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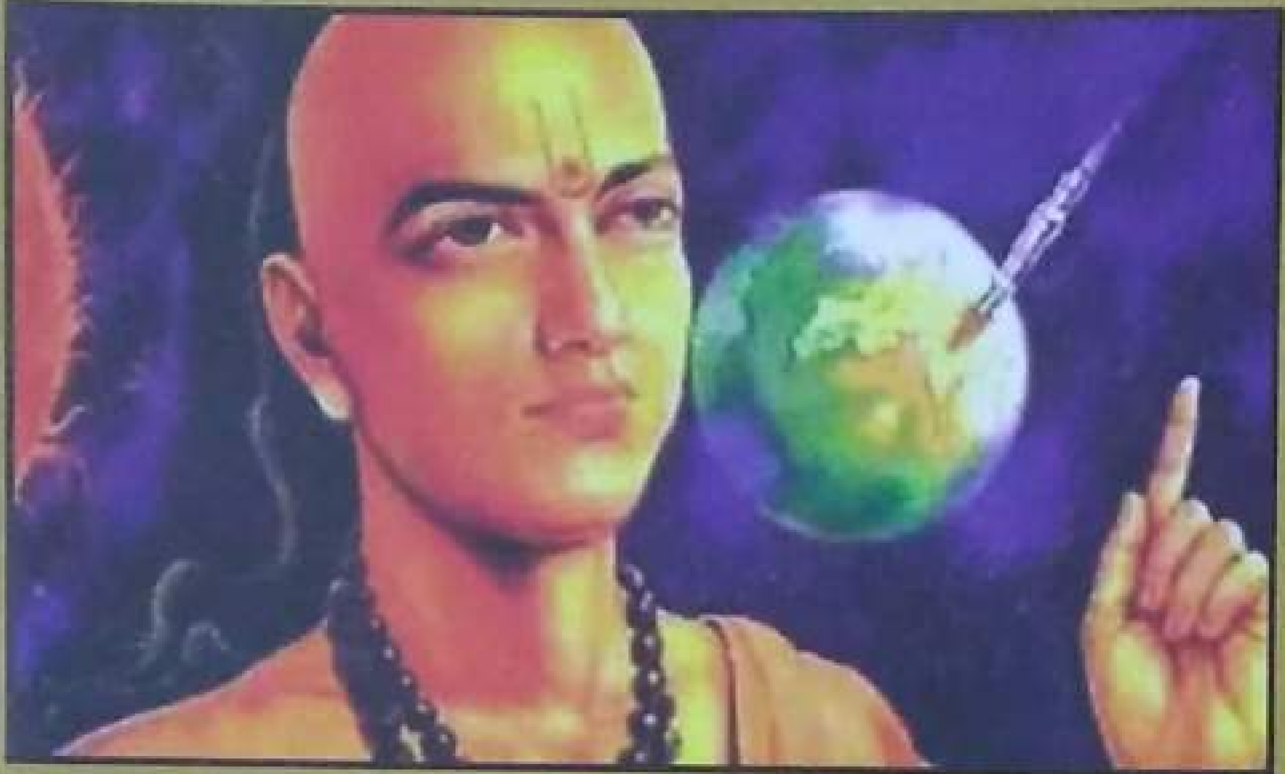
GEOMETRY & MATHEMATICS IN ANCIENT INDIA

Detailed description of general geometry in ancient India & Vedic geometry related with the fire altars and variety of sacrificial structures & Arithmetics and mensuration explained with examples of problems.



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INTRODUCTION

Sri. Aurobindo on the great heritage of Bharath!

Sanskrit ought still to have a future as a language of the learned and it will not be a good day for India when the ancient tongue ceases entirely to written or spoken.

The Vedic Rishies may not have yoked the lightning to their chariots, nor weighed the Sun and the stars, nor materialised all the destructive forces in nature to aid them in massacre and dominatin, but they had measured and fathomed all the heavens and earths within us, they had cast their plummet into the incoscient and the subconsciousness and the superconsciences they had read the riddle of death and found secret of immortality they had sought for and discovered the one and known and worshipped him in the glories of his light and purity and wisdom and power.

.... But in India at a very early time the spiritual and cultural unity was made complete and became the very stuff of the life of all this great surge of humanity between the Himalayas and the two seas. Invasion and foreign rule, the Greek, the Parthian and the Hun, the robust vigour of Islam, the levelling stead roller heaviness of the British occupation and the British system, the enormous pressure of the Occident have not been able to derive or crush the ancient sol out of the body her Vedic Rishis made for her.

India of the ages is not dead nor has she spoken her last creative word; she lives and has still something to do for herself and the human peoples. And that which must seek how to awake is not an anglicised oriental people, docile pupil of the West and boomed to repeat the cycle of the Occident's success and failure, but still the ancient imemorable Shakti recovering her deepest self, lifting her head higher towards the supreme sources of light and strength turning to discover the complete meaning and a vaster from her Dharma.

Mother India is not a piece of earth, she is a power, a Godhead, for all nations have such a Devi supporting their separate existence and keeping it in being, such beings are as real and more permanently real that the men tey influence but they belong to higher plane, are part of the cosmic consciousness and being and act here on earth by shaping the human consiousness on which they exercise their influence, it is natural for man who sees only his own consscíousness individual national or racial at work and does not see what works upon it and shapes it, to think that all is created by him and there is nothing cosmic and great behind it.

