

This conclusion appears as though it is obtained by direct observation by the scientist from north pole. However it is surprising that the knowledge on extrapolation of the known facts to get the unknown facts is brilliant.

Descriptions of arctic circle is given by Aryabhatta as a place that covered with ice. The golden colour of the circle is also metioned. This appears to be a knowledge impossible to be gathered without direct experience or intuition. It is wonderful because without seeing the lcoation(?) he has commented about north pole which is in full agreement with modern geography. It is a fact that the golden colour is due to the reflection of Sun's long range rays on the ice. Because the Sun never comes above the poles, only long ranging golden sunrays fall on surface of ice showing permanently the golden colour. Exact words of Aryabhatta gives this explanation on arctic circle with details.

मेरुर्योजनमात्रः प्रभाकरो हिमवता परिक्षिप्तः ।
नन्दनवनस्य मध्ये रत्नमयः सर्वतो वृत्तः ।।

Meruryojanamaathra: prabhaakaro himavathaa parikshipta: nandanavanasya madhye ratna maya: sarvathovruttha:

The arctic circle is known as Meru, it is only 1 yojana in area, covered with ice, it is the midst of Nandana vana and full of gold (ratna) and it is perfectly circular (Aryabhateeyam 4-11)

The experience of an observer when he moves from north pole down is mentioned by Lallacharya. Reason has been mathematically given using the concept of latitude changes during the movement of the observer, from north down to the equator.

सुरासुराणां विषयाद् यतोयतो यथा यथा गच्छति यश्च कश्चन ।
स तत्र तत्रोन्नतमृक्षमण्डलं तथा तथा पश्यति खान्नतं ध्रुवम् ।।
भवेद् खमध्याद् ध्रुवकस्य या नतिर्भपञ्चरस्योन्नतिरेव साथवा ।
स्वलम्बभागा: स्वषडंशगोहता नरापसार: फलयोजनैर्भवेत् ।।

Surasuraanaam vishayaad yatho yatho yathaayathaa gacchathi yascha kaschana sa thathra thathronna thamruksha mandalam thathaa thathaa pasyathi khaannatham dhruvam bhaveth khamadhyaath dhruvakasya yaa nathirbhapancharasyonaathi reva saathavaa svalambabhaagaa: svashadamsagohathaa: naraapasaara: phalayojanairbhaveth

Whosoever proceeds from the north pole or south pole towards the equator, he would observe the celestial sphere gradually rising and celestial pole more and more depressed from his zenith. Depression of the pole from the zenith being the same as the elevation of the celestial sphere, the number of degrees in colatidue of a place multiplied by 9 1/6 gives the corresponding length in yojana (Sishyadhi vruddhi Tantra 18-6,7)

Varahamihiara has mentioned the angle of vision of the Sun for the observer in north pole and described the period of longest duration of days in the north pole:

मिथुनान्ते च कुवृत्तादंशचतुर्विंशति विहायोच्चैः।
भ्रमति हि रविरमराणां समोपरिष्टात्तदावन्त्याम् ।।

Mithunaanthe cha kuvrutthadamsa chathurvimsathi vihaayochhai
bramathi: hiraviramaraanaam samoparishtaatthadaavanthyaam

During the month of Mithuna (June-July) the Sun is seen at 24° to those who are at the north pole. Those who are at Avanti can see the Sun above their head (Panchasiddhantika 13-10)

In north pole the day time is maximum during Mesha, Vrushabha, and Mithuna months, wheras from the month of Karkitaka, night increases.

षष्टिर्नाड्यस्तस्मिन् सकृदुदितो दृश्यते दिवसनाथः ।
परतः परतो बहुतरमाषण्मासादिति सुमेरौ ।।

Shashtirnaadyasthasmin sakrududithо drusyathe divasanaatha:
paratha: paratho bhahutharamaashanmaasaadithi sumerow

At that latitude (north pole), the Sun can be visible even throughout a day, north and north of this place, the Sun may not set more than one day, unlike at the north pole it does not set for 6 months at a stretch (Panchasiddhantika 13-22).

