INDIAN SCIENTIFIC HERITAGE

DR. N. GOPALAKRISHNAN

FOREWORD BY DR. R.A. MASHELKAR, F.R.S.

Indian Institute of Scientific Heritage
Thiruvananthapuram

Heritage Publication Series -14

About the author.......

The author, Dr. N. Gopalakrishnan was born on 20th November, 1955. He took his M.Sc. Pharm. Chem (1978); M.Sc. Chemistry (1979); M.A. Industrial Sociology (1985) Degree in Journalism (1987) and Ph.D. in Biochemistry. He has been awarded the D. Lit (2002) for his outstanding contribution through the study of the scientific heritage of India. He is a Senior Scientist, in the Regional Research Laboratory, Council of Scientific and Industrial Research (CSIR) and Hon. Director of the Indian Institute of Scientific Heritage. He is the recipient of D.V. Memorial Award (1985), Gardners Award (1988), Dhingra Memorial Award and Gold medal (1990) and again D.V. Memorial Award (1993) for the achievements in the field of scientific research. He has been awarded the first NCSTC award for the popularisation of science, by the DST, Govt. of India, in 1988. He is also the recipient of the prestigious Canadian International Development Agency (CIDA) Fellowship of the Government of Canada in 1993 and was a visiting scientist in the University of Alberta, Edmonton, Canada and visited many universities in USA, Canada and many middle east countries for delivering lectures. He has six patents and fifty research papers in the scientific studies.

He has also received four awards for the literary works and has 41 books to his credit, both in scientific and cultural subjects and many popular articles on Indian Scientific Heritage.

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Thiruvananthapuram - 695 018

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Indian Scientific Heritage

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Published by :

Indian Institute of Scientific Heritage (IISH)
Registered Charitable Trust 328/99/IV
Ushus, Estate Road, Pappanamcode
Trivandrum - 695 018 (Ph. 490149)

Rs. 250/-   (US $ 25)

(For Members Rs. 125/-)

Printed at:

Sree Printers (DTP, Offset & Screenprinting)
Ind. Estate, Pappanamcode, TVM - 19, Ph. 490135

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Beneficiary : IISH Trivandrum
Ac No : 57020795171
IFSC : SBIN0070030
Bank : SBI industrial estate, papanamcode
Trivandrum-19

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PREFACE

THE AIM OF THIS STUDY IS

NOT TO TELL THAT PUSHPAKAVIMAANA IS MODERN AEROPLANE
NOT TO PROVE THAT AGNEYAASTRA IS THE MODERN ATOM BOMB
NOT TO SAY THAT THE VEDAS ARE THE SOURCE OF ALL KNOWLEDGE
NOT TO CLAIM THAT ANCIENT INDIAN RISHIS KNEW EVERY SCIENCE
NOT TO DECLARE THAT INDIA IS THE HOMELAND OF ALL KNOWLEDGE
NOT TO INTERPRET THAT ALL SANSKRIT BOOKS CONTAIN SCIENCE
NOT TO DESCRIBE THAT ALL INDIAN PRACTICES WERE/ARE SCIENCE !

BUT TO INFORM THE WORLD THAT ANCIENT INDIAN SCIENTISTS HAVE INVENTED, DISCOVERED AND SCIENTIFICALLY DOCUMENTED MANY THEORETICAL AND APPLIED SCIENTIFIC AND TECHNOLOGICAL KNOWLEDGE WHICH HAS THE SAME QUALITIES AND QUALIFICATIONS AS THAT OF MODERN SCIENCE AND TECHNOLOGY

My Sincere thanks to Prof. R. Vasudevan Potti, Professor in Sanskrit (Retd.) for his guidance in writing this book. Also I express my gratitude to Dr. R. A. Mashelkar, Director General CSIR & Secretary, Govt. of India, Department of Scientific & Industrial Research, for the inspiring foreword, written to this book.

In loving memory and with the blessings of my parents Smt. K. Satyabhama and Sri. S. Narayana Rao, I submit this book to the feet of the great Rishis of my motherland Bharath (India)

28-2-2000 **DR. N. GOPALAKRISHNAN**

FOREWORD

India has been a repository of wealth since the days of yore-wealth; not only nature's bounty in the form of minerals, rivers, seas and forests; and not only in its rich spiritual and cultural heritage that is known and acknowledged the world over - but also its intellectual wealth. Not many in the present generation, steeped in the modern concepts of science and technology, can appreciate that some of the developments hailed as major advancements and milestones today may have in reality been discovered and developed ages ago. Many in the world know of India in terms of its philosophy, mysticism, architecture, sculpture, performing arts and the like. Few are aware that India was also the fountainhead of important foundational scientific developments and approaches. The reason for this lack of appreciation is that there had been no major research or account of the panorama of scientific work initiated in ancient times, a part of Indian tradition that nurtures the culture of today.

Major contributions to science from India range from the decimal place value counting system and the zero to the holistic philosophy and practice of medicine, Ayurveda, distinctly different from the western medicine. Many developments in mathematics attribute to the west belong to India. Original discoveries in astronomy, chemistry, metallurgy, plant systems on one hand and work on other hand constitute this rich heritage of innovation and creativity. During the 12th to 18th century, over ten thousand books were written in India on science and technology. Indian manuscripts were translated into Persian and Arabic registering flow of knowledge from India to the outside world.

It is important that a systematic approach be made to identify the specific strengths in Science & Technology that were prevalent in ancient times from the wealth of ancient literature available today. Further work is then necessary to decipher the relevant information and then extract more useful information that could be relevant in the current context. This knowledge could, in addition, be extremely useful to establish prior art in the context of resolving intellectual property infringement disputes. In this volume, Dr. Gopalakrishnan, who has basic training in several aspects of chemistry, pharmaceutical chemistry and biochemistry, had undertaken this task. It is indeed a very daunting task and it is particularly heartening that a trained scientist has taken up this work. He has concentrated on the basic science related skills especially in the areas of mathematics, astronomy and metallurgy.

This volume provides a glimpse of the wealth of information and knowledge in three specific sciences. This should instil a sense of pride in all of us and also the feeling of rededication to rebuild that innovative India again. I wish to congratulate Dr. Gopalakrishnan for this painstaking effort. I do hope many more scientists would be enthused into carrying out similar work, which should result in a greater awareness of the great knowledge and wisdom prevalent in India since ancient times

R. A. Mashelkar, F.R.S.

30 May 1999

Director General, CSIR &
Secretary to Govt. of India

PERIOD OF IMPORTANT BOOKS

Aryabhatiya of Aryabhata I (499 A.D)
Brahma-sphuta-siddhanta of Brahmagupta (628 A.D)
Ganitha sara sangraha of Mahavira (850 A.D)
Graha-chara-nibandhana of Haridatta
Graha-chara-nibandhana-sangraha (932 A.D)
Khanda-khadyaka of Brahmagupta (628 A.D)
Karanapadhati (1417 AD)
Karna-prakasa of Brahmadeva (1092 A.D)
Karna-ratna of Deva (689 A.D)
Karana-tilaka of Vijayanandi (966 A.D)
Laghu-Bhaskariya of Bhaskara I (629 A.D)
Laghumanasa of Manjulacharya (932 A.D)
Lilavati of Bhaskaracharya II (1150 AD)
Makaranda-sarani of Makaranda (1478 A.D)
Maha-Bhaskariya of Bhaskara I (629 A.D)
Mallikarjuna suri (1178 A.D)
Maha-siddhanta of Aryabhata II (950 A.D)
Pancha-siddhantika of Varahamihira (587 A.D)
Parameswaracharya (1431 A.D)
Patiganita of Sridharacharya (750 A.D)
Sishya-dhi-vrudhi Tantra of Lalla (749 A.D)
Siddhanta-sekhara of Sripati (1039 A.D)
Siddhanta-siromani of Bhaskara II (1150 A.D)
Sphunta-nirnaya-tantra of Acyuta (1621 A.D)
Vatesvara-siddhanta of Vatesvara (904 A.D)

CHAPTER - I : INTRODUCTION

Modern concepts in ancient Indian sciences is a subject for scholars on all branches of modern and ancient sciences. Many scholars have put in claims that all the modern scientific knowledge existed in ancient India. There is another view which says that there were only philosophy and spirituality in India, and no science in its modern sense was prevalent here. Whatever may be the claims and counter arguments, the facts are to be brought into limelight, by following an unbiased approach with true rational vision and scientific temper.

Sanskrit texts are the prime sources of much of the ancient Indian knowledge. To search the modern scientific concepts in them, one has to go deep into the scientific Sanskrit literature, both orginal and their commentaries. A lot of information available from the archeological observations can be used as additional and advantageous support for the scientific literary knowledge. The literature based subject knowledge is distributed mainly in mathematics, astronomy, metallurgy, medicinal and health sciences, sculpture, civil engineering, agriculture, etc. Some of this knowledge can have its modified and applied field also, which might have escaped notice in the literature. I.e. There are many scientific concepts which transfers from generation to generation in the course of their applications by the practitioners. For example, the metallurgical skill of the ancient Indians can be established from the thousands of metal and alloy samples collected from the archeological excavation sites from all over India. The information available in the ancient literature is far less than what could be gathered from the technological grade of metal samples collected from these sites. Similarly medicinal concepts can be demonstrated even now by the systematic approach followed in the Ayurvedic and siddha branches of applied sciences and medical systems. The information given in

Charaka samhita and Susrutha samhita can thus be proved on the applied field. Much more important information is left out with the practitioners of old generations. On civil engineering and sculpture, the skills are evident and available, if we make a survey of the Indian historical monuments. These constructions can tell us millennia old stories of Indian civil engineering. Ancient Indian capabilities can be seen in Taxila, Mohanjodaro, Lothal, Harappa, Cave temples and various other structures. Almost similar is the approach in agricultural science content of the practices followed in Indian villages, even now.

One cannot ascertain the level of excellence, in the subjects, attained by our ancestors unless one study our ancient literature with the help and parameters of modern sciences.

To prove the ancient Indian authority on mathematics, astronomy and metallurgy one can cite Sanskrit literature as the main source and to some extent the regional language books of ancient origin. Scientific information is distributed in the Sanskrit literature written from the Vedic periods onwards. Rigveda, Yajurveda and Atharvaveda belong to this class of literature. Among the Vedangas, the fifth Vedanga, Jyothisha and the Sulbasutra part of the sixth Vedanga, Kalpasutra, carry standard mathematics and astronomy. Qualitatively and quantitatively, the content of mathematics and astronomy are very high in these literature.

At present the so called Indian mathematics and astronomy at their superficial knowledge level are mainly with a few astrologers. They know very little of the preliminary aspects of these subjects. Exceptionally, a few among them may be endowed with a deeper knowledge of the subjects. Majority of the scholars of modern astronomy and mathematics are unaware of the depth of wisdom of ancient knowledge as a whole. This is evident from the modern books on the subjects published in India and abroad, which ignore the ancient Indian contributions. Thus it is clear that the basic science like mathematics, astronomy and the technological aspects of the metallurgical knowledge were treated as extinct when viewed from the sphere of knowledge of these scholars. The modern concepts in them are to be brought to light to expose, Indian heritage, to the world. Hence the study is important to revitalise this knowledge both for theoretical and applied use in modern life. So, in this book Mathematics, Astronomy and Metallurgy are focussed.

Mathematics :

The layman's knowledge of Indian mathematics stays only on two Indian contributions to mathematics. Many also claim that these 'two achievements' i.e the discovery of the zero (0) and the discovery of Pythagorus theorem are great! The most unfortunate reality is that we have not substantiated our claims by submitting proofs for even these two. If we search through the ancient Indian Sanskrit literature, hundreds and thousands of modern scientific information bits, on mathematics written during the Vedic period and afterwards can be seen. Not merely zero and Pythagorus theorem! Bringing all of them to light is a Herculean task. Demonstrating with few examples on a variety of subject areas need not be very difficult. But the important and crucial requirement to claim Indian contributions is that one has to search almost all important texts. In this book such an approach is followed. The authentic submission of the systematic knowledge with proof of the date of its origin and development can throw light on the glory of our heritage in mathematics.

Astronomy:

This was a subject used mainly for the astrology. Even though astronomy itself is eclipsed with astrology, no effort has been made to bring out the astronomical science and the applied mathematical content of the astronomy. One can never avoid wondering if the in-depth astronomical knowledge of Indians were pondered. The books of Aryabhatta, Bhaskara, Lallacharya, Manjulacharya, Vateswara, and others are fortunately available with English translations which are published by reputed scientific organisations and Universities. Unfortunately no effort has been made to popularise the astronomical content in these books. Hence, in this study effort has been taken to bring into light the modern concepts of the scientific content in the ancient astronomical thoughts.

Metallurgy

Mathematics and astronomy are considered as the theoretical branch of sciences, which have the basic knowledge created by the brain storming works of the ancient Indians. However the metallurgical knowledge has more technology content than what are present in the other two branches. The science and technology of metallurgy need the geological knowledge on the minerals and ores, chemistry of the composition of the minerals and metals, the physics of high temperature processing, the technology of the kiln and furnace making, the knowledge on handling the molten metals and the peculiar compositional structure of the smooth alloy making, etc., Casting of metal ingots and alloys in various shapes also needs the knowledge of making the moulds using suitable sand. The list of the expertise on the technological background of metal processing is thus unending.....

The mathematical and astronomical explanations available in the Sanskrit books, are compared with the latest knowledge on the subject given in modern text books. Ancient metallurgical knowledge and facts described in the Sanskrit books are substantiated with the archeological evidences available as metals and alloys from various archeological sites.

Search was carried out to know whether Indian knowledge on these fields have ever been taken out from this country, by foreign travellers or scholars, through the historical facts. In this part of the study important books taken from India, and those available in the libraries of foreign countries are listed. The works of the western scholars who threw light on the ancient scientific knowledge and the period are also listed.

Another important fact presented about the Indian scientists, is their period or that of their writings, using suitable data as quotations from their books. This is important because for the first claim on discovery, the year of book and the author are to be supported with proof. Especially when 'Chronosensitivity' of the Indian writers are always doubted and there is also a misconception that Indians rarely give the period correctly on any historical incident or writings. Scientifically, the date for the metal products and mines is quoted from the published data available on the C_{14} study of the archeological samples.

CHAPTER II : IMPORTANT LITERATURE ON ANCIENT INDIAN SCIENCES

There are a number of ancient Indian scientific texts on various subjects and also research papers based on these. Scientific books and some spiritual books have also to be included in the list, for searching the modern concepts in ancient sciences. Many scientific books written in Indian languages other than Sanskrit also contain ancient sciences. Even if one looks into Sanskrit books only, detailed presentation of all literature available in India and abroad may not be possible. Random scanning is adopted here to get the depth of literature available. Based on the important and well known literature available during that period, an inference can be made on the knowledge and concepts mentioned by Indian scientists. Only, famous scientific books are surveyed here. They are selected based on their past and present popularity and also their science content. Their popularity in ancient period was ascertained based on the number of commentaries written on them. Two criteria are important in scientific discoveries, other than the qualification of the person(s) behind it; 'What' is discovered and 'when' the discovery was made. Credit goes to first discoverer and hence more ancient books carry special importance because its author is the 'first discoverer' of the scientific idea. Hence those books are given prominence in the literature survey presented in this book.

Mentioned here are literature of great significance in the mathematical and astronomical knowledge during the last three millennia. They are classified according to chronological order described in the bibliography publised by Indian National Science Academy, New Delhi[1]. Literature is arranged in seven groups based on an arbitrarily fixed periods.Vedic Period (before 1000 BC), 1000-500 BC, 500- 00BC, 00 - 500 AD, 500 - 1000 AD, 1000 - 1500 AD and post 1500 AD.

Vedic Period (before 1000 BC)

Vedas and Brahmanas are the important sources of astronomical and mathematical information during the Vedic period. Numbers given in Rig Veda, the oldest literature of the mankind, are well known. Age of Vedas is calculated by different scholars in different ways. However a recent publication based on an astronomical observation, noted in the 10th mandala of Rig Veda and also in Taitireeya Brahmana, (*Bruhaspati prathamam jayamana... thishyam nakshatram abhisambabhoova:*

which means the planet Jupiter moved first after occulting the tishya star and it appeared as new born) throws much light to the calculation of the period of Rig Veda[2]. Based on this line it has been suggested that 10th mandala (which is known, written later than the other mandalas) might have been written when such an observation was made. On modern astronomical calculations, it was found by researchers that, this occulting might have occurred during BC 4275 +/-75. Hence any scientific information available in Rig Veda is nearly 6 millennia old. Like Rig Veda, Yajur Vedic texts also carry lot of mathematical information. Arithmetic and geometric progression are given in Yajurveda. Taitireeya Samhita, Satapatha Brahmana, Tandya Brahmana, Gopatha Brahmana, etc., also contain systematic presentations of mathematics of great historical importance. A few of them are also quoted in the text. So, for the oldest mathematical knowledge Vedas and Brahmanas are to be searched, particularly for numbers, progressions, products, sums, differences, digits and other mathematical calculations. Some information were also given intermixed with spiritual notes.

1000 BC - 500 BC

Some of these literature were reproduced/reprinted with English translations by some Indian and foreign scientific Institutions. Prominent among them are the Sulbasutras, which are very important sources of geometrical and arithmetical knowledge. There are differences in the opinions among Western and Indian scholars on the period of these books. Many evidences brought out recently prove that periods of these writings can be compared with that of the Vedas. They were written in the same language style adopted in Rig and Yajur Vedas. Here, Sulbasutras have been included with literature written during 1000-500 BC, because Indian National Science Academy followed this period for them[3].

Boudhayana sulba sutra (800 - 500 BC) is said to be the oldest of all Sulbasutras. It comprises of 525 sutras/ verses which are divided into three chapters. The first chapter consists of 116 sutras explaining geometrical rules meant for the construction of fire altars. The second chapter and the third consist of 86 and 323 sutras respectively and almost all of them give details on fire altars. These works are of great importance due to the scientific and mathematical approach followed in the construction of fire altars, with the help of theorems and real measurements. A number of foreign and Indian mathematicians have gone deep into the subject matter of these sutras.

Apastamba sulbsutra is also supposed to be written between 800 - 500 BC. It consists of 223 sutras for the construction of sacrificial altars using geometrical rules and mathematical ideas. It is broadly divided into six sections or Patalas. Rules mainly describe various types of vedic fire altars, their spatial magnitudes, relative positions and other constructional details. They provide geometrical propositions and rules for achieving these parameters. Many foreign mathematicians have written commentaries on the subject matter of this book. Burk, A. Cajori, F., Cantor. M. Levei, B. Loschhorn, K., Gaston Milhound, Muller, C., Thibaut, G., Smith, D.E.,Vogt, H., Winternitz, M., Zenthen, H.G. are the prominent among the foreign scholars who studied Apastamba sulba sutras and wrote commentaries and research papers. Indian scholars have also studied this book to bring out the geometrical science content in it.

Katyayana sulbasutra belongs to sage Katyayana. The work consists of 102 sutras in six khandikas. It also deals with the subject of geometry of fire altars.

Manava Sulbasutra is said to be of later origin. It also discusses many rituals and geometry of altar making, similar to those mentioned in the three Sulbasutras noted above. However in this book the mathematical contents are deep, clear and more specific. Quantitatively and qualitatively mathematical content of Manva sulbasutra has great importance, compared to other books.

500 BC - 00 BC

Unlike, the periods of Vedic literature and Sulbasutras, there is not much ambiguity on the period of books belonging to this era, because of high chronological credibility during Buddha, Jaina, Maurya periods and also clarity of the period of the books written by Kautilya, Panini, Pingalacharya and others.

Laghadha's contribution to Jyothisastra is well known. Many books of later origin in astronomy were a continuation, by way of addition to Lagadha's books, on astronomy. Vedanga Jyothisha (three books), also known as Jyothisha Vedanga, is his contribution. It is said that there are four versions of Jyothisha Vedanga; Rig, Yajus, Sama and Atharva corresponding to the four Vedas. Sama Jyothisha is lost and the other texts are available. These three are said to be the oldest available Indian astronomical books. Hence Laghadha is sometimes called the father of Indian astronomy. Even though these Jyothisha books contain only a few verses, a number of studies have been reported by Indian and Western scholars on the subject matter.

Suryapannatti, or Suryaprajapati (300 BC) is a Jaina astronomical work. The principal source of information of this book is Malayagiri's commentary, Suryaprajapati-vrutti. Bhadrabahu appears to be the author of the former book. It follows the subject matter of Yajusha and Archa Jyothisha, in following a five year lunisolar cycle. It also incorporates many puranic and other concepts some of which are non-scientific.

00 - 500 AD

Books written during this period is chronologically well defined. Another advantage of this period is that, mathematicians and astronomers started systematically plotting positions of planets comparable with modern approach. Planetary positions, coincided with the date of birth of the authors or date of their books, were given in their writings. Old versions of Paulisa siddhanta, Romaka siddhanta, Surya siddhanta, Pitamaha siddhanta, etc., were written during this period. A few other well known scientific books of this period are Vruddha Garaga samhita written in the first century AD. This is one of the earliest astronomical samhitas. It is said to have twelve chapters, whereas only 11 are now available. All the eleven chapters discuss on planetary sciences and some astrological aspects. The calculations are dealt within a reasonable level of accuracy and scientific approach.