Use of Average:

Determination of the average of different values, for a set of measurements, is followed to get more accurate results. Average values are determined for many common calculations. Bhaskaracharya II says thus on the use of average in Lilavati:

गणयित्वा विस्तारं बहुषुस्थानेषु तध्युतिर्भाज्या
स्थानकमित्या सममितिरेवं दैर्घ्ये च वेधे च

Ganayithva visthaaram bahushusthaneshu thadyuthirbhaayyaa
sthaanakamithyaa samamithirevam dairgye cha vedhe cha

(For length, breadth and depth) the measurements should be taken at many places and the sum should be divided by the number of times (places) the measurement is taken.

Ganesa the commentator gives the merits of determining average value of data as follows:

यथायथा बहुषु स्थानेषु विस्तराधिकं गण्यते
तथातथा सममिति सूक्ष्म सूक्ष्म तरा स्यात्

Yathaayathaa bahushu sthaaneshu vistharaadhikam ganyathe
thathaathathaa samamithi sookshma sookshma tharaa syaath

Whenever the measurements are taken at more and more places, then more and more accuracy for values will be obtained.

Brahmagupta has used the method of finding out the average of the surface area for accurately calculating the volume of the cylindrical structures.

Ratio and proportion:

Determining ratio among various parameters is common in mathematical and statistical calculations. This is an important criterion for understanding relative quality / quantity with respect to other parameters. When we say one out of every three persons knows Sanskrit, it gives a ratio of 1:3. This type of mathematical presentation is thought to be the contribution of modern mathematics. Explanation taken from Bhaskaracharya's commentary for Aryabhateeya (2.26 (5)) is given below:

अष्टौ दान्तास्त्रयो दम्या इति गाव: प्रकीर्तिता: ।
एकाग्रस्य सहस्रस्य कति दान्ता: कतीतरे ।।

Ashtow daanthaa sithryo damyaa ithi gaava: prakeerthi thaa:
ekaagrasya sahasrasya kathi daanthaa: katheetharai:

(Out of 11 cattle) Eight are tamed and 3 are to be tamed and (how many are) to be tamed) if the number of cows is 1001?

Here the ratio between the tamed and untamed is 8:3. From this, the number of cattles tamed can be calculated by taking the product of 1001 and 8 and dividing by 11. The same is applicable for determining the number of untamed animals. Calculations using ratio/proportion were known to Indians at least from Bhaskara's period. Descriptions given in the available literature say that this subject is dealt with in Bhakshali manuscripts also. If that is also taken into consideration, the history of ratio starts further back, to about 2000 years.

Permutations and combinations:

When a variety of parameters of variables are available for making the maximum number of combinations, it can be possible in different ways. Number of ways possible under a particular combination can be calculated using the principles of permutations and combinations. In modern mathematics permutations and combinations play a very useful role to get benefits of space, time, and energy savings in scientific and social activities. This has also contributed on aesthetic beauty and savings in space. Rules and detailed scientific explanations for permutation and combination methods and their principles are seen in Jaina text Bhasvati sutra written during 300 BC. In the text writing numbers with different combinations of the same set of digit was discussed.

In Susrutha samhita (6th century BC), 63 possible combinations are explained from a mixture of 6 rasas (flavours) by distributing the rasas in the order of one, two, three and so on. Sreenivasa Iyengar describes this problem in detail.[46] Sreedharacharya in Patiganita (rule 72) gives an example similar to that given in Susruta Samhita:

कटुकतिक्त कषायाम्ललवण मधुरैः सखे रसैः षड्भिः ।
विदधाति सूपकारो व्यञ्जनमाचक्ष्व कति भेदम् ।।

Katukathiktha kashaayaamla lavana madhurai: sakhe rasai: shadbhi:
vidadhaathi soopakaaro vyanchanamaachakshva kathibhedam

Friend, a cook prepared varieties of food with 6 savours: pungent, bitter, astringent, acid, saline and sweet. Say what is the possible number of varieties of food that can be made with these savours.

The combinations are: with one each of the savour, we can have 6 types. With the combination of any two, we can have 15 types, with the combination of any 3, we get 20, with the combination of any 4, 5 and 6, respectively, we get 15, 6 and 1 types. These are the combinations possible. (The same is the answer for problem given on Susrutha samhita)

Permutation and combination given in above method is, exchanging, one or more from the total set. Another type of possible combinations is substituting the one for other in the total combination without keeping anyone/thing away from the list. Bhaskaracharya II gives an interesting mathematical problem of this type for explaining the permutation combination possibilities, in Lilavati. Lord Siva and Lord Vishnu with various Bhooshanas were compared in different forms when each one of the bhooshanas is substituted by other, without keeping away any one.

पाशाङ्कुशाहि डमरूक कपाल शूलै:
खड्वांगशक्ति शरचापयुतैर्भवन्ति ।
अन्योन्य हस्त कलितै: कतिमूर्तिभेदा:
शंभोहरिरिव गदारि सरोज शंखचक्रै: ।।

Paasankusaabi damarooka kapaala soolai: khadvangasakthi sara chaapayuthairbhavanthi anyonya hastha kalithai: kathimoorthibhedaa: sambhohaririva gadaari saroja sankachakrai:

Pasa, ankusa, serpant, damaru, kapala, soola, khatvanga, sakti, chapa, with these (ten) items how many permutations and combinations are possible for Lord Siva. Similarly with the four items, sanku, chakra, gadha and padma holding in the hands, how many combinations are possible for Lord Vishnu?

It is known for mathematicians that answers for the problems are 1x2x3x4x5x6x7x8x9x10 (i.e 10! read as factorial 10) combinations are possible for Lord Siva and 1x2x3x4 (i.e 4! read as factorial 4) combinations are possible for Lord Vishnu. Thus these two types of permutations and combinations were also brought into application in India.