Meridian equivalent to Greenwich line: In modern astronomy the time is fixed according to the Greenwich meridian line. Degree measurement of longitude are counted from this imaginary line. In ancient India meridian line was well defined. It was the basis of longitude measurement, as Greenwich line is adopted. Meridian line is defined here as follows:

पुर्वापरमधऊर्ध्वं मण्डलमथ दक्षिणोत्तरं चैव ।
क्षितिजं समपार्श्वस्थं भानां यत्रोदयास्तमयौ ।।

Poorvaaparmadha oordhvam mandalamatha dakshinotthаram chaiva
kshithijam samapaarsvastham bhaanaam yathrodayaasthamayow

The vertical circle which passes through the east and west

points is the prime vertical, and the vertical circle passing through the north and south points is the meridian. The circle which goes by the side of the above meridian circles and on which the stars rise and set is the horizon (Aryabhateeyam 4-18)

The meridian line is then defined clearly. Bhaskaracharya I gives the cities which are situated in the meridian. These cities were famous 1500 years ago, during the period of the author.

लङ्कावात्स्यपुरावन्तीस्थानेश्वरसुरालयान् ।
अवगाह्य स्थिता रेखा देशान्तरविधायिनी ।।

Lankaa vaatsyapuraavanthe sthaanesvara suraalayaan
avagaahya sthithaa rekha desaanthara vidhaayinee

That line which passes through Lanka, Vatsyapura, Avanti, Himalaya, and North pole (Suraalaya) is known as the international meridian line (Laghubhaskareeyam 1-23)

Bhaskaracharya I, Varahamihira, Lallacharya and many others have used the words 'international line' (Desantara) which gives the same concept of the international prime meridian passing through Greenwich. Here Lanka is considered exactly 1/4 of the circle of the earth (passing through north south poles) and hence considered in equator. Equator meets at this point with the meridian.

स्थलजलमध्याल्लङ्का भूकक्ष्याया भवेच्चतुर्भागे ।
उज्जयिनी लङ्कायाः तच्चतुरंशे समोत्तरतः ।।

Sthalajalamadhyaallanka bhookashyaayaa: bhavedchathur bhaage
ujjayinee lankaayaa: thachhaathuramse samottharatha:

From the centre of the land and water at a distance of one quarter of the earth's circumference lies Lanka, and from Lanka at a distance of one fourth, there of exactly northwards lies Ujjaini (Aryabhateeyam 4-14).

Ujjaini is 24° north of Lanka where Lanka is at 0° (equator). Since Lanka falls outside Indian geographical boundary and Ujjaini is in the meridian itself, ancient Indians took Avanti i.e Ujjaini, as the reference line of meridian. From equator point of Lanka to north pole it is 90°, one fourth of that angle is 22° 30′. Hence Ujjaini is at this longitude. As per modern astronomical calculation Ujjaini is at 23°!, the observation was accurate. Lanka has also been given importance as a place where both longitude and latitude are zero as described:

"Midday Lanka is the same that at Ujjaini which is north of Lanka on the same longitude. But day time duration is different except when the Sun is in equator " (Panchasiddhantika 13-17)

More information is given on Lanka by Varahamihira. From this the concept of equator also becomes very clear as it is defined now: The position of Lanka is very close to the equator. Here the globe is exactly equal on either sides. 30 natika day and 30 natika night are possible here. (Panchasiddhantika 12-29)

लङ्कोत्तरतोऽवन्ति भूपरिधेः पञ्चदशभागे ।।

Lankottharatho avanthi bhuparidhe panchadasabhaage

Avanti is to the north of Lanka at a distance of one fifteenth of the circumference of earth (Brahmasphuta and siddhantha 21-9)

1/15 th of 360° is 24° which is close to the longitude of Ujjaini (modern value 23° 9′).

When the position of a city is located with reference to another on the basis of meridian, the knowledge of latitude and longitude should be very sharp. Hence the usage, as one fifteenth north to Lanka, might have been estimated only on the basis of the celestial bodies. That calculation was, done accurately. Drawing the prime meridian line is literally explained by Sankaranarayana in his commentary to Laghubhaskareeyam. A

number of towns which fall in the prime meridian has also been cited by him.