Vruddha Vasishta samhita of second century AD is an astronomical treatise in 13 chapters. Subject matter described here is almost similar to that found in books written in India during a later period. i.e. more refined information is present. The book describes real planets and stars, systematically.

Bakshali manuscripts, are manuscripts of great significance, found in Bakshali village, near the city of Peshwar, during the course of excavations in 1881. It was written in birch bark/leaves. Different views prevail on the date of these manuscripts. Approximately the period ranges between 00 and 200 AD. A considerable protion of the manuscripts has been destroyed and only 70 leaves available are preserved now in Bodleian Library, Oxford [4]. Many studies have been carried out on the mathematical contents of these leaves. Fractions, square roots, progressions, income and expenditure, profit and loss commutations, interest, linear - quadratic and intermediate equations, equations of second degree, etc., are present in the Bhakshali manuscripts. It is assumed that many more such leaves (manuscripts) might be available which could have thrown more light on the authentic Indian knowledge in mathematics during the beginning of the Christian era.

Vasishta siddhanta consisting of five chapters and Romaka siddhanta were written during the 3rd century AD. They resemble Suryasiddhanta in their contents, written in 11 chapters. Vasishta and Romaka siddhantas are modified and updated in Panchasiddhantika, by Varahamihira.

Subject standards of early astronomical works were set by comparing that of Surya siddhanta written during 400 AD. This scientific masterpiece was widely accepted and followed through out India. Many scholars are of the opinion that true modern scientific astronomical era had its beginning in India, from Suryasiddhanta. Old ideas were renewed and scientifically modified with a rational approach in the book. However the text available now, the Suryasiddhanta, is said to be of much later origin, possibly written in 700 AD. But the Suryasiddhanta included in Panchasiddhantika is the older book. Original Suryasiddhanta has 14 chapters. It describes almost all subjects dealt in other astronomical books in the same style of presentation. Many commentaries are also written for this book.

The most important among the books on astronomy and mathematics written during this period is Aryabhatta I's Aryabhateeya, written in the last year of the 5th century AD., (499 AD). Not only the content of the book but also the time of its publication have great significance. Aryabhateeya is a celebrated book, separating ancient Indian astronomy and mathematics and modern subjects. The first of its kind, not only in India but also in the world. It can be said that Aryabhateeya is the marking point of a sudden upsurge on the study of the subject with an entirely new prespective. Aryabhateeya has four chapters known as Geetika, Ganita, Kalakriya and Golapadam, describing systematically standard astronomy and mathematics. Many commentaries were written, during the last 15 centuries by Indian and foreign mathematicians and astronomers, on this book. Almost all are available even now. All these commentaries are also source of novel theorems, applications and equations. Thus, Aryabhatta gave a role model for both mathematicians and astronomers, for developing the subject, through a novel approach he followed in Aryabhateeya.

500 AD - 1000 AD

There are a number of scientific books written during this period. The date of writing can also be determined from planetary data given in them. This period of five hundred years can be called the golden age of development of Indian astronomy and mathematics (period may be replotted as 499 - 1000 AD to include Aryabhateeya of Aryabhatta I also) because the rate of growth of theoretical and appllied knowledge was in its peak.

Varahamihira (505 AD) is a well known astrologer and astronomer having an in-depth applied knowledge of mathematics. His contributions to the world of modern mathematics and astronomy stand unique. Bruhatjjataka is written in 25 chapters dealing mainly astrology with astronomy. Bruhatsamhita is an encyclopedian composition containing tremendous volume of data from the astronomers who lived during and an earlier period to Varahamihira. It attracted great attention from astronomers around the world. Panchasiddhantika is a combination of five earlier siddhantas. Paulisa-Romaka-Vasishta-Saura-and Pitamaha siddhantas. Some have the opinion that all the five siddhantas included in Panchasiddhantika are different from the old siddhantas known by these names.

Bhaskaracharya I is another renowned mathematician astronomer who made break through in both the subjects. The major contributions of Bhaskaracharya I are the Aryabhateeya bhashya written as a commentary to Aryabhateeya, Laghubhaskareeya and Mahabhaskareeya. The Bhaskara bhashya of Aryabhateeya, consists of detailed explanation for each and every stanza given in Aryabhateeya. More over all the mathematical and astronomical data are explained using innumerable illustrations and examples. This book deserves great appreciation because it is the most systematically written oldest commentary, in the modern style, for presenting mathematical problems. Laghubhaskareeya consists of eight chapters and it is said to be a simplified version of Mahabhaskareeya. Mahabhaskareeya is also known as Bruhatbhaskareeya. It deals exclusively with the astronomical treatise and consists of eight chapters. It is the earliest known work to have systematically dealt with methodologies for determining positions of planets using mathematical calculations.

Brahmagupta, has written a famous mathematical and astronomical book of that millennia in 628 AD, known as Brahmasphuta siddhanta. It is also known as Brahmasiddhanta, and consists of twenty four chapters. Chapter 22 of this book has great significance because it explains in detail the instruments used for astronomical calculations. Hence this chapter is separetely known as Yantraddhyaya. This appears to be the first book in which the yantras (instruments) for collecting the astronomical data are systematically classified and explained. Of course some instruments were known earlier, as globe-golayantra is mentioned in Aryabhateeya. It is said that this work had considerably influenced even the renaissance of Arabian astronomy and mathematics in the eighth century AD[5]. Khandakhadyaka or Khandakhadya was another great contribution of Brahmagupta. This book was composed in Saka year 587 (665 AD) and consists of eight chapters on astronomy.

Lallacharya was a renowned but less quoted genius in astronomy, who can be equalled with Brahmagupta, Aryabhatta and Bhaskaracharya. His period is supposed to be 638 AD. (Some say 768 AD). Lallacharya's astronomical work appeares to be less known to the students and scholars of ancient Indian astronomy. Two masterpieces of Lallcharya are Sishyadhi vruddhi Tantra and Khandakhadyaka - paddhati. The former has 24 chapters. In this book the subject matter is arranged exactly similar to the modern astronomical books, including a chapter devoted wholly for the questions and mathematical problems given to the students of astronomy, as a part of examination. Lallacharya got his inspiration probably from Aryabhatta's masterpiece. Khandakhadyaka-paddhati is a commentary for Brahmagupta's work, Brahmasphuta siddhanta.

Manjulacharya's Laghumanasa (662 AD) is a famous book dealing with astronomy. The subject matter dealt in this book is highly complex. Understanding the subject even with the help of a commentary is difficult. It is said to be the abridged version of his own book, Brahmamanasa. Laghumanasa consist of six chapters, discussing the mathematical calculations of various parameters related to planets.

Brahmasiddhanta also known as Sakalyasamhita is an astronomical treatise of Sakalya (821 AD). The book, consisting of 764 verses, is written in six chapters. The main subject in all the chapters is the time measurements and its principles.

In 850 AD Mahaviracharya wrote a mathematical book namely Ganitasarasangraha, in five chapters. This jaina work is one of the most quoted earliest mathematical books.

Pruthudakaswamin (864 AD) has to his credit two books which are commentaries known as Brahma siddhanta vasanabhashya and Khandakhadyaka vivarana. Both these are considered highly authoritative books in astronomy, giving many novel ideas.

Vateswara siddhanta was written in 880 AD by Vateswaracharya. It is a treatise in fifteen chapters, focussing on deep astronomy and applied mathematics.

Aryabhatta II (950 AD) has contributed, highly thought provoking theorems and calculations in astronomy and mathematics, in the eighteen chapters of Aryabhatta siddhanta. Focus is mainly for explaining planetary motions using mathematics.

Sridharacharya (991 AD) is noted for his outstanding contribution in mathematics. The book written by him Ganitasara or Patiganita, because of its popularity, is also known by more than eight titles, including Sreedhareeleelavati. It gives a marvellous account of mathematics in a very systematic way with rules followed by worked out problems. It is a model mathematical book, comparable with modern texts in all respect.

1000 AD - 1500 AD

Stabilised development of various branches of science occurred during this period. The spread of the scientific knowledge from India to other parts of world also occurred. Again, it was a period of in-depth search for better methodology and diversified applications in mathematics and astronomy. It is important to note here that many Sanskrit books were translated to foreign languages. Scientific knowledge available in Sanskrit books also spread to different regions in India through scholars who conducted pilgrimage from south to north and north to south of India. Kerala's contribution on the subjects has great importance. Similar to that of Kerala's contribution, scientific texts were also written in other Indian States. A uniform growth on scientific observations/studies and literature could be seen, throughout India. Some of the scientific Sanskrit books were reproduced in local languages like Tamil, Malayalam, Orriya, etc. either in original form or as commentaries. Many manuscripts in Sanskrit and other Indian languages were taken to different countries, by travellers and scholars. It is also to be remembered that this period was contemporaneous with the renaissance in Europe from where modern scientific knowledge started spreading the world over. International voyages to different countries were conducted by Europeans. They could directly or indirectly take these manuscripts from India. A large number of texts and manuscripts were reproduced during this period. Since the number of original and commentary books written during this period were very large, only a few important books are quoted here.

Sripati has written three books, Dhikoti (1039AD) Siddhantasekhara (1039AD) and Druvamanasa (1056 AD). Among these the most popular one is Siddhanta sekhara. It consists of twenty chapters giving details on topics in astronomy. The last chapter is prasnadhyaya in which answers to questions are given. The nineteenth chapter is Yantraddhyaya which describes experimental procedure for astronomical parameters.

Brahmadeva Ganaka (1092 AD) has contributed an astronomical book entitled Karanaprakasa. It consists of ten chapters describing methods of calculating latitudes and longitudes of places. It quotes profusely from the works of Lallacharya and Aryabhatta.

Bhasvati or Bhasvatikarana is an astronomical book written in eight chapters by Satananda (1099 AD). References of all astronomical measurements are given from his birth place, Puri in Orissa. It contains novel ideas on latitudes and longitudes.

Bhaskaracharya II (1114 AD) is one of the greatest ancient mathematicians of this millennia. His name has also appeared in Mathematica Britannica. No other ancient Indian mathematician has written such a wide variety of astronomical and mathematical books as Bhaskaracharya II. Bijaganita, consisting of eleven chapters, is a famous authentic book on algebra, written by him. Explanations on various algebraic calculations are systematically dealt in this book. Bijopanaya is the second book in which Bhaskaracharya has dealt with the subject of giving corrections for astronomical paths of planets. Lilavati is the third well known book consisting of thirteen chapters devoted fully for mathematics. Numerous commentaries by Indian and Western scholars on Lilavati are available. The fourth contribution of the author is Siddhanta siromoni written in 1150 AD. This is a famous astronomical work divided into two parts. The first part has twelve chapters and the second part fourteen chapters. Karanakutuhala is the fifth book of Bhaskaracharya written in 1183 AD. It is known by many titles and deals with astronomical details.

Mahadevi, or Mahadevisarini is a small book written in 1316 AD by Mahadeva Bhatta. It deals with methods for calculating mean and true positions of planets described in fortythree verses. The epochal positions are given for mean Aries. The book contains tabulated information for calculating mean planets. These tables simplify the calculations commonly used till then in calendar making.

Yantraraja or yantrarajagama is a book exclusively dealing with the subject of methods and instruments for determining astronomical parameters through experiments, directly. The book was written in 1320 AD by Mahendra Suri. It is also available in other titles like sadyantra, suyantragama, etc., The work is compiled in five chapters. The subject is presented very systematically giving first the calculations (designing) for setting up Yantra (Ganita), then its composition (ghatana), draft drawing (yantrarachana), making prototype (yantrasodhana) and analysis of data obtained using the real Yantra (Vicharana). This approach of dealing with technical problems, supposed to be known only for modern science, is followed in this ancient book.

Ganitakaumudi (1365 AD) is a book on mathematics, written by Narayana Pandita, son of Nrusimha Daivajna. It consists of fourteen chapters. The author has also written another book Beejaganitavatamsa, a mathematical work in algebra.

Parameswara (1340 AD) a student of Rudra has to his credit eleven books dealing with astronomy and mathematics. Bhattadipika, Drigganita, Goladipika, Golasthana, Grahanastaka, Laghubhaskareeyavyakhya, Lilavativyakhya, Siddhantadipika and Suryasiddhantavyakhya are the famous among his books. Parameswaracharya's Drugganita has given a new dimension for mathematical approach to astronomy. Hence he is also known as Drugganita Parameswaracharya.

Neelakantha Somasutwan (1465 - 1545 AD) has written a number of books. All of them are equally famous and deal with the twin subjects of astronomy and mathematics. Aryabhatiyabhashya is his commentary for Aryabhateeya. Chandrachaya ganita deals with methods of calculating time during night from measurements of the shadow cast by the moon light. Chandra chayaganita and Golasara are the other two books dealing with calculations pertaining to the moon. Siddhanta darpana and Tantrasangraha deal with complex astronomy. Of these two, the latter consists of eight chapters (430 stanzas) for computing many useful parameters in astronomy. The subject is also adopted in astrology. However, Somasutwan has only dealt with astronomy and astronomical mathematics in these books and not astrology. Many theorems at present ascribed to Western scholars were described by him centuries before, the period of these Western scholars. The beauty of the book Tantra sangraha is that each and evey one of the subject connected with mathematics and astronomy is dealt systematically with illustrations. For making sure that, students pick up the subject, worked out mathematical problems are given with explanations.

Brahmasiddhanta tika, Narmadatika and Yantraratnavali are three important books written by Padmanabha whose period is approximately fixed after Bhaskaracharya II. All the three books are on astronomical instruments. Observations on polar stars have been dealt in detail in the third book, other than the astronomical instruments.

Yuktibhasha or Ganita Yuktibhasha is an anonymous treatise of arithmetics and mensuration written during 1475 AD. This book is said to be an exposition of Nilakanta's Tantrasangraha and it is divided into seven chapters. This is one of the best mathematical books available in Malayalam after, Puthumana Somayaji's Karanapaddhati. Yukti bhasha has its place with Tantrasangraha and Karanapaddhati in describing novel theorems, methods and approaches in mathematics.

Karanapaddhati is a book of high standard modern mathematics and astronomy written by Puthumana Somayaji in the 15th century. It has been estimated that Somayaji composed the book in 1428 AD. However there is difference of opinion on this period, which is discussed elsewhere. Karanapaddhati has ten chapters and the sixth chapter deals with mathematics and in the other chapters astronomy is given.

In the 15th century, Kesava Daivajna wrote an astronomy book on Surya siddhanta namely Siddhanta Laghukramanika, having nine chapters.

In 1438 AD another astronomical book Makaranda was written. It deals with the subject of preparing the calendars. This book consists of tabular information based on Suryasiddhanta.

1500 AD (Renaissance period and later)

During this period, many Sanskrit and other Indian language books on science were translated to the foreign languages, especially European languages and English. Many manuscripts in palm leaves were also transported to foreign countries. A list of such manuscripts is given separately. Large number of scientific books were written within a short span of 200 years ranging between 1400 -1600 AD. Many of them are available now. Given here are only important books written in the post 1500 AD era.

Jnanaraja in 1503 AD has written a book on astronomical geography namely, Siddhantasundara. This book consists of eighteen chapters.

Tippanna is a less known astronomer in the ancient Indian list. He wrote a book Uparagadarpana in 1507 AD. Significantly, this writing gives time of eclipses, occurring in each year during a span of 1020 years from 1429 Saka (1507 AD) to 2449 Saka (2527 AD). It consists of seventeen sections each describing, 60 years cycle.

Grahatantra is an astronomical work of Venkatayajwa written between 1527 - 1627 AD. It has eight chapters on common astronomical subjects.

Grahalaghava is one among the four books written by Ganesa Daivajna in 1507 AD. The astronomical work consists of fourteen chapters. It gives novel astronomical information. Grahalaghava vrutti and Pratodayantra are two other books written by him. The latter deals with astronomical instruments.

Krishnadaivajna, wrote Karanakaustubha in 1653 AD. It is said that this book is a small portion of Tantraratna. Many observations given in the book are similar to the contents of the books written during the same period.

Siddhanta tatwa vivekika or Siddhanta tatwa is a famous book written in 1658 AD by Kamalakara. This is one among the seven books written by him. It consists of fourteen chapters. The subject matter is based on Suryasidhanta. In the first part focus is given to time measurements. Instrument for determining the sankranti, centripetal forces, cause of circular motion, apparent motion of celestial bodies, etc., are also described. Malayendu, in 1659 AD wrote two books on astronomical instruments, namely Yantra rajarachana and Yantrarajatika. The former is a useful book for the building of astronomical instruments and the latter is a guide for the same purpose. The latter is a commentary for the book written by Mahendra Suri.

There are many books written after this period during when modern science started its fast development in other countries. It does not attract much attention when similar scientific literature were produced the world over in different languages. Hence description of those scientific Sanskrit literature produced during post renaissance and industrial revolution periods are not given here.

Other than pure scientific literature ancient books on religion and spirituality also carried a lot of scientific matter. Some of the information given in those books are included in the text. Some Puranas carry information on astronomy and mathematics. Similarly, dharmasastras too carry sociological information in which some of the scientific contents present are in no way less significant. The majority of these books fall in the period between 500 BC to 200 AD. Their periods of origin are reckoned mainly based on opinions of scholars. Scientific information in Kautilya's Arthasastra are of great importance. Similarly Pingalacharya's Chandasastra is an important source of information utilised in mathematical science. Only important and much focussed books are brought to light here to consider the standard scientific information.

The aim of bringing the above selected books into limelight is to prove that enough scientific literaure carrying modern scientific concepts were present in ancient India. A steady and systematic growth of literature and their scientific contents on all branches of theoretical and applied sciences took place in ancient India.

CHAPTER III : AUTHORS OF ANCIENT INDIAN SCIENTIFIC BOOKS AND THEIR PERIODS

Whenever ancient documents are surveyed, an important information collected, is the period of the literature. The data should follow authentic proof, when claims are put forth on first discovery. In the West, Christian era is taken as the point of reference. Till the 19th century, Christian era was not accepted in India. The majority of the books followed Saka era or Kali era and in some cases even the regional era. Based on these calculations, the approach of chronosensitivity of ancient Indian books and authors is to be followed. There was no ambiguity among the authors/ scientists of ancient India, on the reference point of these era and their sharp beginning date. Hence accuracy of the period is also sharp. Almost all the astronomical and mathematical books of ancient Indian orign has in them the date of the author, period of writing or the exact position of the celestial bodies so that one can get the period.

Two most commonly followed era by almost all astronomers and mathematicians of ancient India were Saka era and Kali era. Beginning of Kali era is based on the positions of the planets, the Sun and the moon. The date when all planets were almost in a line according to the two most accepted books; Suryasiddhanta and Aryabhateeya was taken as the reference for the beginning of Kali era. This positions were also approved according to Brahmasphuta siddhanta and other texts. The date can be calculated for intrapolation to Christian era in 3rd millennia BC. Position of the planets, the Sun and the moon on the first day of Kaliyuga (era) is given below:

Planet	Suryasiddanta	Aryabhateeya	Brahmasphutasiddanta
Sun	360° 0′ 0″	360° 0′ 0″	360° 0′ 0″
Moon	360° 0′ 0″	360° 0′ 0″	360° 0′ 0″
Mars	360° 0′ 0″	360° 0′ 0″	359° 3′ 50″
Mercury	360° 0′ 0″	360° 0′ 0″	357° 24′ 29″
Jupiter	360° 0′ 0″	360° 0′ 0″	359° 27′ 36″
Venus	360° 0′ 0″	360° 0′ 0″	358° 42′ 14″
Saturn	360° 0′ 0″	360° 0′ 0″	358° 46′ 34″

(In all astronomical books 360° is represented as 0°)

This day falls on BC 3102 February 17th midnight according to Aryabhatta and 3102 February 18th at sunrise according to other astronomers. As mentioned earlier, astronomers fix the period of importance, from this date. Even though there may be minor differences on the calculation of position of planets noted above among some ancient Indian astronomers, the date remain the same for counting the beginning of Kali era. Hence fixing birth year/ date or date on which a book was written, do not vary.

It is important to mention here that positions of planets and corresponding dates were not fixed arbitrarily, but accurately calculated as it is done in modern science. This is obvious from the calculations given by Aryabhatta for the year 499 AD March 21 (the date during when Aryabhateeya: the celebrated mathematical-astronomy book was completed). This position and the modern intrapolated/calculated values agree marvellously. Methodology and corrections followed by other astronomers also remarkably agreed, when counter checked with modern methods, in calculating the date they wanted to project. Hence even cross checking of day/date mentioned in scientific Sanskrit literature is possible by comparing with the planetary positions obtained and calculating back that date. However details of all authors have not been included, here.

The year of birth of Aryabhatta I is known to us with precision based on a verse in the Aryabhateeya (3-10):

षष्ट्यब्दानां षष्टिर्यदा व्यतीतास्त्रयश्च युगपादाः ।
त्र्याधिक विंशतिरब्दास्तदेह मम जन्मनोऽतीताः ।।

Shashtyabdaanaam shashtiryada vyateethaasthrayascha yugapaada:
thryadhika vimsathirabdaasthadeha mama janmanoftheethaa:

When sixty times sixty years and three quarter Yugas had elapsed (of current Yuga) twenty three years have then passed since my (Aryabhatta's) birth.