Percentage (%) application :

In modern mathematics many commercial and financial transactions are referred to with respect to a base number of 100. Hence it is known as per cent. The equivalent word in Sanskrit is satamanam. References were made for presenting financial or other mathematical data with a standard reference to 100. Thousands of years ago it was practised in India. This was a subject available not only from the mathematical texts, but also referred to in the Dharma sastras and puranas. Hence the use of the concept of % has commenced atleast two and half a thousand years ago, i.e during the period of Dharma sastras and puranas. Detailed mathematical discussion with examples on the use of the % is given in this text under the topic of 'loans'.

Interest calculation:

Description and application of interest are also given under the subtitle of loan. Here the subject is brought only for explaining the use of percentage in olden days. A problem from Siddhanta sekhara quoted in Patiganita (rule 48):

मासेन शतस्य फलं पञ्चैको भाव्यकेर्धिमय वृत्तो ।
लेखकपादो वर्षे पञ्चाधिक नवशतीमिश्रम् ।।

Maasena sathasya phalam panchaiko bhavyakefrdhamaya vruttho
lekhakapaado varshe panchaadika navasatheemisram

The rate of interest being 5% per month, the commission of surety 1% per month, fee for accountant ½% and charges of the scribe 1/4% per month, certain sum amounts to 905 a year. Find the capital, the interest and the shares of the surety?

It can be seen that the use of % was very handy for describing different financial parameters. Another important information that can be elucidated from the above problem is, the systematic and modern approach that existed in India even in financial transactions. i.e the concept of surety, accounting charges/fee, scribing charges, etc., being separately levied in the financial business as in modern times. One among many quotations, from Dharma sastra will throw light on, how the financial transaction fall in the rules of those texts. Vishnu smruti gives this quotation.

सपादपणा धर्म्या मासवृद्धिः । पणशतस्य पञ्चपणा व्यावहारिकी ।।

Sapaadapanaa dharmya maasavuddhi:
pana sathasya panchapanaa vyaavahaarikee

The dharmic rate of interest is 1.25% per month (for common transactions connected with the household loans) for commercial purposes it can be upto 5% per month.

Partnership, shares and profit sharing:

In modern social system and economical transactions, the partnership business and investment in shares are common. Sharing the principal amount for business, is the concept of partnership. Many business come up and expand through partnerships. In this business a part or total profit is shared among the partners/shareholders. There will be a predetermined ratio for sharing profits/produce based on the principal amount shared for the business by share holders. This is a known practice in the modern business. In ancient India, the literature says that such sharing business was common. Almost all the modern concepts like handling charges, commission, etc. were included in the expenditure part of the profit, other than the charges for writing documents and that for office work. The concept of share business in ancient India was similar to the modern system. A few examples are given here on the mathematical part of it. Sreedharacharya (Patiganita 59.1) gives the rules for sharing profit in partnership business.

स्वयुतिहत प्रक्षोपात्फलेन हन्त्यान्पृथक फलावाप्त्यै ।।

Svayuthi hatha prakshopaath phalena
hanthyaan pruthak phalaavaapthyai

To obtain individual share of profit for partners in the produce, the seeds contributed by partners as divided by their sum should be severally multiplied by the product.

The same is the modern approach of sharing the profit for each shareholder. Bhaskaracharya I in his commentary (119.6) gives the following mathematical exercise on sharing profit when partners collaborate in a business.

समवायकास्तु वणिजः पञ्चैकैकोत्तराधि मूलधनाः ।
लाभः सहस्रसंख्यो वद कस्मै तत्र किं देयम् ।।

Samavaayakaasthu vanija: panchaikaikottharaadhi mooladhanaa:
laabha: sahasra sankhyo vada kasmai thathra kim deyam

Five partners collaborate in a business. The capital invested by them are (in the ratio) one and the same number increasing successively by one (i.e 1,2,3,4, & 5) respectively. Profit that accrued amounts to 1000. Say what should be given to whom.

In this problem Bhaskaracharya I gives details on partnership of a business and also the basis of sharing profit on a certain 'ratio'. It is known that each of the individual share is multiplied with the total profit and divided by the sum of the ratios (i.e. 15). He has given another problem of the same nature with ratio in fractions (119.7)

अर्धेन तृतीयेन प्रक्षेपेणाष्टमेन ये वणिजः
सप्ततिरेकेनोना लाभस्तेषां कियान् कस्य ।।

Ardhena thrutheeyena prakshepenaashtamena ye vanija:
sapthathirekenonaa laabhastheshaam kiyaan kasya

The combined profit of three merchants whose investments are in the ratio of ½, 1/3, and 1/8 respectively, amounts to 70 minus 1, what is whose profit?

Here the problem has to be solved by converting the fractions into whole numbers and then sharing the profit as mentioned above. Yet another problem given by Sreedharacharya in Patiganita (Rule 59.1 example 72)

एकस्यार्ध प्रस्थास्त्र्यंशोऽन्यस्यापरस्य नवभागः ।
सप्तदशशतानि फलं पृथक् पृथक् किं भवेत्तेषाम् ।।

Ekasyaardha prasthasthryamsonyasyaaparasya nava bhaaga:
sapthadasasathaani phalam pruthak pruthak kim bhavetheshaam

Half (½) prastha is the contribution of one partner, 1/3 prastha that of another and 1/9, that of the third. If 1700 prastha is the produce, say what are their shares, separately.

Solution of the problem is as mentioned above. The concept of profit sharing in a systematic way governed by mathematical rules is clearly given in these problems.