लङ्कायामेकं शङ्कुकीलं प्रतिष्ठाप्य तेनैकं सूत्राग्रं बद्ध्वा
पुनर्मेरोरुपरि तदग्रमन्यत् बद्ध्वा यथा यथा दृश्यते तद्वत्
भूमावपि काचिद्रेखा लङ्कातः खरपुरतः मेरुमस्तकानवगाह्य
स्थिता सा पुनरत्रा-देशान्तरविधायिनी स्यात् ।।

Lankaayaamekam sankukeelam prathishtaapya thenaikam soothraagram baddhvaa punarmerorupari thadagramanyath baddhvda yathaayathaa drusyathe thadvath bhoomaavapi kaachidrekha lankaatha: kharapuratha..... merumasthakaanavagaahya sthithaa saa punarathra-desaanthara vidhaayini syaath

Fix a pole in Lanka, tie a thread on that, take the other end to the North pole, tie it there also, then one can see the line of the thread passing through Lanka, Khara city, Kharapuri,..... and so many other countries upto the top of Meru. This is the international meridian line (Sankaranarayana on Laghubhaskareeya I - 23)

He has mentioned the importance of this line. Countries on either side of this line is marked as east or west, depending on where they are situated with respect to this meridian. At present explanations are given in longitudinal degree E or W, marked whether they are east or west of Greenwich international line. Exactly similar explanation has been given by Sankaranaryana:

अथेदानीं समरेखायाः पूर्वेण स्थितोऽहमथवा पश्चिमेन स्थितोऽहमिति पूर्वमथ
पुनस्तस्यामेव रेखायां स्थितोहमित्युक्तवान् ।

Athedaaneem samarekhaayaa: poorvena sthitho ahamathavaa paschimena sthithoahamithi pooravmatha punasthasyaameva rekhaayaam sthithohamithyukthavaan.

When comparing with the meridian, people say that I am

standing on the east of it or west of it. Some people can also say that I am standing on the line. (Sankaranaryana in Laghubhaskareeyam 1-29)

तस्या: पूर्वपश्चिमदिक्स्थानां स्वदेशा रेखान्तरयोजनै: एतत्कर्तव्यमेव !

Thasyaa: poorvapaschimadiksthaanaam svadesaa
rekhaantharayojanai: ethath karthavyameva

This (meridian) has east and west. On either sides of the line, countries are marked based on this meridian, and the distance in yojana, that is the way for finding the longitude (Sankaranarayana in Laghybhaskareeya 1-23)

Cities on Meridian: Cities which fall on the meridian line were cited in Laghubhaskareeya. It is interesting to note that names of more cities situated on this line are given by Bhaskaracharya I in Mahabhaskareeya (2-12).

लङ्कात: खरनगरं सितोडगेहं पाणाटो मिसितपुरी तथा तपर्णी
उत्तुङ्गस्सितवरनामधेयशैलोलक्ष्मीवतपुरमपि वात्स्यगुल्मसंज्ञम्
विख्याता वननगरी तथाह्यवन्ती स्थानेशो मुदितजनस्तथा च मेरु:
अध्वाख्य: करणविधिस्तु मध्यमानामेतेषु प्रतिवसतां न विद्यते स:

Lankaatha kharanagaram sithodageham paanaato mishitha puri
thathaa thaparnee utthunga sithavara naamadheya sailo lakshmee
vathapuramapi vatsyaguthma samjnam vikhyaathaa vananagaree
thathaahyavanthee sthaaneso muditha janasthathaa cha meru:
adhvaakhya: karana vidhisthu madhyamaanaametheshu
prathivasathaam na vidhyathe sa:

From Lanka towards north on the meridian line we have the following places, Kharanagara, Sitorugheha, Panata, Misithapuri, Taparni, the lofty mountain called Sitavara, the wealthy town called Vatsyagulma, the well known Vananagari, Avanti, Sthanesa and then Meru which is inhabited by happy

people, for those who reside there correction of longitude does not matter.

The same lines are also repeated with minor modifications by Lallacharya, Vateswara and Sripathy. The in-depth knowledge about the meridian and the places through which the meridian passes has been accurately defined. Another important point given is that, for the north pole there, cannot be any longitudinal definition or correction. In the above line Bhaskaracharya has mentioned this information also.

Latitude and lines of latitudes: Parallel to the equator, lines are drawn towards the north and south pole. It is noted in degree north for lines drawn above the equator, and degree south for lines drawn below the equator. Cities and places are located based on latitude and longitude. Just as Greenwich line is taken as the standard of reference now, in ancient India, Ujjaini line was used as the meridian for the reference to longitudes. Similarly latitude are also explained:

पूर्वापरादिग्लग्नं क्षितिजादक्षाग्रयोश्च लग्नं यत् !