This shows that when Kali year 3600 years elapsed, Aryabhatta I was 23 years of age. This Kali year (when added to 3102 BC) corresponds to 499 AD. I.e. Aryabhatta was born in 476 AD. Gupta king, Buddhagupta reigned in Pataliputra from 476 AD to 496 AD. To be more precise, 3600 years of Kali era came to an end on Sunday March 21 499 AD at mean noon at Lanka and Ujjaini. At the time of mean Sun's entrance into the sign of Aries, birth of Aryabhatta took place i.e at Mesha Sankaranti on March 21, 476 AD . It is a wonderful exposure to know the level of accuracy attained about 1500 years ago, when it is presumed that there was no telescope available to see the positions of planets. The positions given by Aryabhatta I is comparable with modern values for March 21, 499 AD when he was exactly 23 years old.

Planet	Aryabhateeya	Aryabhatta Siddhanta	Modern value
Sun	360° 00′ 0″	360° 00′ 0″	359° 42′ 5″
Moon	280° 48′ 0″	280° 48′ 0″	. 280° 24′ 52″
Moon's apogee	35° 42′ 0″	35° 42′ 0″	35° 24′ 38″
Moon's asc. node	352° 12′ 0″	352° 12′ 0″	352° 02′ 26″
Mars	7° 12′ 0″	7° 12′ 0″	6° 52′ 45″
Mercury	186° 00′ 0″	180° 00′ 0″	183° 09′ 51″
Jupiter	187° 12′ 0″	186° 00′ 0″	187° 10′ 47″
Venus	356° 24′ 0″	356° 24′ 0″	356° 07′ 51″
Saturn	49° 12′ 0″	49° 12′ 0″	48° 21′ 13″

Can one expect a perfection, on calculating planetory positions, beyond this level of accuracy?

It is obvious that these values agree with the modern data. Thus the date of Aryabhatta and his book could be verified. Many similar examples can be quoted.

Another approach was followed by some other scholars. Varahamihira, the famous Indian astronomer belonged to the 6th century AD, in his book, Panchasiddhantika (I.8,9), has taken cut of date for epoch to commutate correction as Saka era 427 (based on Paulisa siddhanta) which corresponds to 505 AD.

सप्ताश्विवेद संख्य शककालमपास्य चैत्रशुक्लादौ ।
अर्धस्तमिते भानौ यवनपुरे सोमदिवसाध्य: ।।
मासीकृते समासे द्विष्ठे सप्ताहते/ष्टभागपक्षै: ।
लब्धैर्युतो/धिमासैस्त्रिंशत् घ्नस्तिथियुतो द्विष्ट: ।।

Sapthaasviveda sankhya sakakalamapaasya chaithrasukladow
ardhasthamithe bhaanow yavanapure somadivasaadhya:
maaseekruthe samaase dvishte sapthaahathefashtabhaagapakshai:
labdhairyuthofdhimaasaisthrimsath ghnasthithiyutho dvishta:

Deduct 427 from Saka year (elapsed) of the time taken. Multiply the remainder by 12. Add the months gone, counting from Chaitra. Put this result in two places. In one place, multiply it by 7, divide by 228 and take the quotient which constitute the intercalary months. Add this to the result kept in the place......

This was the practice in ancient Indian astronomical manuals (Karana granta), to take a contemporary date near the composition of the work, answering to certain specification, as the cut off date. It is thus presumed that Pancha siddhantika was composed in 505 AD. Supporting this fact, In Brahmasphuta siddhanta it is mentioned that '*navaadhika panchasamkhysake Varahamihiracaryo divam gatah*' which means in Saka 509. Varahamihira attained heavens. The reference given in another book can be used for finding out the year on which Varahamihira passed away (as 587 AD). In another book, Bruhatjjatakam (26.1) it is thus mentioned:

आदित्यदास-तनयस्तदवाप्तबोधः कापित्तकः सवितृलब्ध वरप्रसादः
अवन्तिको मुनिमतान्यवलोक्य सम्यग् होरां वराहमिहिरो रुचिरं चकार ।।

Aadithyadaasa thanayas thadavaaptha bodha:
kaapittaka: savithrlabda varaprasaada:
avantiko munimathaanyavalokya samyak
horaam varaahamihiro ruchiram chakaara

Varahamihira was the son of Adhityadasa, he learned the sastra from his father, his native place was Kapithaka; he was blessed by Lord Sun. He later resided in Avanti (Ujjaini) and composed Hora sastram. The biographical details are also given.

The date of Bhaskaracharya I is well known, which has been arrived at, based on his explanation given in the 9th stanza of 1st chapter of Aryabhateeya bhashya book:

कल्पादेरब्दनिराधदयम् अब्दराशिरितीरितः। खाग्न्यद्रिरामार्करसवसुरन्ध्रेन्दवः ।।

Kalpaaderabdaniraadhadayam abdaraasiritheeritha:

khagnyadriraamaar karasa vasu randrendava:

Since the beginning of current Kalpa, number of years elapsed is thus zero, three, seven, three, twelve, six, eight, nine, one (proceeding from right to left) years. This corresponds to 1986123730 years. He also repeats in the same stanza that the number of years elapsed since the beginning of current Kalpa at the commencement of Kaliyuga according to Aryabhatta is 6 Manvantara + 27 and 3/4 Mahayuga which is equal to (6 x 72) + 27 and 3/4 x 4320000 = 1986120000 years. This number is subtrated from the years given by Bhaskaracharya I. I.e. 1986123730 - 1986120000 = 3730 of Kali year which corresponds to 629 AD and that is the period of Bhaskaracharya I.

Devacharya, the son of Gojanna the author of Karanaratna calculated the epoch date for computation of ahargana (the number of days elapsed after the beginning of Saka year) and mean longitude of planet in the beginning of Saka year 611 which corresponds to 689 AD. It shows the author flourished in the 7th century AD, 'and composed Karanaratna in 689 AD, exactly 60 years after Bhaskara I wrote his commentary for Aryabhateeya, and 24 years after Brahmagupta wrote Khandakhadyaka'. As quoted by Devacharya in his book Karanaratna (1. 5-8)

शकवर्ष रुद्ररसै रहितं रवि संगुणं सगतमासम् .....

Sakavarsha rudrarasai rahitam ravi sangunam sagatha maasam

Diminish the current Saka year (for finding out ahargana) by 611 then multiply by 12........ the Saka year 611 is thus obtained which is 689 AD.

Lallacharya the author of Sishyadhi vruddhi Tantra belonged to 748 AD, which is deduced as follows. In Sishyadhi vruddhi Tantra, it is thus mentioned: Subtract 420 from the Saka year elapsed. Multiply the remainder severally by 25, 114, 96, 47 and 153. Divide each product by 250. The quotients in minutes should be subtracted respectively from mean longitude of moon, its apogee and node. Jupiter and the sighroccha of Venus. Again multiply the above remainder severally by 48, 20 and 420. Divide each product by 250. Add the quotients in minutes respectively to mean longitude of Mars, Saturn and the Sighroccha of Mercury.

The number 250 signifies the years elapsed after Aryabhatta's period. It shows that Lallacharya corrected, the position of planets, with the constants given by Aryabhatta, by his own observations. According to the formula, 250 years after 420 Saka (Aryabhatta's time) is Saka year 670 (i.e. 748 AD). So in 748 AD Lallacharya composed Sishyadhi vruddhi Tantra.

Manjulacharya, author of Laghumanasa belonged to the last century of first millennia in 932 AD. This could be elucidated by the epoch date calculations adopted by him in Laghumanasa. Beginning of Chaitra in Saka year 854, which corresponds to Saturday, at noon, on March 10, 932 AD, Laghumanasa was completed. In Laghumanasa chapter 1 and 2, positions of planets have been given and that date was taken as the epoch era. On Saturday noȯn on Saka year 854 (932 AD), the ascending nodes of the planets on the Mesha Sankranti day are:

Moon 249° 56′	Mars 40°	Mercury 20°
Jupiter 80°	Venus 60°	Saturn 100°

Above positions of planets when astronomically calculated agree well with modern estimations. Thus the period can be concluded for Manjulacharya's Laghumanasa as 932 AD.

कृतशरवसुमितशाके चैत्रादौ सौरिवासरमध्याह्ने .....

Kruthasaravasumitha saake chaitradow sowrivaasaramaddhyaahne.....

Sreedharacharya, author of Patiganita book, lived around 991 AD. There is clear evidence for the period of Sridharacharya, in one of his writings, namely Nyayakandal. This book was said to be written by the request of a scholar namely Pandudasa. At the end of the Nyayakandal information of the year is given:

त्र्यधिकदशोत्तर नवशतशकाब्दे न्यायकन्दली रचिता ।
श्रीपाण्डुदास याचितभट्टश्रीधरेणेयम् ।।

Thryadhika dasotthara navasatha sakaabde nyaayakandalee rachithaa
sreepaandu daasa yaachitha bhattasreedhareneyam

In Saka year 913, was this Nyayakandal composed at the request of Sri Pandudasa, by Sridhara.

According to scholars this Sridhara wrote the Patiganita. The year is Saka 913 (991 AD). There is another opinion based on the observations made in the books of Mahavira (whose date is clearly known as 850 AD), in which a number of mathematical problems, rules and quotations with Sridhara's name are reproduced. Ganitasara sangraha and Misravyavahara of Mahavira too contain quotations from Sridharacharya's Patiganita. These two books also give evidences for an earlier period for Sridharacharya. Hence some scholars also fix Sridharacharya's period around 750 AD.

This date is becoming more and more acceptable for the historians of ancient Indian mathematics, becuase more proofs have been made available from more texts, now.

Vateswara, the renowed astronomer and mathematician was born in 880 AD. In Vateswara siddhanta (I. sec. 1.vs.21) he states his year of birth and age at the time of composing this book.

शकेन्द्र कालद्भुजशून्य कुञ्जरैरभूदतीतैर्मम जन्म हायनैः ।
अकारि राद्धान्तमितैः स्वजन्मनो मया जिनाब्दर्द्युसदामनुग्रहात् ।।

Sakendra kaaladbhuyasoonya kunjarairabhoodatheethairmama janmahaayanai:
akaari raaddhanthamithai: swajanmano mayaa jinabdardyu sadamanugrahaath

When 802 years had elapsed since the commencement of Saka era, my birth took place and when 24 years have passed since my birth, this book (Vateswara siddhanta) was written by me by the grace of heavenly bodies.

Obviously it is clear that he was born in Saka 802 (880 AD) and Vateswara siddhanta was written in 904 AD.

Bhaskaracharya II has also given his date of birth by himself. (Goladhyaya, in Siddhanta siromani, verses 58-63)

रसगुणपूर्णमही समशकनृपसमयोƒभवन्ममोत्पत्तिः।
रसगुण वर्षेण मया सिद्धान्तशिरोमणिः रचितः।।
गणितस्कन्धसंदर्भेƒदभ्रदर्भाग्रधीमतः।
उचितोƒनुचितो यत्मे धर्ष्ट्यं तत् क्षम्यतां विदः।।

Rasagunapoornamahee samasakanrupasamayoabhavanmamotpatthi:
rasaguna varshena mayaa siddhantasiromani rachitha:
ganithaskandha samdarbheada bhradarbhaagradheematha:
ychithoanuchitho yanme dharshtyam thath kshamyathaam vida:

From this "words of rasagunapooornamahee" of bhoothasankhya it is clear that his year of birth was Saka year 1036 (1114 AD).

Another method of citing the period of authors can be seen in the case of Mallikarjuna. He has not mentioned his date of birth anywhere in his writings. But he used 1100 and 1107 Saka years for illustrating astronomical rules. He also used the same numbers, for calculating ahargana in his commentary on Suryasiddhanta and also for illustrating the rule for vyatipata. For determining the ahargana in his own words called Thithichakra, 1100 Saka year was subtracted from, total number of Saka years. Hence it can be concluded that the book was composed in the Saka Year 1100 i.e 1178 AD.

Madhavacharya was a famous astronomer of Sangamagrama (modern Irinjalakkuda of Kerala state). Madhava was the teacher of Parameswara (1455 AD), the promulgator of Drugganita school of astronomy. Nilakanta Somayaji (1444-1545 AD) referes to Madhavacharya with Parameswaracharya. On the date of Madhavacharya, the book Sphutachandronnati gives a general indication. For the calculation of mean Sun it has been asked to subtract from current Kaliday, the Khanda 1502008 and 5180 anomalistic cycles of the moon. This would show that the work was composed at that time. This date is 1502008 days of Kali (i.e Kali era 4112 which is 1010 AD + 5180 anomalistic cycles of the moon (Which is equal to 390 years) i.e. 1010 + 390 equal to 1400 AD. Madhava's recently identified work Aganita gives another clue on his date, indicating deductive years for computation of planets:

शकाब्दान् नरलोको(1320)नाद् राघवैर्धीत्सुना कुजः ।
दिव्यलोको(1318)ननीलाग्रै सतत्वज्ञैभजिते बुधः ।।
नवलोको(1340)नसाराङ्गैर्गजै राप्ते गुरुर्भवेत् ।
हेमपुण्यो(1158)नशाकाब्दान् सारवैर्गेर्वरैर्भृगुः ।।

Sakaabdan naralokonaad raaghavairdheetsunaa kuja:
divyalokonaneelagrai- sathatvajonibhajithe budha:
navalokonasaaraangai rgajairaaptae gururbhaveth
hemapunyonasaakaabdaan saaravairgervarairbhrugu:

Deductive years for different planets; Mars, etc are Saka 1320, 1318, 1340, 1158, 1301, 1276 corresponds to 1398, 1396, 1418, 1236, 1379 and 1354 AD. If the date of composition of the book Sphutachandrapti, is taken as the largest deductive year mentioned, it is 1418 AD. This period agrees with both explanations and observations given above.

Indisputable evidences are available regarding the date of astronomer Sankara, Nilakanta's pupil, in the commentary on his teacher's work. He has pointed out in the first and the last verses, of Tanthrasangraha, the chronograms specifying dates of commencement and completion of the work. After giving the natural meaning of the first verse, one has to go to the ka ta pa yadi number system to get the number from Sanskrit letters: (Golasara of Nilakanta Somayajee P. xxiii)

'हे विष्णो निहित। कृत्स्नम्' (16,80,548) जगत् त्वय्येव कारणे ।
ज्योतिषां ज्योतिषे तस्मै नमो नारायणाय ते ।।

He vishno nihithaa kruthsnam jagath thvayyeva kaarane
jyothishaam jyothishe thasmai namo naraayanaya the

The first part for Kali day and the second together with the first, invocation to the Lord. While explaining the above according to Sankara Varman:

आचार्येण इमं श्लोकं आदितो ब्रुवता प्रथमपादेन प्रबन्धारम्भदिन
कल्यहर्गणश्च अक्षरसंख्यया उपदिष्टः

Aachaaryena imam slokam aaditho bruvathaa prathamapaadena prabhandhaaramabhadina kalyaharganascha akshara sankhyaya upadishta:

The two Kali dates 1680548 and 1680553 work out the exact date in Kali year 4601 Mina 26 and 4602 Mesha 1, occurring in April 1500 AD. In Siddhanta darpana (verse 18) Nilakanta's own commentary thereon gives the year and actual date of birth in ka ta pa yadi number system, commonly used by Kerala astronomers.

कलिसन्ध्याष्टमांशे स्वशतांशद्ये गते ततः ।
धनुर्मिथुनयोर्मध्ये प्रायशस्त्वयने उभे ।।

Kalisankhyaashtamaamse svasathaamsadye gate thatha:
dhanurmithunayormadhye praayasastwayane ubhe:

Nilakanta says he was born in Kali date 16,60,181. This works out to 1443 AD, December (Kali year 4545 in the month of Vruschika)

Puthumana Somayajee, one of the well known astronomer mathematicians of Kerala wrote his book in 1353 Saka year. This information is based on the writings of Govindabhatta's Mathematical index. This has been reproduced by another astronomer known as Vadakkumkoor. The verses are as follows:

नवीनविपिने महीमखभुजां मणि: सोमयाज्युदारगणकोऽत्रय:
समभवच्च तेनामुना।
व्यलेखि सुदृगुत्तमा करणपद्धति: संस्कृता त्रिपञ्चाशिखिभूमित
प्रथित शाकसंवत्सरे ।।

Naveenavipine maheemakhabhujaam mani:
somayaajyudaaraganakoathraya: samabhavachha thenaamunaa
vyalekhee sudruguthamaa karanapaddhathi:
samskruthaa thripanchasikhibhoomitha prathitha saaka
samvatsare.

In Naveenavipinam (putuvanam) a Brahmana scholar, renowned mathematician namely, Somayajee, and by him the Sanskrit book Karanapaddhati was written in the Saka year 1353 (1431 AD). This indicates correctly the year of the book.

Laghubhaskareeyam with Sankaranarayana's commentary (P. xv) gives details on Sankaranarayana, the commentator of Bhaskareeya. Sankaranarayana described the process of finding out the ahargana by referring to the origin of Saka era and says *evam sakaabdaha punariha chandra-randra-muni sankyaya asmabhivagataha.* The year in which he wrote the commentary is denoted by these bhootha sankhya numbers: *Chandra-randra-muni* corresponds to 791 of Saka year (869 AD). Another evidence given by Sankaranarayana, in the end of chapter four is *anga-ruthua-ambara-nanda-veda-manu-bhiryate dinaanam gane* which corresponds to 1449066 Kali day i.e 866 AD.

Achyuta, who composed astronomical and mathematical books, was a Keralite and his date of birth and period of composing the work could be elucidated, based on the writing of his student Melpattur Narayana Bhattatiri. The date of demise of Achyuta as given in Katapayadi number system is: vi-dya-tma-sva-ra-sa-rpat which numbers 17,24,514 Kali days and corresponds to 1621 AD. His year of birth was also given by Narayana Bhattatiri in Katapayadi number as 1560 AD. Sputa nirnaya and rasigolasphutaniti are two famous books of Achyuta.

From these examples, it is clear that, the period of birth of the astronomers or during when books were written can be concluded based on the correct saka or kali year and the scientific facts given with great accuracy. In modern approach this proof on the period is inevitable to ascertain the authenticity of the works and their periods.

CHAPTER IV : MANUSCRIPTS OF ANCIENT INDIAN LITERATURE AVAILABLE IN FOREIGN LIBRARIES

From the first part of the literature survey it can be concluded that many books containing innovative scientific knowledge were written and available in ancient India. It is also important to search whether any important information has gone from India which has become the source of inspiration and knowledge, for the foreigners to claim Indian knowledge as their discoveries. The scanning of such migration of literature during the renaissance period, is of great significance because it was the beginning of Modern Scientific Era in Europe.

It is believed that some of the scientific ideas which emerged from Europe during the 14th and the 15th centuries, had their inception in India. Many visitors and travellers including scholars came here for learning and trade during this period. Some of these visitors also stayed here and learned the basic and applied sciences from Indian scholars. Some of them had also taken many literature from India. They might have handed over these manuscripts to the kings, the philosophers and scientists of their country, on their return. Those who worked with these ideas and information might have put in a claim as their original contributions. Is it true? If it is a fact how much truth is in it? How much of Indian information might have contributed to the development of the basic and fundamentals of modern sciences in Europe and other countries. Some of the literature might have been preserved by individuals as their personl collections and some might have also been perished. All those naturally escape the listing. For all those are lost no proofs can be submitted, except extrapolating the observed facts and searching on the books of travellers. Presented here are some titles of books. Only a few important and well known books having great scientific value are listed here. (In the parenthesis are given the year of copying the manuscripts. It is not the period of original manuscript written by its author. If period of copying is unknown it has left without mentioning). Name of the Institute is given where they are at present available.

Verzeichnisse der Sanskrit Handschriften. von. Weber. Berlin

Aryabhateeya (1689) of Aryabhatta I. Aryabhatta siddhanta of Aryabhatta II. Beejaganita, Siddhanta siromani (1653), Lilavati, (1624) Karanakutuhala of Bhaskaracharya II. Brahmagupta's Brahmasphutasiddhanta. Two Manuscripts of Dasaprakarana of Desabala (1481 and 1401). Makarandavivarana of Divakara. Ganesa Daivajna's Grahalaghava. Gangadhara's Ganitamrutasagari (1641). Jnanaraja's Siddhanta sundara (1600). Lagadha's Vedanga Jyothisha (1772). Ganita tatwachintamani (original manuscript by Lakshmidasa 1501). Khandakhadyaka vivarana of Pruthudakaswamin. Romakasiddhanta (unknown author and date). Somasiddhanta of Somanatha (1789). Suryaprakashabijavyakhya of Suryadasa (1538). Two manuscripts of Suryasiddhanta (1621 and 1783).

Catalogue of Sanskrit Mss. in British Museum C. Bandall

Siddhanta siromani vasanabhashya. Vedanga Jyothisha of Lagadha. Yantraratnavali of Padmanabha (1615). Ganitasara of Sridharacharya (1608) and Suryasiddhanta (1828).

Catalogue Memmoire des manuscripts Sanskrit at Palis de la bibliotheca nationale. A. Cbaton. Paris.

Aryabhatta siddhanta, Beejaganita (1793), Siddhanta siromani in Bengali, Karanakutuhala of Bhaskaracharya II. Brahmasphuta siddhanta of Brahmagupta, Gooddarthaprakasika of Ranganatha (1840) and Suryasiddhanta (1790).

American Oriental Society, New Haven, Connecticut.