Loans and Interests:

In modern business, loans are as important as partnership and sharing of profits. Other than business purposes, loans are also availed for various social/personal purposes. Dharmasastra literature says that availing loans were common in ancient India. Financial transactions (vyavahara) relating the principal, wealth, interest and related documentations, guarantee, etc., under the control of the king or the finance minister or a Government agency. Hence the rules and regulations connected with financial transactions were not considered as individual problems. All those rules and regulations were codified in the books coming under the Arthasastra (economics). They are part of Dharma sastra, where the subject matter deals only with the guidelines. Almost all the Dharmasastras give detailed descriptions on loans and their terms and conditions [48]. Vishnu smruti (100 BC), a well known Dharma sastra book says thus on the necessity of availing loans.

कुटुम्बार्थमशक्तेन गृहीतं व्याधि तेन वा
उपप्लावनिमित्तम् च विद्यादापत्कृतं तत् ।
कन्यावैवाहिकं चैव प्रेतकार्येषु यत्कृतं एतत्
सर्वं प्रदातव्यं कुटुम्बेन कृतं प्रभो ।।

Kutumbaarthamasakthena gruheetham vyaadhithena vaa
upaplava nimittham cha vidyaathaapalkrutham thath
kanyaavaivahikam chaiva prethakaaryeshu yathkrutham ethath
sarvam pradaathavyam kutumbena kruthan prabho

Loans are taken for meeting the expenditure connected with economic problems due to family burden, health problems, treatment, education, expenditure during accident, marriage of daughter, for performing rituals connected with the demise of the family members, etc.,

The same is applicable now also! When money is given for the above or for any other purposes, interest can be levied from the borrower. Charging interest on loan can be of four types:

कायिका कालिका चैव कारिता च तथापरा ।
चक्रवृद्धिश्च शास्त्रेषु तस्य वृद्धिश्चतुर्विधा ।।

Kaayikaa kaalikaa chaiva kaarithaa cha thathaaparaa
chakravruddischa saasthreshu thasya vruddhischathurvidhaa

The interest can be charged as manual work equivalent to the money, materials, depending on the period of loan, money on fixed rates and, at compound interest in approved rates.

Rules of charging interest:

In the banking systems, interest on loan amount can be charged on monthly, quarterly, half yearly and annual basis. In ancient India similar procedure was followed. Loan was given to the borrower for varying periods and the interest was also charged at different rates for different periods. Vishnu smruti says thus:

ऊर्ध्वं संवत्सरात् तस्य तद्धनं वृद्धिमाप्नुयात् ।
ऊर्ध्वं मासत्रयात् तस्य तद्धनं वृद्धिमाप्नुयात् ।।

Oordhvam samvatsaraath thasya vruddhimaapnuyaath
oordhvam maasathrayaatthasya thaddhanam vruddhimaapnuyaath

For some money lenders, the wealth is increased (by charging interest) on per annum basis, for some others, it increases on trimonthly basis.

Katyayana Dharma sastra says interest can be charged on loan amount, annually.

अकृतामपि वत्सरातिक्रमेण यथाविहितम् ।

Akruthaamapi vatsaraathikramena yathaavihitham

Whatever may be the rate annual interest, can also be charged

Narada Dharma sastra advocates charging interest on half yearly basis, also

अनाकारिणमप्यूर्ध्वं वत्सरार्धाभिर्वर्धते ।

Anaakaarinamapyoordhvam vatsaraardhaabhirvardhathe

The generally accepted rate of interest on monthly basis is given in Vishnu smruti. However it advocates the need of mutual agreement between the borrower and the lender before finalising the rate of interest. This rule is similar to that, followed at present.

अथ उत्तमर्णः अधमर्णाद्यथा दत्तमर्थं गृह्णीयात् ।
द्विकं त्रिकं चतुष्कं पञ्चकं च शतं प्रतिमासं ।।

Atha utthamarna: adhamarnaadyathaa datthamartham gruhneeyaath
dvikam thrikam chathushkam panchakam cha satham prathimaasam

The loans can be given and taken between borrower and lender. Generally charged interest rates are 2, 3, 4, or 5% per month.

Here, on an annual basis the interest rate comes to 24%, 36%, 48% and 60%. All these rates of interest fall within the accepted/followed rates by public or private - approved or unapproved - banks/ financial institutions, now.

In Kautilya's Arthasastra - dharmic direction for charging interest from common man, for social/domestic need and for business purposes, aiming at making further profits is given:

सपादपणा धर्म्या मासवृद्धिः । पणशतस्य पञ्चपणा व्यावहारिकी ।।....

Sa paadapanaa dharmyaa maasavruddhi:
panassathasya panchapanaa vyaavaharikee

Reasonable (dharmic) rate of interest is 1.25% per month (i.e 15% per annum) on the transactions with common man for non commercial purposes. But for commercial purposes (for making profit out of it) interest rate can be 5% per month.

Vyasa Dharma sastra gives a clear guideline for charging interest from poor men by fixing a maximum limit for the rate of interest.

निराधाने द्विकशतं मासलाभ उदाहृत: ।

Niraadhane dvikasatham maasalaabha udaahrutha:

Interest rate chargeable from a poor man can be (not be more than) 2% per month.

Charging additional amount as penalty interest, when the loan is not repaid on due date was also allowed earlier as it is now. Thus says Narada smruti:

याच्यमानमदत्तम् चेद्वर्धते पञ्चकं शतम् ।

Yaachyamanamadattham chetdvardhathe panchakam satham

If the interest and loan amont were not repaid on due date, the rate of interest can be increased to 5% (the highest permitted rate of interest fixed by any Dharma sastras).