Pooraaparaadiglagnam kshithijaadakshaagrayoscha lagnam yath

The circle which passes through the east and the west points and meets at distances equal to latitudes from the horizon.....(Aryabhateeyam 4-19a)

There are circular imaginary lines drawn from east to west. Each line has been referred in degrees. Above the equator its parallel lines are drawn with total angular space of 90°. Similarly below the equator, to south pole also, are divided into 90° latitude lines. In a globe it is clearly marked. Marking in the globe has been described: The latitude lines are marked in degrees on the spherical surface of golayantra (globe) (Brahmasphuta siddhanta 21-50- quoted earlier)

In ancient India, the line passing through Lanka was also

taken as the reference for the latitude, where the equator meets the meridian. At this point both longitude and latitude are zero. Lallacharya says thus:

लङ्कावृत्ते मध्यस्थिते भुवो यत्कुजं तदुद्वृत्तम् तेन न तत्र चरदलं सदा समत्वं च दिवसनिशो:

Lankaavruthe madhyasthithe bhuvo yathkujam thaduddvruthham thena na thathra charadalam sadaa samathvam cha divasaniso:

Along the latitude in Lanka, the line passes through the middle of the earth here the day and night can be equal. (Sidhyadhi vruddhi Tantra 16-12)

Vateswara has given exactly the measurement of latitude for some places and also the importance of those places:

अक्षश्रवणाभ्यस्ता त्रिज्या वा द्वादशोद्धृता भवति ।
अयनान्तगे रवाविह दिनदलभानौ दशपुरे च ।।

Akshasravanaabhyasthaa thrijya vaa dvaadaso ddhruthaa bhavathi ayanaanthage ravaaviha dinadala bhanow dasapure cha

But here at Anandapura and also at Dasapura, when the Sun is at the summer solstice, the same dhruti of the Sun of the midday is equal to the radius multiplied by the Palakarna and divided by 12. (Vateswara siddhanta 3 (9)-48)

Dasapura is the modern Mandasor in Madhyapradesh whose latitude is 24° 03 i.e equal to the declination of the earth. Vateswara could locate a city whose latitude is exactly equal to the declination of the earth. It is a proof of the efficiency of astronomical calculation and knowledge.

Calculation of latitude of places: Vateswara has given the following method for the calculation of latitude by connecting other parameters of the place.

पलभार्कवर्गगुणितौ त्रिज्यावर्गौ पलश्रवणकृत्या भक्तावावाप्तमूले
पलजीवालम्बजीवे स्त:

Palabhaarkavargagunithow thrijyaavargow palasravana
kruthyaa bhakthaavavaaptha moole palayeevaalmbageee stha:

Multiply the square of radius by the square of the palabha (palabha is the equinoctial midday shadow) and by the square of 12 and divide each product by the square of Palakarna (hypotenuse of the equinoctial midday shadow). Square root of the resulting quotients are the Rsine of the latitude and Rsine of the colatitude. (Vateswara siddhanta 3(2)-1)

Vateswara has described twenty methods for the determination of latitude and colatitude of places using various parameters.

In Mahabhaskareeya, Bhaskara I has given mathematical problems for the calculation of latitude from other parameters.

भास्करे मिथुनपर्यवसाने शर्वरी त्रिगुणसप्तघटी स्यात्
अक्षचापगणितं वद तस्मिन् लम्बकेन सहितं विगण्य ।

Bhaskare mithunaparyavasaane sarvaree thriguna
sapthaghatee syaath akshachaapaganitham
vada thasmin lambakena sahitham viganya

The Sun being at the end of the sign Gemini, the length of the night is 21 ghatis calculate and give out the latitude and colatitude of the place. (Assuming that the obliquity of the ecliptic as 23½°. (The latitude comes to 61° 45') (Mahabhaskareeya 8-25)

Parallax in latitude : Whenever celestial bodies have been located based on the latitude, parallax exists. This is known as the nati in ancient Indian astronomy. The condition for parallax has been described by Vateswara:

विक्रिभक्रान्तिभवांशगणैर्यदाऽक्षो न समो हि तदाऽवनतिर्भवेत्

Vithribha kraanthi bhavaamsaganairyadaa
aksho na samo hi thadaa avanathirbhaveth

When the local latitudes are not equal to the degrees of the northern declination of the central ecliptic point, only then nati, i.e parallax in latitude, exists. (Vateswara Siddhanta 5(1)-4a).