Aryabhateeya, Baba Daivajna's Panjangasiddhi (1649), Lilavati (1698) and Siddhanta siromani and Ganitadhyaya (1621). Goladhyaya (1835) Karanakutuhala and Siddhanta siromani vasanabhashya. Makarandavivarana of Divakara (1796, 1800 and 1865). Ganesadaivajna's Grahalaghava and his Laghuchintamani (1730). Haridatta's Ganitanamamala (1764). Vedanga Jyothisha of Lagadha. Mahendrasuri's yantraraja (1765). Yantrarajatika of Malayendu (1743). Brahmasiddhanta of Sakalya in German script. Bhasvati of Satananda (1822)

Florentine Sanskrit Mss. Th. Aufrecht. Leipzig.

Lilavati, Karanakutuhala, Siddhanta siromani vasanabhashya (1790). Ganesa's Tithimanjari (1740). Grahalaghava of Ganesadaivajna (1795). Yantraratnavali of Padmanabha (1796).

Library of India office, A. B. Keith, London

Aryabhateeya manuscript in Malayalam. Beejaganita and Lilavati in Telugu. Aryabhateeya sutrabhashya by Bhaskaracharya I, Karanakutuhala, Mahaviracharya's Ganitasarasangraha (1800). Nrusimha's Siddhanta siromani vasana varttika (1800) (Goladhyaya). Bhasvati of Satananda. Ganitasara of Sridharacharya. Suryasiddhanta (1700 and other periods).

Library of India office, Kulius, Eggling, London.

Beejaganita in Bengali (1790). Lilavati (1596, 1673, and many other copies in different Indian languages). Siddhanta siromani (1688) Ganitadhyaya (1790) Goladhyaya (1660, 1688, 1790 and 1800), Karanakutuhala (1510, 1673, 1708 and many other periods). Siddhanta siromanitika with vasanabhashya (1750, 1799). Brahmadeva Ganaka's Karanaprakasha (1752), Brahmasphuta siddhanta (1621). Yantrachintamani vivarana of Chakradhara. Ganesa Daivajna's Bruhattithi Chintamani (1658, 1797 and 1812) and his Grahalaghava (1796). Ganitamruta sagari of Gangadhara (1780). Grahapradeepika an anonymous work (1650). Haridatta's Ganitanamamala (1600). Jnanaraja's Siddhanta sundara (1652 and 1782). Ganitasarasangraha of Malayendu. Grahalaghavatika of Mallari (1520). Siddhanta sarvabhauma of Muniswara (1792). Narayanapandita's Ganitakaumudi. Siddhanta siromani vasana varrtika of Nrusimha (1700 and 1751). Brahmasiddhanta vasanabhashya of Pruthudakaswamin. Panjangapatra of Ramadaivajna (1807) and Yantradipika (1761) of the same author. Sadasivabhatta's Laghukarana (1700). Goodarthaprakasha of Ranganatha (1792). Brahmasiddhanta of Sakalya (1791). Bhasvati of Satananda. Ganitasara of Sridharacharya, Suryaprakasha bijavyakhya of Suryadasa, Suryasiddhanta (1792), Brahmasamhita of Varahamihira (1749, 1813, 1650, 1750) as complete and also as fragments belonging to unknown periods. Yallayya's Aryabhateeya vyakhya (1810)

Universitats Bibliothek zu Leipzig. Theodor Aufrecht. Leipzig

Beejaganita (1780), Lilavati (1676 and 1700), Ganithadhyaya (1690), Karanakutuhala (1678). Bhasvati Ratnadipika (1750) by

Achuta Bhatta. Karanakutuhala tika of Ekanatha (1647 as part). Ganitamrutasagari of Gangadhara (1676), Kautuka - lilavati (1780) of Rama. Ganitadhyayatika of Ramakrishna (1700) and his Bijaprabhoda.

The library of Trinity college Cambridge. Th. Aufrecht

Aryabhatta siddhanta, Ganesadaivajna's Grahalaghava, Ganita Tatwachintamani of Lakshmidasa (1600). Ukarakhya grantha of Nayanasukhopadhyaya (1803)

Many other manuscripts have also been cited in different bibliographies. However the above mentioned can give an overall idea of the number of hand written ancient scientific manuscripts that have gone from India. There can also be many manuscripts which might not have been taken to any libraries or institutes, by the people who have taken them from here. Some of them might also have got destroyed due to the poor olden preservation techniques. In India, the copying, of manuscripts on palm leves in different languages used to be a custom, and it was part of rituals. This type of copying could not have been done in the foreign Institutes where these manuscripts were stored. The prime reason for this, is the lack of knowledge on the script and language.

There were many foreign scholars who learned Sanskrit from India and abroad. Details available from the literature show that this might have been one of the probable route through which foreign scholars/scientists got Indian knowledge. History of such transfer of knowledge, in the beginning of modern scientific era, in astronomy and mathematics have been discussed, further.

Foreign Scholars Who Studied Indian Scientific Contributions

From the prehistoric periods Indian continent was easily approchable for many foreigners . India, the cradle of culture and civilization with thousands of years of scientific and spiritual development , on almost every branch of knowledge on human life, played a very important role in the development of various human endeavours. Historical evidences are available on the trasfer of knowledge millennia before Christ. In the 6th century BC, the Achaemenian Empire and the Greco Bactrian Kingdom provided an effective bridge between India and the Mediterranean world. S.N. Sen discusses this in detail[6]. The same author gives details on transmission of scientific ideas from India to foreign countries in ancient and medieval times[7]. It is believed that Pythagorus could have learnt geometry from India.[8] From the technological knowledge transfer side, the metallic ingot exchanges through Lothal ports before 2000 BC is an astonishing information for the historians of the Indian scientific achievements[9]. This shows that travelling, exchange of ideas and materials of scientific and technological importance were common during the pre-Christian period. It is also noteworthy that application of practical geometry from Sulbasutras thrived here during this period[10]. Varieties of Vedic rituals were so common and prevalent. (One of the attractive dogma of Buddhism was that, it stood against animal sacrifice in common Vedic rituals conducted during these periods.) Thus the geometry of these altar constructions should also have been in the finger tips for many priests throughout India, which were practically learned by the foreign scholars and travellers during their visits[11]. A little before or during the beginning of the Christian era, commercial relations were established between India and Ptolemic Egypt and Romes' eastern empire. Some Egyptian and Roman words were borrowed and used in the ancient Indian Astronomical books and vice versa. This transfer appears to be another set of proof of exchanges between India and Europe[12]. Instances of transmission of Indian medical prescriptions and theories have been noticed in the Hippocratic collections, Plato's Timaeus and in the writing of Roman physicians and encyclopaedias such as Celsus, Scribonius Largus, Plyni and Dioscorides which were dated beween 500 BC and 200 BC[13]. The return of Alexander the Great from India could have facilitated the transmission of more information on the Indian civilisation to the rest of the world. Valuables, including 30 pounds of woortz steel, were given to Alexander and his army on their return[14]. This could have been another milestone in exposing the land and its rich scientific and metallurgical heritage, to the rest of the world. Much more knowledge could also have been taken or given in the form of manuscripts of scientific texts. This information naturally finds its way to other countries. Babylonian astronomical parameters can be seen in Varahamihira's Panchasiddhantika[15]. In 150 AD, an astronomer namely Yavaneswara translated into Sanskrit, a Greek astrological text which was written in Alexandria[16]. Original of the above text is lost and details are mentioned by Sphutadwija's writings of the 3rd century AD, which has been preserved. These exchanges were not limited between the European countries and India but also have crossed the Himalayas and gone to China. Abundant literature exist on the subject of Sino-Indian religious and cultural exchanges beginning from the time of Yuch-Chin Dharmaraksha of 3rd century AD and the Kashmirian Kumarajiva of the 4th Century[17]. During this period many Buddhist scholars from India and China actively engaged in the translation of canonical and non canonical texts including a large number of works concerning Brahminical (in Chinese it is pronounced as po-lo-mon) astronomy, mathematics, pharmacy and logics. Records of these activites have been preserved in the catalogues of the Sui and Thang dynasties. Records of Thang dynasty indicate the existence of an active school of Indian astronomers at Chang Nan, engaged in teaching and propagation of Indian Navagraha system of astronomy. These records also say that a number of Chinese astronomers, all bearing the title of Chhuthan (a Chinese transcription of Hindu name; Gautama) compiled astronomical treatises based on Indian elements and translated Sanskrit calendrical work to Chinese [18]. Notable examples are the two Chinese calanders : Chui-Chi-li and Tai-yen calenders. Needham, in his book, Science and culture in China, had observed that, paradoxical as it might appear, Chinese owed to Indian Chhuthan, HsiT, the greatest collection of their ancient and medieval astronomical fragments [19].

Sanskrit scientific works are also known to have played an important role in the foundation of Arabic scientific literature between 750 - 850 AD. Indian works on fables, military and veterinary sciences, medicine and astronomy were in this way incorporated into the corpus of Arabic literature. Arabic astronomical renaissance was possibly brought about by the translation of Indian siddhantas with which the names of al-Fazari (750 AD) and Ya'kub ibn Tariq (770 AD) are intimately associated. Al Fazari was possibly the earliest translator of the Brahmasphuta siddhanta and Ya'kub ibn Tariq, the Khandakhadyaka [20]. Who has prepared the earliest Sanskrit translation to Arabic, Sindhjind and Arkand zijcs still remains unknown. However it is clear that there were earlier translated works from Sanskrit to Arabic like these books[21]. Besides direct translations, several Arabic astronomical tables of Indian origin made use of Indian astronomical elements side by side with those derived from Greek works. Al Khwarizmi's famous astronomical tables of this nature and in their revised version prepared by Maslama al Majrit, exerted considerable influence among medieval astronomic circles in Spain [22]. These tables were translated into Latin by Adelard of bath in the 12th century and only this Latin version have come down to others. Likewise al Khwarizmi's arithmetics, the Arabic version, based on Indian sources, was lost long ago. But the Latin translation under the title Algorithmi de numero Indorum was discovered in Cambridge University Library by prince Baldassare

Boncompagni and published by him in his Truttai d' arithmatics (1857)[23].

Among ancient and medieval scholars who contributed in translating Sanskrit scientific works, the most outstanding was Abu Muhammad bin, Ahmad al biruni (973-1050AD)[24]. Besides his descriptions on India in Kitab fi Tahquq ma li 1 Hind, he composed more than twenty books on India, including a number of works as translations of Sanskrit text to Arabic. These translations included astronomical calculations found in Varahamihira's Brahmasiddhanta, etc., He is known to have also prepared a Sanskrit version of Ptolemy's Almagest, Euclid's elements and one of his own works on astrolabe. He spent about ten years in North West India, collected a large number of Sanskrit manuscripts and learned Sanskrit to undertake such ambitious programmes of translation by himself with the aid of pandits and dictionaries. Infact we hardly come across another Indologist of the stamp of A1 biruni until the 19th century.

Similar observations can be made on the transfer of technological knowhow in the metallurgical sector also. Many foreign travellers collected large amount of metal samples, spent days together in Rajastan and near by places, where metals were extracted, for learning the process. Thus after acquiring the first hand knowledge, they went back from India. Details are discussed in the metalergical section.

Studies During the Beginning of Modern Scientific Era

Modern studies on astronomy and mathematics have their beginning from the publications, of Giovanni Dominique Cassini, the results of his studies on Sanskrit manuscripts in the Memoirs de-1 academe Royale des Sciences [25] (the memires originally appeared in De La Laubere's *Relation de Siam II*. This is in French and published during 1691 - 1699. The manuscripts were taken from India to Paris in 1687 by M. De la Laubere who had been to Siam on an embassy by Louis XIV [26]. The paper contained rules for calculating mean motions of the Sun and moon. Cassini gave exact translations in French of several rules given in these manuscripts along with the commentary. He showed that Hindus have taken their year length to be 365 days 6 hours 12 minutes and 56 seconds (based on Aryabhatta's observations). The epoch astronomical constants which started on Saturday, March 21, 638 AD and other information that specifically mentioned in Indian Sanskrit literature were also translated by him. Details on ahargana, as specifically calculated were also given based on Indian methods. James Burges conjuctured that the above Siamese manuscript was probably on the Paulisa siddhanta [27]. He explained the scientific matter under the title: Burges James notes on Hindu astronomy and the history of our knowledge of it, in the Journal of the Royal Asiatic Society.

The work carried out by Le Gentil was commendable. He was a distinguished French scholar and astronomer who visited India in 1769 to observe the transit of Venus and utilised this opportunity to study Hindu astronomy [28]. He obtained scientific information from the Brahmin astronomers and faithfully recorded them along with his own explanations and mathematical calculations which he mentioned as '........ Based on oral instructions received from Tamil astronomers and calendar makers ....' . This was written in French and published in 'Histore de 1' Academe Royale des Sciences[28] in 1772, as a series of articles. This work was very important for the earlier exposures on Indian astronomy in a European language. For the Europeans this processing of computation of solar and lunar eclipses was an entirely new information. Some of the elements of Suryasiddhanta and Laghuaryasiddhanta were incorporated in this work. In 1777, Robert Barker of Bengal military service, described the masonry astronomical instruments of the Benares

Observatory with drawings and published in English as: 'Brahmin's Observatory at Benares' in the Journal of Philosophical Transactions of the Royal Society [29]. Descriptions of Jai Singh's observatories at Jaipur, Delhi, Ujjaini and other places also appeared in Bernoulli's German and French translations of Tieffenthaler's Description of India. In Latin it was published as Das Patr Joseph Tieffenthalers historisch-geographische, in 1785 - 1789 and another publication in German, Bescgheribung von Hindustan [30]. In French, descriptions historique et geographique de 1' inde (3 vols- 1786 - 89) were again published.

'Traite de 1' Astronomie indienne et orientale' the scientific article summarising earlier findings of Cassini and Le Gentil and utilising new manuscripts accumulated in Paris (which remained unnoticed) Historie de 1' Astronomie ancienne was published in 1787 [31]. Panchanga siromani manuscript was sent from India in 1750 by M. Joseph de Lisle to the French Jesuit Father Xavier du Champ and transmitted to the noted Jesuit astronomer, Father Gaubil in China who was unable to study it, sent it back to M. De Lisle. His commentaries and calculations were marked with great scholarship on Hindu astronomy. This work could generate a lot of interest among European astronomers. Based on this, there appeared a series of articles in 1790 in the Transactions of Royal Society of Edinburgh (vol 2)[32] and in the Asiatic Researches (vol 2) published by the Asiatic Society of Bengal [33]. In two of the articles, William Jones discussed the chronology of the Hindus in which astronomical elements were pressed into service. John Playfair a mathematician- geologist and natural philosopher, published a long dissertation on the astronomy in 1790 with a title 'Remarks on the astronomy of Brahmins'. It appeared in the Transactions of the Royal Society of Edinburgh[34] giving relevant passages directly from the Sanskrit astronomical books. His quotations, reproduced here show that, in Europe, this knowledge could definitely win an astounding appreciation on the achievements of ancient Indian Scientists:

"The constructions and these tables imply a great knowledge of geometry, arithmetic and even of the theoretical part of astronomy. But what, without doubt is to be accounted, the greatest refinement in this system, is the hypothesis employed in calculating the equation of the centre for the Sun, moon and planets that of a circular orbit having a double eccentricity or having its centre in the middle between the earth and the point about which the angular motion is uniform. If to this we add the great extent of geometrical knowledge required to combine this and the other principles of their astronomy together and to deduce from them the just conclusion; the possession of a calculus equivalent to trigonometry and lastly their approximation to the quadrature of the circle, we shall be astonished at the magnitude of that body of science which must have enlightened the inhabitants of India in some remote age and which whatever it may have communicated to the Western nations appeares to have received another from them ...."

In 1789, Samuel Davis undertook study and analysis of a copy of Suryasiddhanta and published his papers in Asiatic researchers [35]. Brahmasiddhanta, Vishnudharmottara purana, Paulisa - Soma - Vasishta - Arya - Romaka - Parasara - Arsa - Siddhantas and Sakalya samhita, Grahalaghava and Siddhantarahasya and Makaranda Sarani, were critically studied and included in his report. Later, Davis came out with another important paper dealing with the 12 year cycle of Jupiter in which he used the statements contained in the Suryasiddhanta, Siddhanta siromani and Jyotishavedanga[36]. He translated with the help of Pandits (their names are not mentioned), Bijaganita and Lilavati of Bhaskaracharya II. But he did not publish them.

Similar to what was explained above, first hand knowledge and research/study materials were obtained for the European scholars, who studied the ancient Indian mathematics and astronomy. Some more names of those famous scholars and their study reports are given below. (The list is presented in alphabetical order of their names and not on the chronological order).

Bailey Jean Sylvan Traite de 1' Astronomie Indienne et orientale (1787)

Bell, E. T. Mahavira's Diophatine system (1946)

Bentley, J. Historical view of the Hindu Astronomy 113 - 17, Calcutta 1823. B. On the Hindu system of astronomy and their connection with history in ancient and modern times (1823). C. On the antiquity of the Suryasiddhanta and the formation of the astronomical cycles therein contained (1819)

Brennand, W., Hindu astronomy (1896)

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Burgess, E., On the origin of the lunar division of the zodiac presented in the Nakshatra system of the Hindus (1862).

Burgess J., The sines of arcs in the Panchasiddhantika (1891)

Burrow Reuben. The proof that Hindus knew the Binomial theorem (1790)

Burk, A. (edited and published) Apastamba sulbasutra with German translation, notes and comments with the title: Das Apastamba sulba sutra ZDMG, 55,543-591,1901; 56, 327-391, 1902.

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D. Note to an essay on the notions of Hindu astronomers, concerning the precision of equinoxes (1816) Miscellaneous Essays Published in 1837: Notes on illustrations: A. Astronomy of Brahmagupta B. Brahmasiddhanta title of Brahmagupta's astronomy. C. Verification of the text of Brahmagupta's treatise on astronomy D. Age of Brahmagupta inferred from the astronomical data (1837)

Drenckhahn Friedrich: Sur Zirkulatur des Quadrats und Quadratur des Kreises in den Sulba sutras (1936).

Fleet.J., Some Hindu values of the dimensions of earth JRAS 463-470, 1912. B. Aryabhatta system of expressing numbers JRAS 109-126, 1911. C. Tables for finding the mean place of Saturn JARS 741, 1915. D. The ancient Indian water clock (1915)

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Hornle, A.F.R. Notices under History of Asiatic Society for 1882. B. Verhandlungen des Sriebenten Internationale Orientalisten Congress (1886) C. The Bakshali manuscript (1888).

Hunter, W., Some account of the astronomical labour of Jayasimha (1799)

Jacobi Herman, Methods and tables for verifying Hindu dates, Thithis, Eclipses and Nakshatras (1888)

Jones Sir William, A supplement to the study of Indian chronology (1807).

Kaye, G.R., Hindu astronomy (1924). B. Two Aryabhattas (1910). C. Ancient Hindu Spherical Astronomy (1919). D. Notes on Indian Mathematics (1908). E. The Bakshali manuscripts (1912)

Le Gentil, memories sur 1' Inde' Histore de 1' Acad. Royal des Sciences, annee 1772. B. Mem de math et. phys 1776.

Levi, B. observazioni et Congetture sopra geometrica degli Indiani.

Loschhorn, K, Uber das Alter des Pythagoraischen Lenhrsatzes

ZMU 33, 183, 1902.

Milhaud Gaston, La geometrie d' Apastamba RGS 21, 521, 520, 1910.

Muller, C. La mathematik der Sulbasutra AGM 7, 173 - 204, Hamburg University 1929

Needam. J. Science and civilization in China (1954)

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Potts, D. H., Solution of Diphantine system proposed by Bhaskara (1946)

Rodet, L. L Algebre d'al-Kharizmi et les methods Indiennes at Grecques, (1878), B. Sur la veritable signification de la notation numerique inventee par Aryabhatta (1880)

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Spottiswoode, W., Notes on the supposed discovery of the principles of differential calculus by an Indian astronomer (1860). On Suryasiddhanta and Hindu method for calculating eclipses (1863)

Strachey, E., Bijaganita: on the Algebra of the Hindus (1813)

Suter, H.G., Uber vielecksformel in Bhascara's Lilavati (1905) B. Eine Indische Methode der Berchnung der Kugelober flache (1908)

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Vogt H., Der Pythagorische Lehrsatz in der altestwn geometrie der inder SG Section 84, 3, (1906) B. Haben die alten Inder den Pythagorischen Lehrasatz und das Irrationale gekannt (1907)

Warren.J., Kala sankalita - a collection of memoris on the various modes according to which the nations of southern India divide time (1825)

Weber.A., Uber den Veda-Kalender Namens Jyothisam (1862) B. Uber den auf der Konigl. Bibl. Zu. Berlin befindlichen codex der Suryaprajapati (1868)

Weissenborn. H., Das trapez bei Euclid, Heron and Brahmagupta (1879)

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Winter. H.J.J., et al., Concerning the Persian version of Lilavati (1952)

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Whish, C.M., De la Notation Alphabetique des Indies (1835). B. On the Hindu quadrature of the circle and the infinite series of the proportion of the circumference to diameter exhibited in four Sastras, The Tantrasamgraha, Yuktibhasha, Karnapaddhati and Sadratnamala, (1835)

Zeuthen, H.G., Theorem de Pythagore origne de la geometrie scientifique CRCIP/11. Geneve, (1904) B. Sur Aarithmetique geometrique des Greecs et des Indies BM, Series 3, 5, 97 - 112, (1904.)