At the same time, if the lender is not taking the money back (or if the money is deposited for getting interest, with the financial institutions) when the borrower gives it, the procedure followed in such a situation is also made very clear in these books. Yogeswara says:

दीयमानम् तं गृह्णाति प्रयुक्तम् य: स्वकम् धनम् ।
मध्यस्थस्थापितम् तत् स्यात् वर्धते न तत: परम् ।।

Deeyamaanam tham grahnathi prayuktham ya: svakam dhanam
madhyastha sthaapitham thath syaat vardhathe na thatha: param

If the lender is not taking money (the deposit) back from borrower, interest need not be paid by the borrower (after due date). But it should be informed through some mediators (as a proof that the borrower was ready to repay the loan/deposit)

From the above rules on loan, interst, period and other parameters, one can understand that giving and taking loans for commercial and domestic purposes were prevalent in ancient India. Even though there can be varying opinions on the date of some of these Dharma sastra books, the date of Kautilaya's Arthasastra is well known, i.e the 3rd century BC. In the same spirit of Dharma sastras, Kautilya had given terms and conditions of banking system that existed in India.

Bakshali manuscripts written in the first/second century AD, is said to contain a lot of information on this subject. Shortly after this period, scholars like Aryabhatta, Bhaskaracharya, Brahmagupta and others have systematically contributed to this branch of knowledge, as a separate chapter in ganita. From among many examples, two of Bhaskaracharya I are given (120.10 and 115.2) from his Bhashya to Aryabhateeya.

(शतस्यार्धचतुष्क) मासप्रयुक्तस्य वृद्धिरर्धपञ्चका रूपका
पञ्चशतो दशमास प्रयुक्तस्य का वृद्धिरिति ?

(sathasyaardhachathushka) maasaprayukthasya vruddhirardhapanchakaa roopakaa panchasatho dasamaasa prayukthasya kaa vryddhirithi?

If 4½ rupakas be the increase (interest) per 100 rupakas for 5½ Months. What will be the increase on 500 rupakas for 10 months?

This is similar to a modern mathematical problem ! The problem can be solved by finding out interest for one month and then finding out for 10 months, to arrive at the interest for

500 rupakas. (It can be noted that the word Rupee in its present form rupa has its origin at least more than 1½ a millennia ago.

पञ्चविंशतेर्मासिकी वृद्धिर्न ज्ञायते। या पञ्चविंशतेर्मासिकी वृद्धिःसा तेनैवार्धेणान्यत्र प्रयुक्ता, सह वृद्धा पञ्चाभिर्मासैर्दृष्टा रूपत्रय पञ्चभागोनं ।
तत्रेच्छामो ज्ञातुं का पञ्चविंशतेर्मासिकी वृद्धिः ?
का वा पञ्चविंशतिर्वृद्धिः ? पञ्चमास प्रयुक्ताया वृद्धिरिति ? ।।

Panchavimsathermasikee vruddhirna jnaayathe /ya panchavimsathermaasikee vruddhi: saa thenaivaardhenaanyathra prayukthaa sahavruddhaa panchabhirmasairdrushtaa roopathraya pancha bhagonam/ thathrechhamo jnaathum kaa panchavimsathermasikee vruddhi? kaa vaa panchavimsathirvruddhi:? panchamaasa prayukthaaya vruddhirithi?

Monthly interest on 25 is not known. But the monthly interest on 25 rupakas lent out elsewhere at the same rate (of interest) amounted to 3-1/5 rupakas in 5 months. I want to know the monthly interest on 25 rupakas, as also the interest for 5 months on the interest of 25 rupakas.

Sreedharacharya has given the method for the claculations of various parameters on the interest, etc. (Patiganita rule 51).

.गतकालफलसमासे मासफलैक्योद्धृते भवेत्कालः ।
शतगुणमासफलैक्ये धनयोगहृते शतस्यफलं ।।

Gathakaalaphala samaase maasaphalaikyoddruthe bhavetkaala: sathaguna maasaphalaikye dhanayogahruthe sathasyaphalam

The sum of the interest accrued (on the given bonds) for elapsed months being divided by the sum of the interest for one month gives the time (in months) and 100 times the sum of interests for one month, being divided by the sum of the capital amount, gives rate of interest in percentage per month.

According to this rule, rate of interest is equal to total interest divided by the number of months. That when divided by the principal after multiplying with 100, gives the rate of interest in %. This is the same modern formula used for calculating the interest i.e. PNR/100 = interest. or interest x 100/ principal = rate of interest. From this it is clear that Sreedharacharya is the first mathematician who has given the equation for calculating interest as it is known to modern mathematics.

Compound interest:

Charging two types of interst is followed by modern financial institutions. The simple and the compound interest. Vishnu smruti gives provisions for charging at compound interest rate also, for the loans. The compound interest is thus defined.

वृद्धेरपि पुनर्वृद्धिश्चक्रवृद्धिरुदाहृता ।

Vruddherapi punarvruddhischakravruddhirudahruthaa

The amount that increases, (as interest) increases further, so as to get a cyclic increase, is called compound interest.