The Parallax in latitude can be calculated as follows:

मध्यमौ क्षितिदलाहतौ हृतौ मध्यमश्रवणयोजनैर्नती ।।

Madhyamou kshithidalaahathow hruthow
madhyamasravanayojanairnathee

Multiply mean drukshepa (Rsine of the Zenith distance of central ecliptic point) of the Sun and Moon by the earth's semi diameter and divide by their own mean distances is yojanas. Then are obtained the parallaxes in latitude (Vateswara Siddhanta (5(2)-5)

The Parallax in latitude can be calculated from the above eqaution for the Sun or the Moon. Another method given by him related with motions of these celestial bodies:

मध्यभुक्तितताडितौ (च वा) दृक्षेपौ तिथिभाग ताडितौ
नती त्रिगुणभाजितौ क्रमात् (सूर्यचन्द्रमसौ: कलागते)

Madhyabhukthithaadithow cha vaa drukshepow thithibhaaga thaadithow
nathee thriguṇabhaajithow kramaath sooryachandramasow: kalaagathe

The Rsine of the zenith distance of the meridian ecliptic point multiplied by 21 and by the difference of the true motion of the Sun and Moon and divided by the difference of their mean motion and the radius gives the parallax in latitude (Sishyadh vruddhi Tantra 7-8)

Sankvarga an astronomical observation and the latitude: Sankvarga of a heavenly body is the distance of the projections of the body on the plane of the horizon from its rising and setting

line. Its relation with latitude is given by Bhaskaracharya I.

अक्षजीवाहत: शङ्कुरिष्टकालसमुद्भव: ।
भाजितो लम्बकेनाथ शङ्क्वग्रं नित्यदक्षिणं ।।
Akshajeevaahatha: sankurishtakaalasamudbhava:
bhaajitho lambakenaatha sankvagram nithyadakshinam

R sine of the altitude for a desired time multiplied by the Rsine of the latitude and divided by Rsine of the colatitude is the Sankvarga. This projection accordingly is always south of the rising - setting line. (Mahabhaskareeya 3-54)

Calculation of distance of the place from Meridian using latitude: Methods were well known to calculate the distance of the place from meridian. The circumference of the earth has been calculated as 3300 yojanas and total degree 360, for calculation.

कुमध्यरेखाविषयं स्वपत्तनादवस्थितं तिर्यगवेत्य योजनम् ।
स्वकीयतत्रत्यपलांशकान्तरं खरामरघ्नं विभजेत् खषड्गुणै: ।।
पलकृतिरथ यावद्वर्गतो योजनानां पदमृजु निजधामक्ष्मार्धयोरन्तरं स्यात् ।
ग्रहगतिहतमेतत् स्पष्टभूवृत्तभक्तं धनमृणमपरैन्द्रयोर्लिप्तिकादि क्षमार्धात् ।।
Kumadhyarekhaa vishayam svapatthanaadavasthitham
thiryagavethya yojanam svakeeya thathra thya palaamsakaantharam
khakharaamaghnam vibhajeth ghashadgunai: palakruthi ratha
yaavadvargatho yojanaanaam padamrujnijadhaamakshmaardha
yorantharam syaath grahagathihathamethath spashta boovruthha
bhaktham dhanamruna maparaindrayorlipthikaardi kshamaardhaath

Ascertain the shortest distance in yojanas between a place on the meridian line and the observer's station. Multiply the degrees in the difference of latitudes of these two places by 3300 and divide by 360. Square the result, subtract it from the square of yojana. Find the square root. This is the distance between the meridian lines passing through the two places. Multiply it by the

mean daily motion of a planet and divide by the correct circumference of the earth. The result in minutes should be added (to the longitude) for places to the west of the meridian line of Lanka and subtracted for places to the east. (Sishyadhi vruddhi Tantra 1-44, 45)

Correction for longitude : Manjulacharya gives the corrections for longitude of places with reference to the prime meridian passing through Avanti:

अवन्तीसमयाम्योदग्रेख पूर्वापराद्वना ।
ग्रहगत्यंशषष्ट्यंशो हतो लिप्तास्वर्णं धनं ।।