CHAPTER V : ANCIENT INDIAN CONTRIBUTIONS IN MATHEMATICS

In ancient India mathematics had an esteemed status among all branches of knowledge. Aptly, ganita was compared to the jewel on the hood of serpents and with eyes among the sense organs. Jyothisha is one among the six Vedangas of Vedic and related literature. Jyothisha include astronomy and mathematics. In modern times also the students of mathematics are expected to learn astronomy as a part of their curriculum. In Vedanga

Jyothisha, it is thus said (Yajusha Jyothisham 4):

यथा शिखा मयूराणां नागानां मणयो यथा।
तद्वद् वेदाङ्गशास्त्राणां गणितं मूर्धनि स्थितम्।।

Yathaa sikhaa mayooraanaam naagaanaam manayo yathaa thaduat vedaangasaasthranaam ganitham moordhani sthitham

Similar to sikha on the head of a peacock and jewel on the hood of a serpent, the status of mathematics is on the forehead of Vedanga sutras.[36]

In modern science, every branch of knowledge studied with the support of mathematics. It is so for all scientific, cultural and social subjects. The ancient Indians had the same opinion on the application of mathematics for various human activities. Thus says Mahaveeracharya (Ganita Sarasangraha 1-9-ii):

लौकीके वैदिके सामायिकेऽपि यः व्यापारस्तत्र सर्वत्र संख्यानुपयुज्यते!
कामतन्त्रेऽर्थशास्त्रे च गान्धर्वे नाटकेऽपि वा सूपशास्त्रे तथा वैद्ये
वास्तु विद्यायादि वस्तुषु बहुभिर्विप्रलाभैः किं त्रैलोक्यसचराचरैः
यत्किञ्चिद्वस्तु तत्सर्वं गणितेन विना न हि ।।

Lowkeeke vaidike saamayike fpi ya: vyaapaarasthathra sarvathra sankhyaanupayujyate kaamathanthre frthasaasthre cha gaandharve naatake fpi vaa soopasaastre thatha vaidye vaasthu vidyadi vastushu bhahubhirvipralaabhai: kim thrailokya sacharaacharai: yatkinchivasthu thatsarvam ganithena vinaa na hi

Mathematics is used in all calculations. It has a role to play in all common social activities, Vedic rituals, commercial and transactional activities, in sex, pure and applied sciences, music, dance, drama, cooking, medical sciences, architecture, etc. Other than the above, for better understanding and application of various branches of knowledge, mathematics is used by learned people. It is said that without the application of mathematics, nothing can exist in three worlds.

As mentioned above, modern science has accepted the principle that for every branch of social and scientific studies, statistical evaluations and accounting are inevitable. This supports the ancient Indian view on the application of mathematics.

Mathematics is said to have a systematic beginning in India, which was connected with the development of Vedic rituals. Mathematics is an important part in Kalpasutras, which is known as Sulbasutras. In the Sulbasutra too, a continuous development and refinement on many geometrical calculations had been taking place from the books of earlier origin to those of later origin. The mathematical knowledge has been developing for achieving better accuracy in results and methods for applications. It is a well known scientific temper attribute that every knowledge has to be continuously refined. Thus say scholars on achieving prefection in mathematics.

कालान्तरे तु संस्कारश्चिन्त्यतां गणकोत्तमैः

Kaalanthare thu samskaaraschinthyathaam ganakotthamai:

During the course of time, great mathematicians should refine the subject (rules and equations on the subject).

This principle obviously declare the rational and scientific approach Which is the same as that of modern scientists. It is not correct to believe that in ancient India all that was taught by Gurus and teachers were transferred to generations 'blindly' without adequate thought input. Both, in theoretical and applied aspects, this approach of refinement of mathematical principles can be seen in concurrence with a steady progress in the depth of knowledge in subject matter. The books on mathematics written from 1000 BC (Sulbasutras) to the later books written in the 18th and 19th centuries AD, this progress in content is clearly visible. The search of modern concepts in mathematics has to commence from the development of number systems.

Numbers :

Rig Veda is the oldest literature, hitherto available, of the human race. It contains fundamentals of the number systems. But a very systematic presentation of numbers can be seen in different texts of Yajurveda (xvii. 24, 25)

एका चमे तिस्रश्चमे पंच चमे सप्त चमे नवचमे एकादश च मे त्रयोदश च मे पञ्चदश चमे सप्तदश चमे नवदश चमे एकविंशतिश्च मे त्रयोविंशतिश्च मे पञ्चविंशतिश्चमे सप्तविंशतिश्चमे नवविंशतिश्चमे एकत्रिंशच्चमे त्रयस्त्रिंशच्चमे......

Eka chame thisraschame pancha chame saptha chame navachame ekaadasa cha me thrayodasa chame panchadasa chame sapthadasachame, navadasa chame ekavimsathischame, thrayovimsathischame panchavimasthi schame sapthavimsathischame navavimsatischame eka thrimsaschame thryasthrimsachha me ....

This is an arithmetic progression of odd numbers starting from 1 and ending in 33, proportionately (with common difference of 2).

Even number progressions are also given in the Yajurveda book (xviii - 23). Presentation of numbers, as a series of multiples of ten is yet another observation noted in Yajurvedic Vajasaneyee samhita (xvii. 2), showing an experienced handling of numbers in mathematics. It gives the proof on the first use of decimal places in writing numbers as early as 2500 BC (a few historians fix this period for Yajurveda and a few others go further back to 4000 BC).

एकं च दशं च दशंच शतंच शतंच सहस्रंच
सहस्रंचायुतं च अयुतं च नियुतं च नियुतं च प्रयुतं च
प्रयूतंचार्बुदंच समुद्रश्च, मध्यम्चान्तश्च परार्धश्चैता ......

Ekam cha dasam cha dasam cha satamchasatam cha sahasram cha sahasram chaayutham cha ayutam cha niyutham cha niyutham *cha prayutham cha prayutham chaarbudam cha samudrascha madhyam chaanthascha paraardhaschaithaa....*

One, ten, hundred, thousand, one lac, one crore upto ten thousand crore is given here. Similar presentations are available in Taitireeya samhita (1.5.11.1), which is another recension of Yajurveda itself.

Use of numbers for presenting data on length, breadth and area of the Yajna saala (sacrificial/ritual hall), can be seen in all the four important Sulbasutras namely, Boudhayana, Apastamba, Katyayana and Manava sulbasutras. Some of these books are chronologically belong to the same period as those of Yajurvedas.

Opinion put forth by professor Neugebauer on the mathematical content in these texts attracts attention of the subject experts in this area "From the time of Samhitas, the Vedic Indians used the decimal scale without the use of symbols. The expressions of numbers of the scale, eka, dasa, ... upto 18th power of 10 were given in Sulba sutras....... The successive placing of dasa, sata, sahasra, etc. is obvious proof for the decimal places..." Says Neugebauer, in his celebrated book, 'The Exact Science in Antiquity' (pp 10, 13-14). In the comment 'without the use of symbols' Neugebauer might have meant only that in the text they have not given the symbols. It cannot be taken as an observation on the non existence of symbols for writing numbers. Because in Rig Veda, it is mentioned 'give thousand cows whose ears are marked with the number eight' (*ashtakarnya*). Marking a number eight shows knowledge in number system.

It is important to note here that when Greeks were using only upto a maximum value Myriad (1000) and Romans, Millie (1000), Indians could go upto 18th power of 10 level during the Vedic period. Dr. Hopkins gives a better picture of the Greek mathematicians of the 1st millennia BC. He says "Before the 6th century BC, all these religious and philosophical ideas of

Pythagorus were current in India." The well known philosopher of ancient Greek, Appolonius has mentioned that Pythagorus went to India and was taught by Brahmins, on the geometrical rules. Not much is known about the European knowledge of mathematics after Pythagorus and Euclid, for nearly a thousand years. It has been told that Leonardo Fibonancii of Pisa spread Hindu numerals in Europe. By 1228 AD, he wrote a book focussing on Indian mathematics namely Liber abaci. Historians say that liber abaci is the stepping stone for the west to the modern mathematics. Evidences are many for this inference. The numerical words penta (pancha), hexa (shasta), septa (sapta), octa (ashta), nona (nava), deca (desa).... penta deca (pancha dasa).... octa deca (ashta dasa)..... etc. are still used in the number systems by the Europeans and the English. All these terms are in Vedas.

Albiruni's book written in 1030 AD, namely Tarik al Hind (Chronicles of India) in which he says that "The numeral signs which we (the people of the west) use are derived from the finest forms of the Hindu signs". A glimpse on the development of mathematics in the second half of the first millennia AD, will definitely give an insight on the real Indian contributions, prior to Leonardo Fibonancii.

In Aryabhatteya 1 to 9th power of 10, places have been mentioned, specifically for the purpose of understanding the rule of writing numbers. (Aryabhateeya 2-2)

एकं दशं च शतं च सहस्रं त्वयुतनियुते तथा प्रयुतम् ।
कोट्यर्बुदं च वृन्दं स्थानात् स्थानं दशगुणं स्यात् ।।

Ekam dasam cha satham cha sahasram thwayuthaniyuthe thathaa prayutham kotyarbudam cha vrundam sthaanath sthaanam dasagunam syaath

One, ten, hundred, thousand, ...to thousand million numbers. From place to place (towards left) each number is ten times to the preceding ones.

Aryabhateeya was famous not only in India but also in Arabia and other countries. Merits of his astronomical and mathematical works were well appreciated the world over. In Arabia the book Aryabhateeya was known as Arjbar - Sindhind which means Aryabhatta's siddhanta. His contributions in mathematics have also been transferred to Arabia and from there to the west. Aryabhatta has also developed another number system which is explained elsewhere in the text.

A systematic presentation of higher/larger numbers can be seen in the texts written after Aryabhateeya. Sreedharacharya in Patiganita (I. 7-8), gives the number from one to ten thousand crore crore i.e 10^{18}, in the order of multiples of ten.

एकं दश शतमस्मात्सहस्रमयुतं ततः परं लक्षम् प्रयुतं कोटिमथार्बुदमब्ज
खर्वं निखर्वं च तस्मान् महासरोजं शङ्कु सरितां पतिं ततस्त्वान्यं
मध्यं परार्द्धमाहुर्यथोत्तरं दशगुणं तज्ज्ञाः ।।

Ekam dasa sathamasmaathsahasramayutham thatha: param laksham. prayutham kotimathaarbudamabja krarvam nikharvam cha thasmaan mahaasarojam sankhu sarithaam pathim thathasthvaanyam madhyam paraardhamaahur yathottharam dasagunam thajnyaa:

The order of starting the number is eka, dasa..... and ending in madhya, parardha. Each number is stated, ten times the preceeding, by those who have knowledge of mathematics.

Further, higher numbers are given seen in Mahaviracharya's (850 AD) book. It gives the number upto 10^{24} (mahakshobha) in Ganita tilaka (55. 2-3) as follows:

एकं दशं शतं सहस्रं दशसहस्रं लक्षं दशलक्षं कोटि दशकोटि
शतकोटि अर्बुद न्यर्बुद खर्व महाखर्व पद्म महापद्म क्षोणि
महाक्षोणि शङ्कु महाशङ्कु क्षिति महाक्षिति क्षोभ महाक्षोभ.....

Ekam dasam satham sahasram dasasahasram laksham dasalaksham koti dasakoti sathakoti arbuda nyarbuda kharva mahakharva padma mahapadma kshoni mahaakshoni sanku mahaasanku kshithi mahakshithi kshobha mahakshobha.........

Yallayya (1480 AD) has given in his Aryabhateeya bhashya the last number upto 10^{29} which is known as bhuri

एक दश शत सहस्र अयुत लक्ष प्रयुत कोटि दशकोटि शतकोटि अर्बुद न्यर्बुद खर्व महाखर्व पद्म महापद्म शङ्कु महाशङ्कु क्षोणि महाक्षोणि क्षिति महाक्षिति क्षोभ महाक्षोभ परार्ध सागर अनन्त चिन्त्य भूरि ..

Eka dasa sahasra ayutha laksha prayutha koti dasakoti satakoti arbuda kharva mahakharva padma mahapadma sanku mahasanku kshoni mahakshoni kshithi mahakshithi kshobha mahakshobha paraardha saagara anantha chinthya bhoori ....

Pavalluri Mallikarjuna starts the number series from one and ends in 10^{36}. This number was called the mahabhuri by him in his book. (Ganita sastra)

A point to be noted here is that rarely a few of the same number terms denote different numbers when used by different mathematicians. Yallayya used bhuri for 10^{29} and Mallikarjuna used bhuri for 10^{35}. (In modern mathematics also billion has two values i.e thousand million (USA) and a million million (UK).

It is clear that writing numbers with many place values has been very common in ancient India even during Vedic period and in the first millennia AD. During this period the Europeans and other country men were only entering the doorsteps of the world of mathematics.

Method of Presenting mathematical data

In ancient India, almost all mathematical data like numerical values, theorems, equations etc., have been presented as slokas (poetical stanzas) which is entirely different from the present style followed in modern mathematics. For easy presentation and keeping the chandas (metres) numbers, three number systems (other than the common Sanskrit system) were adopted by astronomers and mathematicians. This appears to be unique to ancient India. Available literature do not give anything similar to this in other civilisations. However for the understanding of ancient Indian mathematical and astronomical contributions knowledge on all the three number systems viz. Aryabhateeya system, Bhootha sankhya system and Katapayadi system. A glance on all the three systems are given below.

Aryabhateeya number system: This number system was used for the first time by Aryabhatta I in his book Aryabhateeya. In the first chapter of the book known as Geetika Padam, this system is used. The basis of this number system is mentioned by him in the second stanza of the first chapter (Aryabhateeya - 1.2)

वर्गाक्षराणि वर्गे/वर्गे/वर्गाक्षराणि कात् ङ्मौ य:
खद्विनवके स्वरा नव वर्गे/वर्गे नवान्त्यवर्गे

Vargaaksharaani varge fvarge fvargaaksharaani kaath ngmow ya:
khadvi navake swaraa nava varge avarge navaanthya varge ya:

The varga (group/class) letters Ka to Ma are to be placed in the varga (square) places (1st, 100th, 10000th.... etc. places) and avarga letters like ya, ra, la, ... have to be placed in avarga places (10th, 1000th,.....etc. places). (varga letters - ka to ma - have value 1,2,3..... upto 25 and avarga letters - ya to ha - value 30, 40, 50.... upto 100). The 'value' in using this number system is like getting the sum of nga and ma (i.e. 5 + 25) = ya = 30. Nine vowels should be used upto nine place values in the varga (and avarga) places. In the varga and avarga letters, beyond the ninth vowel (place), new symbols can be used.

Sanskrit vowels are as follows: i = 100; u = 10000, ru = 1000000 and so on. Example, cha = 6; chi = 600; chu = 60000. Sum of the values of the letters, with or without vowels give the number. As for gi yi nga sa = gi + yi + nga + sa= 300 + 3000 + 5 + 70 = 3375; Similarly njila = nji + la = 1000 + 50 = 1050.

Using this number system small and large numbers can easily be written. Aryabhatta has used this system for presenting the astronomical and mathematical data. For presenting fractions also this number system can conveniently be used. Thus: Nga, nja, n'a, na mamsaka is 1/5, 1/10, 1/15, 1/20 and 1/25. Mixed fractions can be written as; Jhardham (jha is nine; it's half) = 4½. It appears that only Aryabhatta used this number system and no other mathematician/astronomer has used it for their original contributions, but used in their commentaries on Aryabhateeya.

Bhootha Sankhya System : Bhootha sankhya is one of the most commonly used number systems for presenting data on astronomy and mathematics. It can well combine with the Sanskrit number system. It is said that the earliest work in which Bhootha sankhya is found is Pingalacharya's Chanda sastra (200 BC). Bhootha sankhya can be understood from the following explanations, which are quoted from Sankaranarayana's commentary for Laghubhaskareeyam (1-15,16)

चन्द्रश्शीतांशुरिन्दुश्च चन्द्रमा हिमगु: शशी
एवमादीनि नामानि चन्द्रस्य कथितानि च ।।
रूपमित्येतदेकस्य द्वयोरपि च कीर्त्यते
नयनस्य तु नामानि युग्मं युगलमेव च ।।
यमं च यमलं चैव दस्रौ नासत्य एव च
अश्विनोर्नामधेयत्वात् द्विसंख्येति प्रकीर्तिते ।।
अग्निनामानि यान्यत्र गुणो लोकाश्च पुष्करा:

रामो व्रतं त्रयाणांतु कीर्तितानि बुधैस्सदा ।।
वेदपर्यायशब्दाश्च समुद्रस्य तथैव च
कृतश्चेति चतुर्णांच संख्या सद्भिरुदाहृता ।।
इन्द्रियाणि च भूतानि कामदेवेषवस्तथा
वायुपर्याय शब्दाश्च पञ्चानां तु प्रकीर्तिताः ।।
ऋत्वङ्गरस संज्ञास्युः षण्णां चापि प्रकीर्तिताः
मुनयोगिरिनामानि स्वरपर्याय एवच ।।
सप्तानां गणितं विद्याद्वसुश्च प्रकृतिस्तथा
नागानां चापिनामानि व्याख्यातानि विदुस्तथा ।।
अष्टानामथ रन्ध्रश्च सुषिरश्छिद्रगौरपि
नन्दशब्दो नवानां तु शास्त्रेऽस्मिन् कथितानि तु ।।
दशानां पङ्क्तिसंज्ञा स्यात् दिगित्येतत्तु
कीर्त्यते रुद्राणां भास्कराणां च विश्वेदेवगणस्यच ।।
मनूनां च सुरेशानां तिथिनामानि कीर्तिताः
एकादशादि पञ्चानामष्टिः षोडश कीर्तिताः ।।
अत्यष्टिरिति सप्तानां सदशानां प्रकीर्त्यते
धृतिरष्टादशाख्या स्यात् एवमत्रोच्यते बुधैः ।।
आकाशस्य च नामानि असन्द्विन्दुश्च कीर्तिता
शून्यस्थानेषु सर्वेषु शास्त्रेऽस्मिन् पठिता बुधैः ।।
अनुक्तानां तु संख्यानां यद्दृष्टं तद्विचिन्त्य
वैकल्पनीयं बुधैरत्र प्रसिद्धं बहुषुश्रुतम् ।।

Chandra seethaamsurinduscha chandramaa himagu: sasee
evamaadeeni naamaani chandrasya kathithaani cha
roopamithyedathekasya dwayorapicha keerthyathe
nayanasya thu namaani yugmam yugalameva cha
yamam cha yamalam chaiva dasrow naasathrya evacha
asvinornamadeyatvaath dvsankhyethi prakeerthithe

agninaamaani yaanyathra guno lokaascha pushkaraa:
raamo vratham thrayaanaamthu keerthithaani budaissadaa.
vedaparyaaya sabdascha samudrasya thadaiva cha
kruthaschethi chathurnaam cha sankhya sadbhirudaahruthaa
indriyaani cha bhoothaani kaamadeveshavasthathaa
vaayu paryaaya sabdaascha panchaanam thu prakeerthithaa:
munayo giri naamaani swaraparyaya eva cha
sapthaanaam ganitham vidyaadvasuscha prakruthisthathaa
naagaanaam chaapi naamaani vyakhyaathaani vidusthathaa
ashtaanaamatha randrascha sushiraschhidra gowrapi
nandasabdo navaanaam thu saasthresmin kathithaani thu
dasaanam pangthisamjna syaath digithyethatthu keerthyathe
rudraanaam bhaskaraanaam cha visvedevaganasyacha
manooonaam cha suresaanaam thithi naamaani keerthithaa:
ekaadasaadi panchaanaamashti: shodasakeerthithaa:
athyashtirithi sapthaanaam sadasaanaam prakeerthyathe
druthirashtaadasaakhyaasyaath evamathro chyathe budhai:
aakaasasya chanamaani asandvinduscha keerthithaa
sooonyasthaaneshu sarveshu sastresmin patithabudhai:
anukthaanaam thu sankhyaanaam yaddrushtam thadvichinthya
vaikalpaneeyam budhairathra prasiddham bahushusrutham

For Moon and its synonyms, roopam, etc value is one. Eyes, its synonyms, ears, yamam, yamalam, nose and aswini, denote number two. Fire, gunas, lokas and Rama denote three. Veda, occean, krutha, etc. denote four. Sense organs (organs in general) bhootha, Kamadeva and air denote five. Seasons, vedanga etc., stand for six. Muni, mountains, swara, etc denote seven. Vasu, serpent, prakruthi, knowledge are for eight. Openings (in human body), hole, nanda, etc. stand for nine. Dikh and pangthi give value of ten. Rudra, soorya, viswedeva, Manu and tithi, respectively stand for 11, 12, 13, 14 and 15. Ashti for 16, athyashti for 17, dhruti for 18, say the learned people. Synonyms of sky are used in the place of zero.

Synonyms of all the above nouns too carry the same value and they are commonly used in ancient books. Wherever required, based on rational thinking one can make numbers. Say for example the stars/ celestial bodies stand for 27, teeth (dantha) for 32, masa (month) for 12, Jina for 24, etc. For arriving at the above values, puranic/Vedic/historical/philosphical or observed facts can be taken as references.