The cyclic increase in the wealth is due to the increase in interest for the interest added to the principal i.e. compound interest: chakravruddhi. Aryabhatta's method for calculating interest appears to be through a different method in which compounding the interest is followed. (Aryabhateeya 2-25)

मूलफलम् सफलं कालमूल गुणमर्धमूलकृतियुक्तम् ।
तन्मूलं मूलार्धोनम् कालहृतं स्वमूलफलम् ।।

Moolaphalam saphalam kaalamoola gunamardha moolakruthiyuktham
thanmoolam moolardhonam kaalaahrutham svamoolaphalam

Multiply the interest on the principal + (plus) the interest on that interest by the time and by the principal. Add square of half the principal. Take square root. Subtract half the principal and divide by period, the result is interest on the principal. This can be mathematically presented as follows:

Interest I =√(PTA + (P/2)² - P/2 ÷ T (where P is principal, I is the interest T is period in months and A, after T months, the interest I amounts to A). Simplified form of interest calculation method is given in Brahmasphuta siddhanta (XII. 15) by Brahmagupta. Bhaskaracharya I gives a problem on compound interest calculation in the Bhashya to Aryabhateeya (114.1)

जानामि शतस्य फलम् न च किन्तु शतस्य यत्फलं सफलं ।
मासैश्चतुर्भिराप्तम् षड् वृद्धिं शतस्य मासोत्थाम् ।।

Jaanaami sathasya phalam na cha kinthu sathasya yathphalam saphalam maasaischathurbbi raaptam shad vruddhim sathasya maasotthaam

I don't know the interest on 100, but I do know that the (monthly) interest on 100 + interest on that interest accruing in four months is 6. Give the monthly interest for 100.

This problem has to be solved using Aryabhatta's equation given above. The same problem has also been requoted by Yallaya and Raghunatha Raja in their mathematical works on the commentary for Aryabhateeya. A detailed accounting in the banking procedure of charging, not only interest but also other commissions is given, in Patiganita of Sreedharacharya. The problem is also given in Siddhanta sekhara and quoted earlier.

The rate of interest being 5% per month, commission of surety 1% per month, fee for accountant is ½% per month and charges of the scribe 1/4% per month. A certain sum amounts to 905 a year. Find the capital, the interest and the shares of the surety, accountants and the scribe.

Here the capital = (100 x 1 x 905) divided by (100 x 1 + (5+1+½+1/4) x 12.

The history of modern banking system appears to have started in USA in 1782, (Philadelphia), The Bank of Hindustan was the first of that sort in India established by Alexander & Company of Calcutta in 1790 (but closed in 1832). But the ancient Indian system stood the test of time!

Bodies in motion:

Studies on bodies in motion is a mathematical problem of great interest for modern mathematicians and astronomers alike, as it was for their counterparts in ancient India. This attracted special attention of astronomers because of occulting of planets and stars and also eclipses. More over this subject has given many puzzling exercises for the students of mathematics. Almost all ancient books on mathematics and astronomy have dealt with this problem of meeting travellers and /or bodies which are moving at different speeds either in the same direction or in opposite directions. From the available literaure it is observed that Aryabhatta I (499 AD) appears to be the first mathematician who put forth the rule clearly for calculating the meeting places, on the basis of time, of moving bodies. It is said that this problem is also discussed in Bhakshali manuscripts. In Aryabhateeya (2-31) the rule is given as follows:

भक्ते विलोमविवरे गतियोगेनानुलोमविवरे द्वौ ।

गत्यन्तरेण लब्धौ द्वियोगकालावतीतैष्यौ ।।

Bhakthe vilomavivare gathiyogenaanulomavivare dvow gathyantharena labdow dviyogakaalaavatheethaishyow

Whenever two bodies are travelling in the opposite directions, the distance between them is to be divided by the sum of their speeds. If they move in the same direction, the distance is to be divided by the difference of their speeds. This gives the time required for meeting of the bodies or the time elapsed after meeting of the moving bodies.

The same rule with examples is also given in the Ganita sara sangraha (326.1a). Patignita (example 83) gives the problem of two travellers walking at different speeds, first in the same and then in the opposite directions.

एकौ ना योजनान्यष्टौ यात्यन्यो योजनद्वयम् ।
योजनानां शतं पन्था: संगम: क्व गमागमे ।।

Ekow naa yojananyashtow yaathyanyo yojanadvayam
yojanaanaan satham panthaa: sangama: kva gamaagame

One man travels at 8 yojana speed per day. Another travels at 2 yojana per day, starting simultaneously from the same place. After reaching the destination, the first man comes back. If the length of the track is 100 yojana. Say where is the meeting place of the two? (One going forward and the other traveller returning).

This problem is not only a mathematical puzzle but also gives information on the curiosity of mathematicians to find out speed of travelling, distance, etc. Solution for the above problem has been discussed by Bhaskaracharya and other mathematicians. Time taken by the first traveller to reach the destination (i.e 100/8 days) and his return after these days is to be calculated separately. The distance travelled by the second, during the above number of days is also to be calculated. Then the distance is to be divided by the sum of speeds.

Two problems of the same nature are given by Bhaskarabhashya to Aryabhateeya (131.1 & 131.2)

सार्धं योजनमेको वलभीतो यात्यसौ दिनेनैव आगच्छति हरूकच्छात्
पादयुतं योजनं ह्यपर: अन्तरमनयोर्दृष्टं त्वष्टावंश योजनानि पथिकेन
वाच्यं योग: कियता कालेनाभूत्तयोर्गणक: ।।

Saardham yojanameko valabheetho yaathyasow dinenaiva aagachhathi harookacchath paadayutham yojanam hyapara: antharamanayordrushtam thvashtaavamsa yojanaani pathikena vaachyam yoga: kiyathaa kalenaabhootthayorganaka:

One man goes from Valabhi at the speed of 1½ yojana a day. Another man comes along the same route from Harukacha at the speed of 1 1/4 yojana a day. The distance between the two is known to be 18 yojanas. Say when will they meet each other.