Avanthee samayaamyodagrekha poorvaaparaadvanaa
grahagathyamsashathtyamso hatho lipthaasvarunam dhanam

By the distance in yojanas of the local place, east or west of the meridian of Avanti, multiply the 60th part of the degrees of the planet's daily motion; subtract the resulting minutes from or add them to the longitude of the planet (according to the place is east/west of meridian of Avanti (Laghumanasa 4-3)

I.e Longitude correction = + or -. D x m/60 min. D = distance in yojanas from Avanti, m = planets daily motion in degrees;

The circumference of the earth is taken here as 3600 yojanas by Manjulacharya. (However other astronomers have correctly mentioned that the circumference of the earth is 3300 yojanas). Addition or subtration is to be done depending upon, the place is in the east or west of the meridian. Even though the calculation method is correct, the circumference of the earth was wrongly taken as 3600 yojanas.

Bhaskaracharya has mentioned that by knowing the degree values of latitude and longitude one can easily find out the distance of the place. Same methodology is adopted now.

समरेखास्वदेशाक्ष विश्लेषान्तरसङ्गुणं वृत्तं स्वदेशजो भूमेर्बाहुश्चक्राशकोद्धृतं ।।

Samarekha svadesaaksha visleshanthara sangunam vrutttham swadesajo bhumerbaahuscha kraamsakoddhrutham

The line drawn parallel (same rekha) to the longitude/ latitude in any country (my country - swadesha) and finding the difference from the meridian (angle) and doing the calculation, one can find out the distance between places on earth. And extrapolating this, one can get the celestial body positions in the sky also. (Laghubhaskareeyam 1-25)

Sankaranarayana further clarifies the above statement in his commentary:

समरेखायां खरनगरप्रभृतिषु नगरेष्वेकस्मिन् ।
अक्षप्रमाणमवगम्य स्वदेशजाक्षेण विश्लेषं कृत्वा यो विशेष
स तावदिह समरेखा स्वदेशाक्षविश्लेषान्तरमित्युच्यते ।

Samarekhaayaam kharanagara prabhruthishu nagareshvekasmin aksha pramaanamavagamya svadesajaakshena visleshaam kruthva yo visesha sa thaavadiha samarekha svadesaaksha visleshaantharamithyuchyathe.

By knowing the longitude of a place and by comparing it with Khara Nagara, or any country that falls in the meridian, and by subtracting it, one can get the parallel line for calculating the difference in longitude of any country (Sankaranarayana's commentary to Laghubhaskareeyam 1-25)

Bhaskaracharya puts forth methods to calculate the longitudes and latitudes of the place from other parameters:

अष्टौलवा: षोडशलिप्तिकोना: पलप्रमाणं प्रवदन्तियस्मिन् ।
छायादिनार्धेऽर्धचतुर्थसंख्या तत्राशु वाच्य: सविता नभस्थ: ।।

Ashtow lavaa: shodasalipthikonaa: palapramaanam pravadanthi yasmin cchayaadinardhe ardhachathurtha sankhyaa thathraasu vaachya: savitha nabhastha:

Quickly say the longitude, and the meridian Sun for the place where the latitude is stated to be 8° Minus 16′ N (i.e 7° 44′) and midday shadow of the gnomon, 3 and 1.5 angula. (Mahabhaskareeya 8-8)

The method of calculating the longitude has also been worked out by Bhaskara I. through a mathematical problem.

सूर्याचन्द्रमसौ तुलाधरगतौ दृष्टो मया तत्वतो
भागैर्द्वादशभिर्द्वयेन च युतौ सूर्यस्य वारोदये ।।
लिप्ताभिः शशिशून्यसागरयुतौ जीवस्य वारे पुनः
शुक्रस्याथ शनैश्चरस्य दिवसे तुल्यौ कियद्भिर्दिनैः

Sooryaachandramasow thulaadharagathow dhrushto mayaa thathvatho bhaagairdvaadasa bhirdvayena cha yuthow sooryasya vaarodaye lithaabhi: sasi soonya saagara yuthow jeevasya vaare puna: sukrasyaatha sanaischarasya aivase thulyow kiyadbhirdinai:

The Sun and the Moon on a Sunday at sunrise are carefully seen by me in Libra. The degrees of their longitude are 12 and 2 respectively ..... After how many days will they assume the same longitude again at Sunrise on a Thursday and Saturday, respectively? (Mahabhakareeya 8-21,22)