Bhrahmagupta, Lallacharya, Bhaskaracharya (I &II) and many others have used this number system in all their scientific contributions. The advantage is that a number of synonyms can be used for writing the data in a poetical stanza style to keep the rhythm/ metre of presentation. Indu (moon) has the number value one. All of its synonyms like Chandra, Tharaanatha, Sasi, ... can conveniently be used for literary beauty of the poetic presentation, where the value 1 is required. An example on the application of bhootha sankhya system; *Vyoma - soonya - sara-adri-indu - ranthra - adri - adri-sara - indavaha* - denote the total number of days in a mahayuga (4320000 years) which is 1577917500, written in bhoothasankhya. A significant point to be noted is that when words are read from left to right, numbers are written from right to left, as it is in modern mathematics. Also, each digit (word) carries the place value too.

Katapayadi number system : This is another system commonly used by astronomers and mathematicians in South India. In a detailed search of literature it is found that use of Katapayadi number was common in the books written by Kerala astronomers and mathematicians. As in bhoothasankhya, where synonyms are used for presenting mathematical data, the letters, in Katapayadi system, with or without vowels play the same role in presenting numbers. It was considered an art in Katapayadi system to make suitable Sanskrit words, using appropriate letters with or without vowels, to present numbers.

In this number system ka to jha, tha to dha (with or without vowels) carry 1 to 9 number values, respectively. Pa to bha (with or without vowels) 1 to 4 values. Ya to ha (with or without vowels) carry 1 to 8 value. Nja, na and all vowels used independently in the beginning are equal to zero. Vowels do not have any value if used with consonants or used independently in between, but at the beginning it stands for 0. In compound consonants the last consonant is to be taken for its number equivalent. Digits are written according to the letters, similar to bhootha sankhya system, i.e from right to left, with place values. Explanation on the use of Katapayadi system of notation is given in Sadratnamala, a book written by Sankaravarman, a famous Keralite mathematician astronomer. Example of the use of Katapayadi number system; *La ku tam* = 113; *A na ntha pu ram* = 21600. (A=0 na = 0 tha = 6 pu = 1 ra = 2)

Kerala Astronomers have used the combination of all the above number systems in their books. More applications of all these number systems are given in the mathematics and astronomy texts.

Discovery and use of Zero

While explaining the number and notations of mathematics, it is interesting to mention the development of numeral 0. It is accepted that zero was discovered by Indians. Pingalacharya's Chandasastra (200 BC) appears to be the first book in which application of Soonya is given for writing numbers as follows:

गायत्रे षड्संख्यामर्धेऽपनीते द्वयङ्के अवशिष्ट स्त्रयस्तेषु
रूपमपनीय द्वयङ्काधः शुन्यं स्थाप्यम् ।।

Gaayathre shadsankhyaamardhe f apaneethe dvayanke
avasishtasthrayastheshu roopamapaneeya dvayankaadha: soonyam sthaapyam

In gayatri chandas, one pada has six letters. When this number is made half, it becomes three (i.e the pada can be divided into two). Remove one from three and make it half to get one. Remove one from it, thus gets the zero (Soonya).

Kane, P.V. [38] in his article, on the decimal notation, which appeared in the journal of Bombay branch of the Royal Asiatic Society (I, 28, PP 159-60 (1953) says that "In view of the discovery of decimal place value concepts in India, it is accepted that 0 as a part of the numerical system is an Indian contribution. Pingalacharya's Chanda sastra first mentioned the word Soonya, describes the rules for calculating the number of long and short syllables in metre of different syllables".

Chanda sastra in the 29th stanza of the 8th chapter says : *Roope soonyam* and in 30th stanza it says: *Dvi soonyam*. Paulisa siddhanta (200 AD) and original Surya siddhanta (400 AD) have also used Soonya and kha (means sunyakasa = space = zero) Vyoma, akasha, ambara in the places of zero in bhootha sankhya.

Commentator Surya Deva has written : *'Kha'ni soonyopalakshithani* which means whenever kha is written it is for 0. खानि शून्योपलक्षितानि

As per the development of the numeral 0 (as it used now), it has undergone only minor changes during the process of its development [39]. The Ancient Kashmiri book (first centruy BC) on Atharvaveda used big circular dotes, sufficiently large, for giving folio numbers. This was first observed by Maurice Bloomfield and Richard Garbe, of Baltimore in 1901, when they were reproducing the Vedic book into films. Bhakshali manuscripts also contain similar small circular dotes representing 0. From available literature one can conclude that the present form of 0 had its birth around the latter half of the first millennia BC, in India. During then, or a near later period using zero in other civilizations is thus explained by Needham, (Journal of Science & Civilization in China). "The Chinese, even upto the

8th century AD, left a gap or some vacant space similar to the Babylonians where 0 was required. A symbol for 0 in the usual circular form appeared only in 1247 AD in the Chinese work: Su Shu Chiu Chang of Chin-Chu-Shao". This refutes the stray claim put forward by some experts that 0 might have been developed in China, by assuming that the growth of astronomy and mathematics in China was at par with that of India during then. Albiruni (973 - 1048) writes that in Indian notation - when zero has to be written it does not have resemblance with ha or : (in Sanskrit).

Calculations with 0:

Results obtained when calculations are done with 0 were well known to ancient Indians. Sreedharacharya's Patiganitha (rule 21) says thus;

क्षेपसमं खं योगे राशिरविकृतः खयोजनापगमे
खस्य गुणनादिके खं संगुणने खेन च खमेव

Kshepasamam kham yoge raasiravikrutha: khayojanaapagame
khasya gunanaadhike kham sangunane khena cha khameva

When a number is added to cipher, the sum is equal to additives, when cipher is added to or subtracted from a number it remains unchanged. In multiplication and other operations, the result is cipher itself.

Rule for division is not given here separetely. Sreedharacharya only says that in other operations also the results are zero. However it is known that when a number is divided by 0, the result is infinity. This has been made very clear in Lilavati by Bhaskaracharya II (page 71 rule 1)

योगे खं क्षेपसमं, वर्गादौ खं, खभाजितो राशि :
खहरः स्यात् खगुणः खं खगुणाश्चिन्त्यश्च शेषविधौ।

Yoge kham kshepasamam, vargaadow kham, khabhaajitho raasi:

khahara: syaath khaguna: kham khagunaaschinthyascha seshavidhow

When 0 is added to a number, value is the number itself. Square of 0 is 0. When a number is divided by 0, the result is infinity (khahara). When multiplied by 0, it becomes 0. Think for their calculation like this.

Bhaskaracharya II, had defined the word Khahara as an endless number equal to infinity (*Anantho rasi*: infinity). Kha (0) hara (dividing) means 'that is divided by zero'.

Sripathi (1039 AD) in Siddhanta sekhara (14-6) gives explanation for the operation with 0 in a better way with more details and clear rules.

विकारमायान्ति धनऋणखानि न शून्य संयोग वियोगतस्तु
शून्याद्धि शुद्धं स्वमृणं क्षयं स्वं वधादिना खं खहरं विभक्ता:

Vikaaramaayaanthi dhanarunakhaani
na soonya samyoga viyogathasthu
soonyaaddhi suddham swamrunam kshayam
swam vadhaadinaa kham khaharam vibhakthaa:

Nothing happens (to the number) when a positive or negative number is added with 0. When +ve and -ve numbers are subtracted from 0, the +ve number becomes negative and -ve number becomes +ve. When multiplied with 0, the values of both +ve and -ve numbers become 0, when divided by 0, it becomes infinity (khahara).

Knowledge about infinity : As mentioned above, infinity was well known for ancient Indian mathematicians. Kha hara is the term used to represent infinity. It is also well defined literarally as *anantho raisi* i.e never ending number. Bhaskaracharya II and others have given clear explanation with examples for infinity. In Bhaskaracharya's Beejaganitha (stanza 20), infinity is thus glorified.

अस्मिन् विकार: खहरे न राशावपि प्रवेष्टेष्वपि नि:सृतेषु
बहुष्वपि स्याल्लयसृष्टिकाले/नन्ते/च्युते भूतगणेषु यद्धत्

Asmin vikara khahare na raasaavapi praveshteshvapi ni: srutheshu bahushvapi syaallaya srushtikaalef nanthef chyuthe bhoothaganeshu yaddhath

Nothing happens to the (huge number) infinity, when any number enters (added) or leaves (subtrated) the infinity. During pralaya many things get dissolved in Mahavishnu and after pralaya, during srushti all those things get out of him. This happens without affecting the lord himself. Like that, whatever number is added to infinity or whatever is subtracted from it, the infinity remains unchanged.

This explanation for infinity is in full agreement with modern mathematics.

Positive and negative numbers and their calculations:

In ancient India knowledge of these fundamental rules on positive and negative numbers were at par with that of present day. Mention has been made about it, while discussing the zero. Brahmagupta and Bhaskaracharya II have given these aspects as early as the 6th century AD. Brahmagupta has given three rules on +ve and -ve numbers in Brahmasphuta siddhanta.

ऋणमृणयोर्धनयोर्घातो धनमृणयोर्धनवधो धनं भवति

Runamrunayordhanayorghatho dhanamrunayordhanavadho dhanam bhavati

When two -ve numbers are multiplied, the resulting number is positive. When a +ve and a -ve number are multiplied, the result is -ve. When two +ve numbers are multiplied the result is +ve.

Later in 1150 AD Bhaskaracharya II has given the rules of +ve and -ve numbers in Beejaganitham, which is a chapter incorporated in his famous astronomical treatise Siddhanta siromani. Sree Krishna Daivajna has written a detailed commentary for Beejaganitham in 1650 AD. Rules for addition and subtration of +ve and -ve numbers are given in Beejaganitham (1.1)

योगे युति: स्यात् क्षययो स्वयोर्वा धनर्णयोरन्तरमेव योग:

Yoge yuthi: syath kshayayo swayorvaa
dhanarunayorantharameva yoga:

One can add and subtract +ve and -ve numbers. The difference between the numbers gives sum of the +ve and -ve numbers.

Krishna Daiwajna, in his commentary on Beejaganitham, gives examples for above calculations with +ve and -ve numbers (1.1).

अत्रप्रथमोदाहरण: -3+-4 योगे जातम् -7 द्वितीये व्यास:
3+4 योगे जातम् +7. तृतीयेन्यास: +3+-4 जातम् -1
चतुर्थ न्यास: 4+-3 योगे जातम् +1

When -3 and -4 are added, the result is -7; secondly when +3 and +4 are added, it is 7, thirdly +3 and -4 are added, it is -1; fourthly +4 and -3 are added, result is 1.

Like this there are three/four types of calculations using +ve and -ve numbers. Says Krishna Daiwajna, in his commentary to the above (1.1)

रूपत्रयं रूपचतुष्टयं च धनं वा सहितं वदाशु

Roopathrayam roopachathushtayam cha
dhanam vaa sahitham vadaasu

Rule for the multiplication of +ve and -ve numbers in Beejaganitham (1.2)

स्वयोरस्वयो: स्वं वध: स्वर्णघाते क्षय:

Swayorasvayo: swam vadha: swarunaghate kshaya:

Among +ve and -ve numbers, when multiplied each other, the result is -ve. Rule of multiplying +ve numbers and -ve numbers among themselves in Beejaganitham (2.2)

धनं धनेनर्णमृणेन निघ्नं द्वयं धनं

Dhanam dhanenarnamrunena nighnam dvayam dhanam

When +ve number multiplied with a +ve number and -ve with a -ve number, the results are +ve numbers.

Krishna Daiwajna makes it clear that, when -ve numbers are multiplied the result is +ve.

अस्वयोर्वध स्वम् *Asvayorvadha swam*

When non +ve numbers are multiplied the +ve number is the result.

Rule for division of +ve and -ve numbers is also given in Beejaganitha (3.2)

भाज्य भाजकयोरुभयोरपि धनत्वे ऋणत्वे कालब्धिर्धनमेव

Bhaajya bhaajakayorubhayorapi
dhanathve runathve kalabdhirdhanameva

+ve and -ve numbers when divided among themselves the results are +ve.

Bhaskaracharya had, not only given the calculations with -ve and +ve numbers among themselves, but also had given rules for determining the squares and square roots of +ve and -ve numbers (Beejaganitham 1.4)

कृति स्वर्णयो: स्वं स्वमूले धनर्णे न मूलक्षयस्यास्ति तस्याकृतित्वात्

Kruthisvarunayo: swam swamoole dhanarne
na moolakshayasyaasthi thasyaakruthithwaath

For +ve and -ve numbers, square is always +ve. Because of the nature of -ve numbers there is no square root (for -ve numbers).

It is interesting to note that the ancient Indians could say that there is no real number as square root for a negative number. In modern mathematics it is an imaginary number. The same explanation is given using the words 'absence of shape': *akrutitwat*.

Number place :

The commonly used number writing method, using number places, is an ancient Indian contribution. I.e. the decimal system. As mentioned in the beginning of the text while discussing number systems the decimal places were well known to ancient Indians from the vedic period onwards. They have defined the place values such as 1st, 10th 100th, 1000th,.......... etc. Ancient Egyptians, Babylonians, Arabs and Chinese followed entirely different systems for writing numbers. Egyptians and Semitic groups followed a method for writing numbers which is in Hieroglyphics system. Separate symbols were used for one, ten, hundred, ...etc. For example, the number 23 is written as 111UU = 1+1+1+10+10.

In Shang Oroacle bone form of Chinese system, separate symbols for one, five to ten, twenty, hundred, etc. were used. For the numbers two, three and four, one is repeated as we write iii for three. Similarly digits were repeated when symbols are not present. Without much improvement the same system was used by the Chinese even upto the 5th century AD. In Greek too, various symbols depicting numbers were used. For writing a larger number, addition procedure was followed in all the non Indian systems. Romans also used symbols for writing numbers and addition method was followed instead of place values.

The number 1548, in Roman system, is written MCCCCCXXXXVIII i.e 1000 + 500 + 40 + 5 + 1 + 1+ 1.

In India the use of symbols without the use of decimal value could be seen in Kharoshthi (250 BC) and Brahmi (250

BC) inscriptions. It disappeared by the 3rd century AD. Inscriptions in Nanaghat (150 BC) and that in Nasik (150 AD) caves and a few other inscriptions namely Kusana (150 AD), Kshatrapas (200 AD) resemble each other [40] where decimal places are clearly used. These numbers are the basis of our modern numeral systems. There is no evidence of the use of the above system of numerals including zero in a decimal place value scale, anywhere in the world, as it was found in India. At the same time the system of numbers with nine symbols from one to nine and the symbol zero, following the basis of decimal places value concept was used in ancient India. According to Sarton[41] "our numerals and the use of zero were invented by the Hindus and transmitted to us through the Arabs and hence known as Arabic numerals which we have given them the name".

Explanations on place values are given by many authors. Aryabhatta says, after mentioning one, ten, hundred, ... upto more than a million, " .... from place to the next place it is the multiple of ten". The place values for digits in a number are mentioned very clearly in Vyasa Bhashya for Yogasutra which is historically an old book written not later than 650 AD. In this book (3-13) place values are thus illustrated.

A woman may be called mother (by her children), daughter (by her father) daughter-in-law (by her mother in law), even though she is the same woman. Similarly even if the digit is the same, depending upon its place, in the number, its value varies. In the unit place the digit has the same value, in 10th place, 10 times the value and in 100th place 100 times the value, is given.

यथा एकरेखा शतस्थाने शतं दशस्थाने दशैवं चैकस्थाने यथा च
एकत्वेपि स्त्री-मात: च उच्यते दुहिता स्वसा च इति

Yathaa ekarekhaa sathasthaane satham dasasthane dasaiam chaikasthaane yathaa cha ekathvepi sthree mathaa cha uchyathe duhithaa svasaa cha ithi

Adi Sankaracharya (700 AD) in Vedanta sutra bhashya (II.2.17) has given similar explanation for writing numbers giving decimal place values.

यथाचैकापि रेखा स्थानान्यत्वेन निविशमानैक
दश शत सहस्रादि शब्द प्रत्यय भेदमनुभवति

Yathaachaikaapi rekha sthaananyathvena nivisamaanaika dasa satha sahasraadi sabda prathyaya bhedhamanubhavathi

One and the same numerical sign when occupying different places is conceived as measuring 1, 10, 100, 1000 etc.

Hence writing the numbers using the decimal places, is an Indian contribution. From the period of Rig Veda, Sanskrit number system was also common here. The Sanskrit number system was followed in the earlier mathematical treatise of Sulbasutras. Hence number places have also been followed.

In Manava Sulbasutra (9.4) the Sanskrit number is given:

नवांगुलसहस्राणि द्वे शते षोडशोत्तरे
Navangula sahasraani dve sathe shodasotthare

The fire altar measures 9216 angula, 9 thousand, 2 hundred and 16.

एकैकस्य सहस्रं स्याच्छते षण्णवतिः परा

In stanza (9.5) such a value is given: *Ekaikasya sahasram syacchatam shannavatihi para*: The area is 1196, 1 thousand, 1 hundred, ninety and six.

एकादश सहस्राणि अङ्गुलानां शतानि षट्
शतम् चैव सहस्राणां क्षेत्रमग्नेर्विधीयते
*Ekaadasa sahasraani angulaanaam sathaani shad
satham chaiva sahasraanaam kshethramagnervidheeyathe*

Area of fire altar is given in stanza (9.6) Area is 111600 sq. Angulas. The period and other details of Sulba sutras are discussed separately. Even in this oldest book one can observe the application of the decimal systems.

In the bhootha sankhya and katapayadi systems also, the number places are well defined and applied. Since bhootha sankhya was known at least as early as 200 BC, it can be assumed that application of number places while writing the numbers, was prevalent during the period.

Use of fractions in mathematical calculations:

The measurements of length, breadth and areas of bricks, sacrificial altars and halls (Yajnasalas) are given in whole numbers, fractions and even in square roots. A few examples are quoted in the geometrical part of this text. [42]

Almost all ancient Indian mathematical texts directly use the fractions in the common application. Bhaskaracharya I in his commentary gives this quotation (123.1)

अर्ध षष्ठं द्वादशभागं चतुर्थभागसंयुक्तं ।
एकत्र कियद्द्रव्यं निर्देश्यं तत्क्रमेणैव ।।

Ardhashashtam dwaadasabhaagam chathurthabhaaga samyuktham ekathra kiyaddravyam nirdesyam thatkramenaiva

$\frac{1}{2}$, $\frac{1}{6}$, $\frac{1}{12}$ and $\frac{1}{4}$ are respectively added together, say what is the aggregate?

Tantrasangraha gives this rule for the addition and subtration of the fractions.

कुर्यात् समच्छिदामेव राशीनां योगमन्तरम्।।

Kuryaat samachidaameva raseenaam yogamantharam

For addition or subtraction, the denominators are to be equalised in a fraction.

In Aryabhateeya bhashy, Bhaskaracharya I (123.3) has given this exercise question, with fractions.

अर्धषड्भागोनं पञ्चांशच्चापि सप्तभागानाम् ।
व्यंश पादोनं वा गणयत कियद् द्रव्यम् ।।

Ardhashadbhagonam panchamsachbhapi sapthabhaagaanaam
vyamsa paadonam vaa ganayatha kiyad dravyam

Calculate O! mathematician what the sum amount to (when added together) $\frac{1}{2}$, $\frac{1}{6}$, $\frac{1}{5}$, $\frac{1}{7}$, $\frac{1}{3}$, $\frac{1}{4}$?

Same principle, which was used earlier is adopted in modern mathematics also. By taking the LCM two or more denominators are equalised and further processed. Aryabhatta I has given the method for operation with fractions (Aryabhateeya 2-27)

छेदा परस्पर हता भवन्ति गुणकार भागहाराणाम् ।।

Chhedaa paraspara hathaa bhavanthi gunakaara bhaagahaaraanaam

The numerators and denominators of multipliers and divisors are multiplied by one another (to simplify the fractions and then to process for addition or subtraction).

Aryabhateeya (2-28) gives this method for the determination of LCM of the denominators of fractions.

छेदगुणं सच्छेदं परस्परं तत् स्वर्णत्वम् ।।

Chhedagunam sachbhedam parasparam thath swarunathvam

Multiply the numerators as also the denominators of each fraction by the denominator of the other fraction; then the fractions are reduced to common denominator.

Making common denominator is like taking the lowest common multiples of two numbers (LCM). Same principle has been explained by Bhramagupta in Brahmasphuta siddhanta (XII.2) and Aryabhatta II in Maha-siddhanta (XV.13). Sreepati in Siddhanta sekhara (XIII.11) has gone deep into this subject. The applied example given by Bhaskaracharya II for the calculations with fractions in Lilavati (2-1) is an impressive one which can be cited for showing the capability of ancient mathematicians to deal with fractional numbers.

द्रम्मार्धत्रिलवद्वयस्य सुमते पादत्रयं यद्भवेत्
तत्पञ्चाँशक षोडशाँशचरण: संप्रार्थितेनार्थिना ।
दत्तो येन वराटका: कति कदर्येणार्पितास्तेन मे ब्रूहित्वं
यदि वेत्सि वत्स गणित जातिं प्रभगाभिधाम्

Drammaardha thrilavadvayasya sumathe paadathrayam yadbhaveth that panchaamsaka shoda saamsa charana: sampraarthithenaa-rthinaa datto yenavaraatakaa: kathi kadaryenarpithastena me broohithvam yadi vetsi vatsaganitha jaathim prabhagaabhidhaam

One man has given to a beggar fraction of 1 dramma (a unit of money). That fraction is one fourth of the one sixth of one fifth of the three fourth of the two third of the half of a dramma. Then tell how much kowdi (a unit fraction of the amount dramma) was given to the beggar?