If T denotes the time of meeting 1½ T+1 1/4 T = 18, giving the time as 6 6/11 days. The second problem is also interesting, It is as follows:

वलभीते याति नरो गंगां दिवसं योजनसार्धं अपरास्त्रिभागहीनं शिवभाग्यपुरात् तदा यति । अष्टौ त्रिगुणानि तयोरन्तरमुक्तं च योजनानि बुधैः एकेन पथा यातौ कियता कालेन संयुक्तौ ।।

Valabheethe yaathi naro gangaam divasam yojanasaardham aparaasthri bhaagaheenam sivabhaagyapuraath thadaa yathi. ashtow thrigunaani thayoranthara muktham cha yojanaani buddhai: ekena pathaa yaathow kiyathaa kaalena samyukthow

One man goes from Valabhi to Ganges at a speed of 1½ yojana a day and at the same time another man proceeds from Sivabhagyapura at a speed of 2/3 yojana a day. Distance between the two places has been stated by the learned to be 24 yojanas. If they travel along the same route, after how long they meet? 1½ T-2/3T = 24 from this period of meeting can be calculated.

Indian mathematicians have dealt not only with the motion in the same and opposite directions, at the same rate of speed, but also calculated the time of meeting when the speed of travellers varies i.e when acceleration is attained. In Patiganita (example III) a problem is given.

त्र्याद्येकोत्तरवृद्ध्या यात्येकः प्रतिदिनं नरस्त्वन्यः ।
दश योजनानि कियता कालेन तयोर्गतिस्तुल्या ।।

Thryaadyekothbhara vruddhya yaathyeka: prathidinam narasthvanya: dasa yojanaani kiyathaa kaalena thayorgathisthulyaa

One man goes with an initial speed 3 yojanas per day and the rate of acceleration 1 yojana per day, and another man goes with the constant speed of 10 yojanas per day. In what time will they cover the same distance?

This problem is solved by the equation n = 2 (u -v)/f+1. Here 'n' is the number of days required for the meeting of the travellers. 'u' is the uniform speed of the second traveller and 'v' the initial speed of the first and 'f' is the acceleration of the first traveller per day. Among the moving bodies two problems are connected with the rate of their motion, i.e with acceleration and without acceleration and in the same direction and in the opposite direction of motion.

There is yet another type of motion in which the moving bodies make forward and backward motions at a fixed speed. If a body moves backward and forward at different speeds, then time taken to reach the destination is to be calculated using different speeds, then time taken to reach the destination is to be calculated using a different formula. Sreedharacharya in patiganita gives the rule (44.1)

On subtracting the backward motion (per day) from the forward motion per day, the true distance travelled per day will be obtained. (Dividing the distance by the true rate of distance travelled per day, one can find out the time). Bhaskaracharya I gives as exercise on this type of motion, in Bhaskarabhashya for Aryabhateeya (118.4)

नागोविंशति हस्तः प्रविशत्यर्धांगुलम् मुहूर्तेन ।
प्रत्येति च पञ्चांशं कतिभिरहोभिर्बिलं प्राप्तम् ।।

Naagovimsathihastha: pravisathyardhaangulam muhurthena prathyethi cha panchaamsam kathi bhirahobhirbilam praaptham

A serpent of 20 cubit long enters into the hole moving forward at the rate of ½ of an angula per muhurta and backward at the rate of 1/5 of an angula per muhurta. In how much time does the snake get into the hole completely?

Here the difference of forward and backward motions gives, the true forward distance travelled per unit time of muhurta. When total length of the serpent is divided by true forward distance, the time taken by the serpent to enter in the hole will be obtained.

A very interesting problem on the same topic is given from another old Sanskrit mathematical book. This is requoted in Patiganita book (example 32)

नागेन्द्रो दिनपञ्चमांशनवमत्र्यंशैः स्वपादान्वितैः
षड्भिर्याति सपादयोजनदलं त्र्यंशोनमर्धान्वितम् ।
प्रत्यायाति च योजनं द्विगुणितं स्वत्र्यंशहीनं सखे
साधहिन च तत्र योजनशतं कालेन केनैष्यति ।।

Naagendro dina panchamamsanavamathryamsai: svapaadaanvithai: shadbhiryaathi sapaadayojanadalam tryamsonamardhaanvitham prathyaayathi cha yojanam dvigunitham svathryamsaheenam sakhe: saardhaahena cha thathra yojanasatham kaalena kenaishyathi

The best amongst the elephants goes forward at a rate of (½ (1+1/4) (1-1/3) (1+½))of a yojana in 6x1/5x1/9x1/3(1+1/4) of a day and comes back at a rate of 2 (1-1/3) yojana in (1+½) days. In how much time will they go to a distance, 100 yojanas?

The rule applied here is the same as that of a body moving forward and backward. True distance travelled, foreward, per day is to be calculated from the difference and on dividing the distance by that value. The same problem appears in Ganita sara sangraha (V. 27)

Progression:

Progression is a chapter of great importance in mathematical calculations. Equations of both arithmetical and geometrical progression play important role in the calculation of many transactions and also estimation of areas and volumes of geometrical figures. This was a subject used for various applications in Indian mathematics too, even from the Vedic period. Arithmetic progression is given in Taitiriya samhita of the Yajurveda (VII. 2, 12 - 17)

एका चमे तिस्रश्चमे पञ्च चमे सप्त च मे नव च म एकादश

च मे त्रयोदश चमे पञ्चदश चमे

Ekaa chame thisraschame pancha chame saptha cha me
nava cha ma ekaadasa cha ma thrayodasa chame pancha dasa
chame

It is the number order 1, 3, 5, 7, 9, 11, 13, 15, ... etc.