Terrestrial longitude : Lallacharya has defined terrestrial longitude directly:

क्रमेण लङ्कोज्जयिनी हिमाचलप्रबद्रेखविषयेषु मध्यमाः ।
भवन्ति पूर्वापरपत्तनेष्वमी ततश्च देशान्तर कर्म संस्कृताः ।।

Kramena lankojjayinee himaachala prabadrekhaa vishayeshu madhyamaa: bhavanthi poorvaaparaa patthaneshvamee thathascha deranthara karma samkruthaa:

The mean longitude of planets calculated are for places on the meridian passing through Lanka, Ujjaini and Himalayas. When to these longitudes are applied, correction for difference

in terrestrial longitude, the results are longitudes for places east or west of the meridian (Sishyadhi vruddhi Tantra 1-42)

Mean longitude calculation: Bhaskaracharya gives the methods for the calculation of mean longitude of planets:

कलीकृतं वा ससमं दिवाकरं स्वगीतिकोक्तैर्भगणै: समाहतम्
भजेत वर्षैर्युगसङ्ख्ययोदितैर्विहङ्गमानां प्रवदन्ति लिप्तिका:

Kaleekrutham sasamam divaakaram saageethikokthair bhaganai:
samahatham bhajetha varshairyuga
sankhyayodithairvihankamaanaam pravadanthi lipthikaa:

Reduce the Sun's mean longitude together with the years elapsed to minutes of arc; Multiply this by the planets own revolution number as said in Gitika pada of Aryabhateeya and divide the product with the number of solar years in a Yuga. The result is the planet's mean longitude in terms of minutes (Mahabhaskareeya 1-9).

The longitude at a particular time was known and the arc through which the planet has revolved, after a certain period and the number of revolution in a Yuga are correlated to get the new position of the planet. This procedure is just like calculating the position based on the starting point, velocity and time duration as followed in modern astronomy.

Calculation of time and its relation with longitude: Time at any place is directly connected with the longitude of that place. This approach is followed in the calculation of time, with reference to Greenwich line. A series of methodology for the calculation of the longitude and time from other parameters have been discussed in astronomical books. Varahamihira in Panchasiddhantika gives this explanation:

यवनान्तरजा नाड्य: सप्ताविन्त्यां त्रिभागसंयुक्ता:
वाराणस्यां त्रिकृति साधनमन्यत्र वक्ष्यामि ।।

Yavanaantharajaa naadya: sapthaavanthyaam
thribhaga samyukthaa: vaaraanasyaam thrikruthi
saadhanamanyathra vakshyaami

Correction to the time of longitude in Yavanapura to get time of longitude of Ujjaini is seven nadikas, and twenty vinadikas and that of Banaras is nine Vinadikas. How to find the correction for longitudes with this? (Panchasiddhantika 3-13)

It is inportant to note that the longitude of a foreign city Yavanapura (present Alexandria) is given here. Modern value is in agreement with this. Here, based on the time given for Ujjaini, the longitude and time for Alexandria are to be calculated.

The east of Greenwich longitude of Yavanapura, Ujjaini and Banaras are 30°, 75° 50′ and 83°. The time differes equivalent in nadi for Ujjaini would be (75° 50′ - 30°) 1/6 = 7.38, whereas Varahamihira has given the value 7.20. Similarly for Banaras (83°-30°), x 1/6 = 8°8′ where as Varahamihira has given 9.0. The level of accuracy is very high for even a place which is outside Indian continent and about 5500 km from Ujjaini.

Linear measurement of Yojana and longitude: Angular measurement is related with the longitude of the place. Correction of the measurements are given above. Here the longitude is connected through linear measurement:

त्रिकृतिघ्नात् खवसु हृताद् योजनपिण्डात् स्वताडिताज्जह्यात्
अक्षद्वयविवरकृति मूल्या: षट्कोद्धृता नाड्य: ।।
Thrikruthaghnaath khavasu hruthaath yojana
pinddaath svathaadithaa jyahyaath akshadvaya
vivarakruthi moolyaa: shatkoddhruthaa naaddya:

Take the distance in yojana between two places for which the line difference for longitude has to be found. Multiply this with 9 and divide by 80 (the result is their distance in degrees)