Answer can be obtained by multiplying all the fractions with the number of Kowdi equal to dramma. i.e. 1/4 x 1/6 x 1/5 x 3/4 x 2/3 x ½ x number of kowdies for one dramma. The work of a mathematical genius is demonstrated in this problem! Here the product of fractions is the product of numerator divided by products of denominators. This methodology for determining the product of fraction is also explained in ancient Sanskrit books. Patiganita rule 33 (i) says:

प्रत्युत्पन्नफलं स्यादंशवधे छेदघात संभवते ।।

Prathyuthpannaphalam syaadamsavadhe chhedaghaatha sambhavathe

The product of a given fraction is obtained on dividing the product of numerators by the product of denominators.

Square, square root and their applications:

Needham, J., in Science and Civilization in China (pp 65-

68) says that the Chinese tried to find out square root (in Chinese language square root is Khifang) by geometrical methods different from that of Aryabhatta and had hardly any success in formalising a rule upto the 12th century AD. Smith D.E in History of mathematics (2, p 148) says thus about the situation in Europe: "In Europe these methods (for finding out the square and square root) did not appear before Cataneo (1546 AD). He gave the method of Aryabhatta for determining the square root."[43].

In India the knowledge on theoretical and applied part of square and square root was atleast as old as Sulba sutras. Uses of square root can be seen in the measurements of length, breadth and area of bricks and the sacrificial halls. Method for finding out square root of 2 and 3 are given in Boudhayana sulba sutra. Dwikarani and Trikarani are the common mathematical terms adopted for square root of 2 and 3. Karani is an earlier term used in all Sulba sutras which was replaced by vargamoolam in the later mathematics books, for square root.

Jaina book on mathematics, Uttaradhyayana sutra, written in 300 BC gives following terms on the mathematical calculations as varga, ghana, vargavarga (4th power) ghana varga (6th power) ghana varga varga (12th power) etc. [44]. Anuyogadwara sutra of 1st century BC gives, 1st square and 2nd square and also their square roots. Aryabhatta in Aryabhateeya (2.4, 5) has given a method for finding out square root of numbers having many digits:

भागं हरेदवर्गान्नित्यं द्विगुणेन वर्गमूलेन ।
वर्गाद्वर्गे शुद्धे लब्धम् स्थानान्तरे मूलम् ।।

Bhaagam haredavargaannithyam dvigunena vargamoolena
vargadvarge suddhe labdham sthaananthare moolam

(Having subtracted the greatest possible square from last odd place and then having written down the square root of the number subtracted in the line of the square root) always divide the even place (standing on the right) by twice the square root. Then, having subtracted the square (of the quotient) from the odd place (standing on the right) set down the quotient at the next place (i.e on the right of the number already written in the line of the square root). This is the square root. (Repeat the process if there are still digits on the right)

Kaye has stated that Aryabhatta's method is algebraic in character and that resembles the method given by Theon of Alexandria [45]. This statement was proved wrong by Clarke, W.E and his colleagues and they declared that Aryabhatta's method is perfect and novel. Similar method has been given by Aryabhatta for finding cube root, also.

अघनाद् भजेद्द्वितीयात् त्रिगुणेन घनस्य मूलवर्गेण ।
वर्गास्त्रिपूर्व गुणित: शोध्य: प्रथमाद् घनश्च घनात् ।।

Aghanaad bhajedwitheeyaath thrigunena ghanasya moola varghena vargaastripoorva gunitha: soddhya: prathamaath ghanascha ghanaath

(Having subtracted the greatest cube from the last cube place and then having written down the cube root of the number subtrated in the line of the cube root) divide the second non cube place (standing on the right of the last cube place) by thrice the square of the cube root (already obtained) (then) subtract from the first non cube place (standing on the right of the second non cube place) the square of the quotient multiplied thrice the previous (cube root); and (then subtract) the cube (of the quotient) from the cube place (standing on the right of the first non cube place) and write down the quotient on the right of the previous cube root in the line of the cube root, and treat this as the new cube root. Repeat the process for larger numbers.

These are the two methods known for finding out the square root and cube root for numbers having any number of digits. The credit of discovering these two methods has to go to

Aryabhatta I. Sreedharacharya gives an important definition for square in Patiganita (rule 24):

सदृशाद्विराशिघातो रूपादिद्विचय पदसमासो (वा) ।
इष्टोनयुतपदवधो वा तदिष्टवर्गान्वितो वर्गः ।।

Sadrusaadviraasighaatho roopaadidvichaya padasamaaso (vaa)
Ishtonayutha padavadho vaa thadishtavargaanvitho varga:

Square is equal to the product of two equal numbers or the sum of as many terms of the series whose first term is 1 and common difference is 2 or the product of the difference and the sum of the given number and an assumed number plus the square of the assumed number.

These can be mathematically represented as:

$n^2 = n \times n$ or $1+3+5+ \ldots n$ terms or $(n-a)(n+a)+a^2$

Similar definition is also given in Ganitasara sangraha (II.29). Sreedharacharya in Patiganita (rule 118) gives a simple method for determining the approximate value of square root of small non squaring numbers:

राशेरमूलदस्याहतस्य वर्गेण केनचिन्महता ।
मूलं शेषेण विना विभजेद् गुणवर्गमूलेन ।।

Raaseramooladasyaahathasya vargena kenachinmahathaa
moolam seshena vinaa vibhaje gunavargamoolena

Of the non square number as multiplied by some large number (having square root) extract the square root, neglecting the remainder and divide that by the square root of the multiplier.

I.e. multiply the number with a large square number and find out the square root of the product. Neglecting remainder, divide that square root by square root of the multiplier. (This gives an approximate square root of small numbers). An example for finding the square root of 10, multiply it with 100 and find out approximate square root of 1000 as 31 (neglect the remainder) divide 31 by the square root of 100 to get 3.1. Approximate value of square root for 10 is obtained. Instead of 100, if large number is used better accuracy would be obtained for square root.

Patiganita (example 99) gives yet another interesting exercise to prove the depth down to which the ancient Indian mathematicians went in search of the application of square root

मूलं शेषात्षष्ट: शेषपदं शेषपञ्चमो दत्त: ।
राशे: शेषस्य पदद्वितयम् रूपाष्टकम् शिष्टम् ।।

Moolam seshaathshashta: seshapadam seshapanchamo dattha:
rase: seshasya padadvithayam roopaashtakam sishtam

A number is diminished by its square root, what remains is diminished by its one sixth, what remains after that is diminished by its square root, what remains after that is diminished by its one fifth and what remains after that is diminished by twice the square root of itself; the residue now left is 8. Find out the number.

Obviously this excellent mathematical problem will stand as a proof of the status of the use of square roots in the 7th century AD in India.

Square and square root of fractions:

Square of numerator divided by the square of denominator is the square of the fraction says Sreedharacharya in Patiganita (34.1)

अंशकृतौ भक्तायाँ छेदनवर्गेण भिन्नवर्गफलं ।

Amsakruthow bhakthaayaam chhedanavargena bhinnavarga phalam

Brahmasphuta siddhanta XII (ii) and Ganita sara sangraha (III.13) also give the same rule for finding square root of fractions. In Aryabhateeyabhashyam, Bhaskaracharya I gives this mathematical exercise for the calculation of square root of a fraction (52.2).

षण्णां सचतुर्थानां त्रयोदशानां (स) चतुर्नवांशानां ।
विगणस्य वर्गमूले वद भट्टसंख्यानुसारेण ।।

Shannaam sachathurthanaam thrayodaasaanaam (sa)
chathurnavaamsaanaam
viganasya varga moole vada bhatta sankhyanusaarena

Calculate in accordance with the Ganita of Bhatta, the square root of 6 + 1/4 and of 13 +4/9 and state the two results.

This mixed fraction is to be converted into simple fraction and square root of numerator and denominator are to be taken. Here Ganita of Bhatta means, following the method given by Aryabhatta I.

Cube and cube root:

Determination of cube and cube root of large numbers was in common practice in olden days. This becomes clear when we go through the methodology and mathematical exercise given in the Sanskrit text books. Aryabhatta thus explains about the mathematical meaning of cube (Aryabhateeya 2-3)

सदृशत्रयसंवर्गो घनस्तथा द्वादशाश्रिः स्यात् ।

Sadrusathraya samvargo ghanasthathaa dwadasasri: syaath

The continuous product of three equals as also the (rectangular) solid having 12 equal edges are called cube.

Similar definition is also given in the Brahmasphuta siddhanta (XVIII. 42), Ganitha sara sangraha (II. 43) and Siddhanta sekhara (XIII. 4) It is interesting that in modern mathematics too the term cube stands for two mathematical meaning as defined in the above stanza. Ghana, in Sanskrit means, a factor of power with the number, multiplied by itself thrice and also a cubical structure. Hence the use of square and cube for two different meanings is an Indian contribution.

Bhaskaracharya II in Liḷavati (1-13) gives an interesting problem for the calculation of cubes and cube roots of a series of numbers in his usual style:

नवघनं त्रिघनस्य घनं तथा कथय पञ्च घनस्य घनं च मे ।
घनपदं च ततोऽपि घनात् सखे यदि घनेऽस्ति घना भवतो मतिः ।।

Navaghanam thrighanasya ghanam thadaa kathaya
panchaganasya ghanam cha me ghanapadam cha thatho fpi
ghanaath sakhe yadi ghane fsthi ghanaa bhavatho mathi;

Tell me O! intelligent, the cube of 9, 3 and 27 and 5, 125 and also the cube root of all these.

Bhaskaracharya I (Bhaskarabhashyam to Aryabhateeya 51.3) gives another excercise for calculating cubes of numbers having double powers.

एकादिनवान्तानां रूपाणां मे घनं पृथक् ब्रूहि ।
अष्टाष्टकवर्गं घनं शतपादकृतेः कृतेश्चापि ।।

Ekaadi navaanthaanam roopaanaam ghanam pruthak broohi
ashtashtakavarga ghanam sathapaadakruthe: krutheschaapi

Tell me separately the cubes of the integral numbers beginning with 1 and ending in 9 and also the cubes of $[(8x8)^2]^3$ and $(25^2)^{23}$.

This problem is an excellent example to show the capability to write the numbers in powers in multiple levels. Such problem is given just to find out the cube root, and was common in those days (529 AD)

Cube and cube root of fractions:

Cube root of fractions have also been determined by the ancient mathematicians. Bhaskaracharya I in his commentary to Aryabhateeya gives this problem (51.4):

षट्पञ्चदशाष्टानां तावद्भागैर्विहीनगणितानाम् ।
घनसंख्यां वद विशदं यदि घनगणिते मतिर्विशदा ।।

Shatpanchadasaashtaanaam thaavadbhaagir viheenaganithaanam
ghanasankhyaam vada yadi ghanaganithe mathirvisadaa

If you have clear understanding of cubing a number, say correctly the cubes of 6, 5, 10 and 8 as respectively diminished by 1/6, 1/5. 1/10 and 1/8.

Here the exercise is to determine the cube of 6-1/6 i.e 5-5/6, similarly 5-1/5, 10-1/10 and 8-1/8. The rule for calculating the answers for the above problem has been well defined by these scholars. In Mahasiddhanta [XV 17(1)] of Aryabhatta II and Ganitasara sangraha (III. 13) it is said that:

अंशस्यघनं विभजेच्छेदस्य घनेन ।

Amsasya ghanam vibhajechhedasya ghanena

Cube of the numerator divided by cube of the denominator gives cube of the fraction.

In Bhaskarabhashyam, a problem for the determination of cube root is given (54.1)

एकादीनां मूलं घनराशीनां पृथक्तु मे ब्रूहि ।
वस्वश्विमुनीन्दूनां घनमूलं गण्यतामाशु ।।

Ekaadeenam moolam ghanaraaseenaam pruthakthu me broohi
vasvaswimuneendoonaam ghanamoolam ganyathaamaasu

Tell me separately the cube root of the cube number 1, etc (2, 3, 4, ...etc.) and also calculate the cube root of 1728.

Bhaskarabhashyam (54.2) for Aryabhateeya gives another exercise for determining the cube root of a larger number.

कृतयमवसुरन्ध्ररसाब्धिरूपरन्ध्रशिवनाग सङ्ख्यस्य ।
मूलं घनस्य सम्यक् वद भट्टशास्त्रानुसारेण ।।

*Kruthayamavasurandravasaabdiroopa randrasvinaga sankhyasya
moolam ghanasya samyakvada bhatta sasthraanusarena*

Correctly state in accordance with the rules prescribed in the Bhatta sastra the cube root of 8291469824.

This knowledge for calculating the cube root of large number is a noteworthy quality of ancient Indian mathematicians. Another complex calculation:

मूलं त्रयोदशानां पञ्चघनांशैस्त्रिशून्यरूपाख्यैः ।
अधिकानां भिन्नाख्यं विगण्यतां सङ्ख्यया सम्यक् ।।

*Moolam thrayodasaanaam panchaghanamsai sthri sooonyarooopaakhyai:
Adhikaanaam bhinnaakhyam viganyathaam sankhyayaa samyak*

Correctly calculate in accordance with the Ganita (of Aryabhatta) the fractional cube root of 13 +/-103/125.

This problem is also given in Bhaskarabhashyam (54.3) Here, two complicated mixed fractions are given (indirectly) for finding out the cube root. The fraction 103/125 is added to, in one of the figures and subtracted from other, and the cube roots are calculated for both mixed fractions. I.e. 13 + 103/125 and 13 - 103/125. An important point worth noting here is the use of +/ - sign in the same problem. It is generally assumed that simultaneous use of + and - in a mathematical statement is of later origin in the West.

Bhaskaracharya II in Lilavati gives this rule (1-13) on cube and square.

वर्गमूलघनः स्वघ्नो वर्गराशेर्घनो भवेत् ।

Varga moolaghana: swagnovargaraaserghano bhaveth

Take the square of the cube of the square root of a number, that is the cube of the number.

Use of Average:

Determination of the average of different values, for a set of measurements, is followed to get more accurate results. Average values are determined for many common calculations. Bhaskaracharya II says thus on the use of average in Lilavati:

गणयित्वा विस्तारं बहुषुस्थानेषु तध्युतिर्भाज्या
स्थानकमित्या सममितिरेवं दैर्घ्ये च वेधे च

Ganayithva visthaaram bahushusthaneshu thadyuthirbhaayyaa
sthaanakamithyaa samamithirevam dairgye cha vedhe cha

(For length, breadth and depth) the measurements should be taken at many places and the sum should be divided by the number of times (places) the measurement is taken.

Ganesa the commentator gives the merits of determining average value of data as follows:

यथायथा बहुषु स्थानेषु विस्तराधिकं गण्यते
तथातथा सममिति सूक्ष्म सूक्ष्म तरा स्यात्

Yathaayathaa bahushu sthaaneshu vistharaadhikam ganyathe
thathaathathaa samamithi sookshma sookshma tharaa syaath

Whenever the measurements are taken at more and more places, then more and more accuracy for values will be obtained.

Brahmagupta has used the method of finding out the average of the surface area for accurately calculating the volume of the cylindrical structures.

Ratio and proportion:

Determining ratio among various parameters is common in mathematical and statistical calculations. This is an important criterion for understanding relative quality / quantity with respect to other parameters. When we say one out of every three persons knows Sanskrit, it gives a ratio of 1:3. This type of mathematical presentation is thought to be the contribution of modern mathematics. Explanation taken from Bhaskaracharya's commentary for Aryabhateeya (2.26 (5)) is given below:

अष्टौ दान्तास्त्रयो दम्या इति गाव: प्रकीर्तिता: ।
एकाग्रस्य सहस्रस्य कति दान्ता: कतीतरे ।।

Ashtow daanthaa sthryo damyaa ithi gaava: prakeerthi thaa:
ekaagrasya sahasrasya kathi daanthaa: katheetharai:

(Out of 11 cattle) Eight are tamed and 3 are to be tamed and (how many are) to be tamed) if the number of cows is 1001?

Here the ratio between the tamed and untamed is 8:3. From this, the number of cattles tamed can be calculated by taking the product of 1001 and 8 and dividing by 11. The same is applicable for determining the number of untamed animals. Calculations using ratio/proportion were known to Indians at least from Bhaskara's period. Descriptions given in the available literature say that this subject is dealt with in Bhakshali manuscripts also. If that is also taken into consideration, the history of ratio starts further back, to about 2000 years.

Permutations and combinations:

When a variety of parameters of variables are available for making the maximum number of combinations, it can be possible in different ways. Number of ways possible under a particular combination can be calculated using the principles of permutations and combinations. In modern mathematics permutations and combinations play a very useful role to get benefits of space, time, and energy savings in scientific and social activities. This has also contributed on aesthetic beauty and savings in space. Rules and detailed scientific explanations for permutation and combination methods and their principles are seen in Jaina text Bhasvati sutra written during 300 BC. In the text writing numbers with different combinations of the same set of digit was discussed.

In Susrutha samhita (6th century BC), 63 possible combinations are explained from a mixture of 6 rasas (flavours) by distributing the rasas in the order of one, two, three and so on. Sreenivasa Iyengar describes this problem in detail.[46] Sreedharacharya in Patiganita (rule 72) gives an example similar to that given in Susruta Samhita:

कटुकतिक्त कषायाम्ललवण मधुरैः सखे रसैः षड्भिः ।
विदधाति सूपकारो व्यञ्जनमाचक्ष्व कति भेदम् ।।

Katukathiktha kashaayaamla lavana madhurai: sakhe rasai: shadbhi:
vidadhaathi soopakaaro vyanchanamaachakshva kathibhedam

Friend, a cook prepared varieties of food with 6 savours: pungent, bitter, astringent, acid, saline and sweet. Say what is the possible number of varieties of food that can be made with these savours.

The combinations are: with one each of the savour, we can have 6 types. With the combination of any two, we can have 15 types, with the combination of any 3, we get 20, with the combination of any 4, 5 and 6, respectively, we get 15, 6 and 1 types. These are the combinations possible. (The same is the answer for problem given on Susrutha samhita)

Permutation and combination given in above method is, exchanging, one or more from the total set. Another type of possible combinations is substituting the one for other in the total combination without keeping anyone/thing away from the list. Bhaskaracharya II gives an interesting mathematical problem of this type for explaining the permutation combination possibilities, in Lilavati. Lord Siva and Lord Vishnu with various Bhooshanas were compared in different forms when each one of the bhooshanas is substituted by other, without keeping away any one.

पाशाङ्कुशाहि डमरूक कपाल शूलैः खड्वांगशक्ति शरचापयुतैर्भवन्ति ।
अन्योन्य हस्त कलितैः कतिमूर्तिभेदाः
शंभोहरेरिव गदारि सरोज शंखचक्रैः ।।

Paasankusaahi damarooka kapaala soolai: khadvangasakthi sara chaapayuthairbhavanthi anyonya hastha kalithai: kathimoorthibhedaa: sambhohariva gadaari saroja sankachakrai:

Pasa, ankusa, serpant, damaru, kapala, soola, khatvanga, sakti, chapa, with these (ten) items how many permutations and combinations are possible for Lord Siva. Similarly with the four items, sanku, chakra, gadha and padma holding in the hands, how many combinations are possible for Lord Vishnu?

It is known for mathematicians that answers for the problems are 1x2x3x4x5x6x7x8x9x10 (i.e 10! read as factorial 10) combinations are possible for Lord Siva and 1x2x3x4 (i.e 4! read as factorial 4) combinations are possible for Lord Vishnu. Thus these two types of permutations and combinations were also brought into application in India.

Percentage (%) application :

In modern mathematics many commercial and financial transactions are referred to with respect to a base number of 100. Hence it is known as per cent. The equivalent word in Sanskrit is satamanam. References were made for presenting financial or other mathematical data with a standard reference to 100. Thousands of years ago it was practised in India. This was a subject available not only from the mathematical texts, but also referred to in the Dharma sastras and puranas. Hence the use of the concept of % has commenced atleast two and half a thousand years ago, i.e during the period of Dharma sastras and puranas. Detailed mathematical discussion with examples on the use of the % is given in this text under the topic of 'loans'.

Interest calculation:

Description and application of interest are also given under the subtitle of loan. Here the subject is brought only for explaining the use of percentage in olden days. A problem from Siddhanta sekhara quoted in Patiganita (rule 48):

मासेन शतस्य फलं पञ्चैको भाव्यकेऽर्धमय वृत्तो ।
लेखकपादो वर्षे पञ्चाधिक नवशतीमिश्रम् ।।

Maasena sathasya phalam panchaiko bhavyake frdhamaya vruttho
lekhakapaado varshe panchaadika navasatheemisram

The rate of interest being 5% per month, the commission of surety 1% per month, fee for accountant ½% and charges of the scribe 1/4% per month, certain sum amounts to 905 a year. Find the capital, the interest and the shares of the surety?

It can be seen that the use of % was very handy for describing different financial parameters. Another important information that can be elucidated from the above problem is, the systematic and modern approach that existed in India even in financial transactions. i.e the concept of surety, accounting charges/fee, scribing charges, etc., being separately levied in the financial business as in modern times. One among many quotations, from Dharma sastra will throw light on, how the financial transaction fall in the rules of those texts. Vishnu smruti gives this quotation [47].

सपादपणा धर्म्या मासवृद्धिः । पणशतस्य पञ्चपणा व्यावहारिकी ।।

Sapaadapanaa dharmya maasavuddhi:
pana sathasya panchapanaa vyaavahaarikee

The dharmic rate of interest is 1.25% per month (for common transactions connected with the household loans) for commercial purposes it can be upto 5% per month.

Partnership, shares and profit sharing:

In modern social system and economical transactions, the partnership business and investment in shares are common. Sharing the principal amount for business, is the concept of partnership. Many business come up and expand through partnerships. In this business a part or total profit is shared among the partners/shareholders. There will be a predetermined ratio for sharing profits/produce based on the principal amount shared for the business by share holders. This is a known practice in the modern business. In ancient India, the literature says that such sharing business was common. Almost all the modern concepts like handling charges, commission, etc. were included in the expenditure part of the profit, other than the charges for writing documents and that for office work. The concept of share business in ancient India was similar to the modern system. A few examples are given here on the mathematical part of it. Sreedharacharya (Patiganita 59.1) gives the rules for sharing profit in partnership business.

स्वयुतिहत प्रक्षोपात्फलेन हन्त्यान्पृथक फलावाप्त्यै ।।

Svayuthi hatha prakshopaath phalena
hanthyaan pruthak phalaavaapthyai

To obtain individual share of profit for partners in the produce, the seeds contributed by partners as divided by their sum should be severally multiplied by the product.

The same is the modern approach of sharing the profit for each shareholder. Bhaskaracharya I in his commentary (119.6) gives the following mathematical exercise on sharing profit when partners collaborate in a business.

समवायकास्तु वणिजः पञ्चैकैकोत्तराधि मूलधनाः ।
लाभः सहस्रसंख्यो वद कस्मै तत्र किं देयम् ।।

*Samavaayakaasthu vanija: panchaikaikottharaadhi mooladhanaa:
laabha: sahasra sankhyo vada kasmai thathra kim deyam*

Five partners collaborate in a business. The capital invested by them are (in the ratio) one and the same number increasing successively by one (i.e 1,2,3,4, & 5) respectively. Profit that accrued amounts to 1000. Say what should be given to whom.

In this problem Bhaskaracharya I gives details on partnership of a business and also the basis of sharing profit on a certain 'ratio'. It is known that each of the individual share is multiplied with the total profit and divided by the sum of the ratios (i.e. 15). He has given another problem of the same nature with ratio in fractions (119.7)

अर्धेन तृतीयेन प्रक्षेपेणाष्टमेन ये वणिजः
सप्ततिरेकेनोना लाभस्तेषां कियान् कस्य ।।

*Ardhena thrutheeyena prakshepenaashtamena ye vanija:
sapthathirekenonaa laabhastheshaam kiyaan kasya*

The combined profit of three merchants whose investments are in the ratio of ½, 1/3, and 1/8 respectively, amounts to 70 minus 1, what is whose profit?

Here the problem has to be solved by converting the fractions into whole numbers and then sharing the profit as mentioned above. Yet another problem given by Sreedharacharya in Patiganita (Rule 59.1 example 72)

एकस्यार्ध प्रस्थास्त्र्यंशोऽन्यस्यापरस्य नवभागः ।
सप्तदशशतानि फलं पृथक् पृथक् किं भवेत्तेषाम् ।।

*Ekasyaardha prasthasthryamsonyasyaaparasya nava.bhaaga:
sapthadasasathaani phalam pruthak pruthak kim bhavetheshaam*

Half (½) prastha is the contribution of one partner, 1/3 prastha that of another and 1/9, that of the third. If 1700 prastha is the produce, say what are their shares, separately.

Solution of the problem is as mentioned above. The concept of profit sharing in a systematic way governed by mathematical rules is clearly given in these problems.

Loans and Interests:

In modern business, loans are as important as partnership and sharing of profits. Other than business purposes, loans are also availed for various social/personal purposes. Dharmasastra literature says that availing loans were common in ancient India. Financial transactions (vyavahara) relating the principal, wealth, interest and related documentations, guarantee, etc., under the control of the king or the finance minister or a Government agency. Hence the rules and regulations connected with financial transactions were not considered as individual problems. All those rules and regulations were codified in the books coming under the Arthasastra (economics). They are part of Dharma sastra, where the subject matter deals only with the guidelines. Almost all the Dharmasastras give detailed descriptions on loans and their terms and conditions [48]. Vishnu smruti (100 BC), a well known Dharma sastra book says thus on the necessity of availing loans.

कुटुम्बार्थमशक्तेन गृहीतं व्याधि तेन वा
उपप्लावनिमित्तम् च विद्यादापत्कृतं तत् ।
कन्यावैवाहिकं चैव प्रेतकार्येषु यत्कृतं एतत्
सर्व प्रदातव्यं कुटुम्बेन कृतं प्रभो ।।

Kutumbaarthamasakthena gruheetham vyaadhithena vaa
upaplava nimittham cha vidyaathaapalkrutham thath
kanyaavaivahikam chaiva prethakaaryeshu yathkrutham ethath
şarvam pradaathavyam kutumbena krutham prabho

Loans are taken for meeting the expenditure connected with economic problems due to family burden, health problems, treatment, education, expenditure during accident, marriage of daughter, for performing rituals connected with the demise of the family members, etc.,

The same is applicable now also! When money is given for the above or for any other purposes, interest can be levied from the borrower. Charging interest on loan can be of four types:

कायिका कालिका चैव कारिता च तथापरा ।
चक्रवृद्धिश्च शास्त्रेषु तस्य वृद्धिश्चतुर्विधा ।।

Kaayikaa kaalikaa chaiva kaarithaa cha thathaaparaa
chakravruddischa saasthreshu thasya vruddhischathurvidhaa

The interest can be charged as manual work equivalent to the money, materials, depending on the period of loan, money on fixed rates and, at compound interest in approved rates.

Rules of charging interest:

In the banking systems, interest on loan amount can be charged on monthly, quarterly, half yearly and annual basis. In ancient India similar procedure was followed. Loan was given to the borrower for varying periods and the interest was also charged at different rates for different periods. Vishnu smruti says thus:

ऊर्ध्वं संवत्सरात् तस्य तद्धनं वृद्धिमाप्नुयात् ।
ऊर्ध्वं मासत्रयात् तस्य तद्धनं वृद्धिमाप्नुयात् ।।

Oordhvam samvatsaraath thasya vruddhimaapnuyaath
oordhvam maasathrayaatthasya thaddhanam vruddhimaapnuyaath

For some money lenders, the wealth is increased (by charging interest) on per annum basis, for some others, it increases on trimonthly basis.

Katyayana Dharma sastra says interest can be charged on loan amount, annually.

अकृतामपि वत्सरातिक्रमेण यथाविहितम् ।

Akruthaamapi vatsaraathikramena yathaavihitham

Whatever may be the rate annual interest, can also be charged

Narada Dharma sastra advocates charging interest on half yearly basis, also

अनाकारिणमप्यूर्ध्वं वत्सरार्धाभिर्वर्धते ।

Anaakaarinamapyoordhvam vatsaraardhaabhirvardhathe

The generally accepted rate of interest on monthly basis is given in Vishnu smruti. However it advocates the need of mutual agreement between the borrower and the lender before finalising the rate of interest. This rule is similar to that, followed at present.

अथ उत्तमर्णः अधमर्णाद्यथा दत्तमर्थं गृह्णीयात् ।
द्विकं त्रिकं चतुष्कं पञ्चकं च शतं प्रतिमासं ।।

Atha utthamarna: adhamarnaadyathaa datthamartham gruhneeyaath
dvikam thrikam chathushkam panchakam cha satham prathimaasam

The loans can be given and taken between borrower and lender. Generally charged interest rates are 2, 3, 4, or 5% per month.

Here, on an annual basis the interest rate comes to 24%, 36%, 48% and 60%. All these rates of interest fall within the accepted/followed rates by public or private - approved or unapproved - banks/ financial institutions, now.

In Kautilya's Arthasastra - dharmic direction for charging interest from common man, for social/domestic need and for business purposes, aiming at making further profits is given:

सपादपणा धर्म्या मासवृद्धिः । पणशतस्य पञ्चपणा व्यावहारिकी ।।....

Sa paadapanaa dharmyaa maasavruddhi:
panassathasya panchapanaa vyaavaharikee

Reasonable (dharmic) rate of interest is 1.25% per month (i.e 15% per annum) on the transactions with common man for non commercial purposes. But for commercial purposes (for making profit out of it) interest rate can be 5% per month.

Vyasa Dharma sastra gives a clear guideline for charging interest from poor men by fixing a maximum limit for the rate of interest.

निराधाने द्विकशतं मासलाभ उदाहृतः ।

Niraadhane dvikasatham maasalaabha udaahrutha:

Interest rate chargeable from a poor man can be (not be more than) 2% per month.

Charging additional amount as penalty interest, when the loan is not repaid on due date was also allowed earlier as it is now. Thus says Narada smruti:

याच्यमानमदत्तम् चेद्वर्धते पञ्चकं शतम् ।

Yaachyamanamadattham chetdvardhathe panchakam satham

If the interest and loan amont were not repaid on due date, the rate of interest can be increased to 5% (the highest permitted rate of interest fixed by any Dharma sastras).

At the same time, if the lender is not taking the money back (or if the money is deposited for getting interest, with the financial institutions) when the borrower gives it, the procedure followed in such a situation is also made very clear in these books. Yogeswara says:

दीयमानम् तं गृह्णाति प्रयुक्तम् यः स्वकम् धनम् ।

मध्यस्थस्थापितम् तत् ·स्यात् वर्धते न ततः परम् ॥

Deeyamaanam tham grahnathi prayuktham ya: svakam dhanam
madhyastha sthaapitham thath syaat vardhathe na thatha: param

If the lender is not taking money (the deposit) back from borrower, interest need not be paid by the borrower (after due date). But it should be informed through some mediators (as a proof that the borrower was ready to repay the loan/deposit)

From the above rules on loan, interst, period and other parameters, one can understand that giving and taking loans for commercial and domestic purposes were prevalent in ancient India. Even though there can be varying opinions on the date of some of these Dharma sastra books, the date of Kautilaya's Arthasastra is well known, i.e the 3rd century BC. In the same spirit of Dharma sastras, Kautilya had given terms and conditions of banking system that existed in India.

Bakshali manuscripts written in the first/second century AD, is said to contain a lot of information on this subject. Shortly after this period, scholars like Aryabhatta, Bhaskaracharya, Brahmagupta and others have systematically contributed to this branch of knowledge, as a separate chapter in ganita. From among many examples, two of Bhaskaracharya I are given (120.10 and 115.2) from his Bhashya to Aryabhateeya.

(शतस्यार्धचतुष्क) मासप्रयुक्तस्य वृद्धिरर्धपञ्चका रूपका
पञ्चशतो दशमास प्रयुक्तस्य का वृद्धिरिति ?

(sathasyaardhachathushka) maasaprayukthasya vruddhirardhapanchakaa
roopakaa panchasatho dasamaasa prayukthasya kaa vryddhirithi?

If 4½ rupakas be the increase (interest) per 100 rupakas for 5½ Months. What will be the increase on 500 rupakas for 10 months?

This is similar to a modern mathematical problem ! Th problem can be solved by finding out interest for one month and then finding out for 10 months, to arrive at the interest for 500 rupakas. (It can be noted that the word Rupee in its present form rupa has its origin at least more than 1½ a millennia ago.

पञ्चविंशतेर्मासिकी वृद्धिर्न ज्ञायते। या पञ्चविंशतेर्मासिकी वृद्धि:सा तेनैवार्धेनान्यत्र प्रयुक्ता, सह वृद्धा पञ्चाभिर्मासैर्दृष्टा रूपत्रय पञ्चभागोनं ।
तत्रेच्छामो ज्ञातुं का पञ्चविंशतेर्मासिकी वृद्धि: ?
का वा पञ्चविंशतिर्वृद्धि: ? पञ्चमास प्रयुक्ताया वृद्धिरिति ? ।।

Panchavimsathermasikee vruddhirna jnaayathe /ya panchavimsathermaasikee vruddhi: saa thenaivaardhenaanyathra prayukthaa sahavruddhaa panchabhirmasairdrushtaa roopathraya pancha bhagonam/ thathrechhamo jnaathum kaa panchavimsathermasikee vruddhi? kaa vaa panchavimsathirvruddhi:? panchamaasa prayukthaaya vruddhirithi?

Monthly interest on 25 is not known. But the monthly interest on 25 rupakas lent out elsewhere at the same rate (of interest) amounted to 3-1/5 rupakas in 5 months. I want to know the monthly interest on 25 rupakas, as also the interest for 5 months on the interest of 25 rupakas.

Sreedharacharya has given the method for the claculations of various parameters on the interest, etc. (Patiganita rule 51).

गतकालफलसमासे मासफलैक्योद्धृते भवेत्काल: ।
शतगुणमासफलैक्ये धनयोगहृते शतस्यफलं ।।

Gathakaalaphala samaase maasaphalaikyoddruthe bhavetkaala: sathaguna maasaphalaikye dhanayogahruthe sathasyaphalam

The sum of the interest accrued (on the given bonds) for elapsed months being divided by the sum of the interest for one month gives the time (in months) and 100 times the sum of interests for one month, being divided by the sum of the capital amount, gives rate of interest in percentage per month.

According to this rule, rate of interest/month is equal to total interest divided by the number of months. That when divided by the principal after multiplying with 100, gives the rate of interest in %. This is the same modern formula used for calculating the interest i.e. PNR/100 = interest. or interest x 100/ principal = rate of interest. From this it is clear that Sreedharacharya is the first mathematician who has given the equation for calculating interest as it is known to modern mathematics.

Compound interest:

Charging two types of interst is followed by modern financial institutions. The simple and the compound interest. Vishnu smruti gives provisions for charging at compound interest rate also, for the loans. The compound interest is thus defined.

वृद्धेरपि पुनर्वृद्धिश्चक्रवृद्धिरुदाहृता ।

Vruddherapi punarvruddhischakravruddhirudahruthaa

The amount that increases, (as interest) increases further, so as to get a cyclic increase, is called compound interest.

The cyclic increase in the wealth is due to the increase in interest for the interest added to the principal i.e. compound interest: chakravruddhi. Aryabhatta's method for calculating interest appears to be through a different method in which compounding the interest is followed. (Aryabhateeya 2-25)

मूलफलम् सफलं कालमूल गुणमर्धमूलकृतियुक्तम् ।
तन्मूलं मूलार्धेनिम् कालहृतं स्वमूलफलम् ।।

Moolaphalam saphalam kaalamoola gunamardha moolakruthiyuktham
thanmoolam moolardh nam kaalaahrutham svamoolaphalam

Multiply the interest on the principal + (plus) the interest on that interest by the time and by the principal. Add square of half the principal. Take square root. Subtract half the principal and divide by period, the result is interest on the principal. This can be mathematically presented as follows:

Interest I = √(PTA + (P/2)² - P/2 ÷ T (where P is principal, I is the interest T is period in months and A, after T months, the interest I amounts to A). Simplified form of interest calculation method is given in Brahmasphuta siddhanta (XII. 15) by Brahmagupta. Bhaskaracharya I gives a problem on compound interest calculation in the Bhashya to Aryabhateeya (114.1)

जानामि शतस्य फलम् न च किन्तु शतस्य यत्फलं सफलं ।
मासैश्चतुर्भिराप्तम् षड् वृद्धिं शतस्य मासोत्थाम् ।।

Jaanaami sathasya phalam na cha kinthu sathasya yathphalam saphalam maasaischathurbhi raaptam shad vruddhim sathasya maasotthaam

I don't know the interest on 100, but I do know that the (monthly) interest on 100 + interest on that interest accruing in four months is 6. Give the monthly interest for 100.

This problem has to be solved using Aryabhatta's equation given above. The same problem has also been requoted by Yallaya and Raghunatha Raja in their mathematical works on the commentary for Aryabhateeya. A detailed accounting in the banking procedure of charging, not only interest but also other commissions is given, in Patiganita of Sreedharacharya. The problem is also given in Siddhanta sekhara and quoted earlier.

The rate of interest being 5% per month, commission of surety 1% per month, fee for accountant is ½% per month and charges of the scribe 1/4% per month. A certain sum amounts to 905 a year. Find the capital, the interest and the shares of the surety, accountants and the scribe.

Here the capital = (100 x 1 x 905) divided by (100 x 1 + (5 + 1 + ½ + 1/4) x 12.

The history of modern banking system appears to have started in USA in 1782, (Philadelphia), The Bank of Hindustan was the first of that sort in India established by Alexander & Company of Calcutta in 1790 (but closed in 1832)[49]. But the ancient Indian system stood the test of time!

Bodies in motion:

Studies on bodies in motion is a mathematical problem of great interest for modern mathematicians and astronomers alike, as it was for their counterparts in ancient India. This attracted special attention of astronomers because of occulting of planets and stars and also eclipses. More over this subject has given many puzzling exercises for the students of mathematics. Almost all ancient books on mathematics and astronomy have dealt with this problem of meeting travellers and /or bodies which are moving at different speeds either in the same direction or in opposite directions. From the available literaure it is observed that Aryabhatta I (499 AD) appears to be the first mathematician who put forth the rule clearly for calculating the meeting places, on the basis of time, of moving bodies. It is said that this problem is also discussed in Bhakshali manuscripts. In Aryabhateeya (2-31) the rule is given as follows:

भक्ते विलोमविवरे गातियोगेनानुलोमविवरे द्वौ ।
गत्यन्तरेण लब्धौ द्वियोगकालावतीतैष्यौ ।।

Bhakthe vilomavivare gathiyogenaanulomavivare dvow
gathyantharena labdow dviyogakaalaavatheethaishyow

Whenever two bodies are travelling in the opposite directions, the distance between them is to be divided by the sum of their speeds. If they move in the same direction, the distance is to be divided by the difference of their speeds. This gives the time required for meeting of the bodies or the time elapsed after meeting of the moving bodies.

The same rule with examples is also given in the Ganita sara sangraha (326.1a). Patignita (example 83) gives the problem of two travellers walking at different speeds, first in the same and then in the opposite directions.

एकौ ना योजनान्यष्टौ यात्यन्यो योजनद्वयम् ।
योजनानां शतं पन्था: संगम: क्व गमागमे ।।

Ekow naa yojananyashtow yaathyanyo yojanadvayam
yojanaanaan satham panthaa: sangama: kva gamaagame

One man travels at 8 yojana speed per day. Another travels at 2 yojana per day, starting simultaneously from the same place. After reaching the destination, the first man comes back. If the length of the track is 100 yojana. Say where is the meeting place of the two? (One going forward and the other traveller returning).

This problem is not only a mathematical puzzle but also gives information on the curiosity of mathematicians to find out speed of travelling, distance, etc. Solution for the above problem has been discussed by Bhaskaracharya and other mathematicians. Time taken by the first traveller to reach the destination (i.e 100/8 days) and his return after these days is to be calculated separately. The distance travelled by the second, during the above number of days is also to be calculated. Then the distance is to be divided by the sum of speeds.

Two problems of the same nature are given by Bhaskarabhashya to Aryabhateeya (131.1 & 131.2)

सार्धं योजनमेको वलभीतो यात्यसौ दिनेनैव आगच्छति हरूकच्छात्
पादयुतं योजनं ह्यपर: अन्तरमनयोर्दृष्टं त्वष्टावंश योजनानि पथिकेन
वाच्यं योग: कियता कालेनाभूत्तयोर्गणक: ।।

Saardham yojanameko valabheetho yaathyasow dinenaiva
aagachhathi harookacchath paadayutham yojanam hyapara:

antharamanayordrushtam thvashtaavamsa yojanaani pathikena vaachyam yoga: kiyathaa kalenaabhootthayorganaka:

One man goes from Valabhi at the speed of 1½ yojana a day. Another man comes along the same route from Harukacha at the speed of 1 1/4 yojana a day. The distance between the two is known to be 18 yojanas. Say when will they meet each other.

If T denotes the time of meeting $1\frac{1}{2}\,T+1\ 1/4\ T = 18$, giving the time as 6 6/11 days. The second problem is also interesting, It is as follows:

वलभीते याति नरो गंगां दिवसं योजनसार्धं अपरास्त्रिभागहीनं
शिवभाग्यपुरात् तदा यति । अष्टौ त्रिगुणानि तयोरन्तरमुक्तं च
योजनानि बुधैः एकेन पथा यातौ कियता कालेन संयुक्तौ ।।

Valabheethe yaathi naro gangaam divasam yojanasaardham aparaasthri bhaagaheenam sivabhaagyapuraath thadaa yathi. ashtow thrigunaani thayoranthara muktham cha yojanaani buddhai: ekena pathaa yaathow kiyathaa kaalena samyukthow

One man goes from Valabhi to Ganges at a speed of 1½ yojana a day and at the same time another man proceeds from Sivabhagyapura at a speed of 2/3 yojana a day. Distance between the two places has been stated by the learned to be 24 yojanas. If they travel along the same route, after how long they meet? $1\frac{1}{2}\,T-2/3T = 24$ from this period of meeting can be calculated.

Indian mathematicians have dealt not only with the motion in the same and opposite directions, at the same rate of speed, but also calculated the time of meeting when the speed of travellers varies i.e when acceleration is attained. In Patiganita (example III) a problem is given.

त्र्याद्येकोत्तरवृद्ध्या यात्येकः प्रतिदिनं नरस्त्वन्यः ।
दश योजनानि कियता कालेन तयोर्गतिस्तुल्या ।।