Another recension of the Yajurveda-Vajasaneyee Madhyandina samhita-also gives the arithmetic progression in even numbers. In another book namely Bruhadevata (500 BC), the result of the sum of arithmetic progression 1+2+3+4+5+.... + 100 is given as 5050. A mathematical progression is compared with an earthern drinking vessel with narrow bottom and steadily increasing upper opening. Patiganita (rule 79) says:

विस्तारोऽल्पोऽधस्तादुपरि महान् स्याद्यथा शरावस्य।

श्रेढीक्षेत्रस्य तथा गच्छसमो लम्बकस्तस्य ।।

Vistaaroalpodhastaadupari mahaan syaadyathaa saraavasya sreddeekshethrasya thathaa gacchasamo lambakasthasya

As in the case of an earthern drinking glass the base of which is smaller and the top wider, so also is the case with a series in progression figures. The altitude (lambaka) of that series figure is equal to the number of terms in the figure.

I.e. a progression generally starts with a small number or set and increases uniformly to large numbers.

A variety of mathematical problems are given in ancient Indian books on the series/progressions. Many of the equations taught in modern mathematics, are given in these books. Patiganita (rule 14.1) gives an equation for calculating the sum of a basic arithmetic progression as follows:

सैकपदाहतपददलमेकादिचयेन भवति संकलितम् ।।

Saikapadaahathapadadalamekaadichayena bhavathi sankalitham

If the first term is unity and common difference is also unity, then the sum is equal to half the number of terms multiplied by total number of terms plus one.

I.e Sn of $1+2+3+...+n = \frac{1}{2} n(n+1)$; where Sn is the sum and n is the number of terms.

A few (among a number of) problems, of different nature connected with determination of various parameters in the series are given to cite examples of the deep knowledge on progressions. Bhaskaracharya I in Aryabhateeyabhashya (105.1) has given this example:

आदिद्वितयं दृष्टं श्रेद्या: प्रवदन्ति चोत्तरं त्रीणि ।

गच्छ: पञ्च निरुक्तो मध्याशेषे धने ब्रूहि ।।

Aadidvithayam drushtam sreddyaa: pravadanthi chottharam threeni gaccha pancha niruktho madhyaaseshe dhane broohi

In a series, first term is 2; the common difference is stated to be 3 and number of terms is 5. Tell me the middle term and the sum of the series.

This example has also been given by Yallayya, Suryadeva and Raghunatha in their commentaries to Aryabhateeya (2.19). In Bhaskarabhashya another example (106.3) is given for finding out the desired term of a series in arithmetic progression.

एकादशोत्तराया: सप्तादे: पञ्चविंशतिर्गच्छ:।
तत्रान्त्योपान्त्यधने वद शीघ्रं विंशतेश्च कियत् ।।

Ekaadasottharaayaa: saptaade: panchavimsathirgacche thathraanthyopaanthya dhane vada seeghram vimsathescha kiyath

In an arithmetic progression, in which the common difference is 11, first term is 7, the number of terms is 25. Quickly say the ultimate and penultimate terms of that series and also say what is the 20th term.

Here the last term and 20th term are to be calculated using the equation: n^{th} term = a+(n-1)d. Where a is the first term and n is the total number of terms and d is the common difference.

When the first term and the last term of a series in an arithmetic progression are given, determining the number of terms is given in this exercise (Bhaskarabhashya example 107.6)

पञ्चभिराद्य: शङ्ख: पञ्चोनशतेन यो भवेदन्त्यम्।
एकादश शंखानां यत्तन्मूल्यं त्वमाचक्ष्व ।।

Panchabhiraadya: sankha: panchonasathena yo bhavedanthyam Ekaadasasankhaanaam yatthanmoolyam thvamaachakshva

Of 11 conch shells, which are arranged in increasing order of their prices (which are in arithmetic progression), the first shell is acquired for 5 and the last for 95. Say what is the price of the total shells.

Initial term (5), last term (95) and total terms (11) are indirectly given in the above problem, as prices and number of shells, respectively. From this the common difference and sum are to be calculated using the equation given earlier.

Bhaskaracharya I also gives examples for finding out the number of terms when first term, sum and the common difference are given (Bhaskarabhashya 109.2)

नवकाष्टौ वृद्धिमुखे यत्र यत्कीर्त्यते धनं क्रमश:
रामाष्टशरं दृष्टं पदप्रमाणं त्वया वाच्यम् ।।

Navakaashtow vruddhimukhe yathra yathkeerthyathe dhanam kramasa:
ramaashtasaram drushtam padapramaanam thvayaa vaachyam

In an arithmetic progression, common difference and the first term are 9 and 8 respectively, the sum is 583. Tell me the number of terms.

Using the first equation, answer can be found out, where Sn = 583, a = 8 and d = 9. This example is given for the application of the rule given in Aryabhateeya (2-19)

इष्टं व्येकं दलितं सपूर्वमुत्तरगुणं समुखमध्यम् ।
इष्टगुणितमिष्ट धनं त्वथवाद्यन्तं पदार्धहतम्

Ishtam vyekam dalitham sapoorvamuttharagunam samukhamadhyam
ishtagunithamishta dhanam thvathavaadyantham padaardhahatham

Diminish the given number of terms by one, then divide by 2, then increase by the number of preceding terms (if any) then multiply by the common difference and increase by the first term of the series. Result is the arithmetic mean (of the given number of terms). This value multiplied by given number of terms is the sum of the given terms. Or, multiply the sum of the first and the last terms by half the number of terms i.e. Arithmetic mean of the series = a + (n-1)½ x d. Where a is the first term, n is the number of terms and d is the common difference.

Sum of the series = n (a+½ (n-1)d) Or ½ n (a+s); s is the last term a and d are as explained above.

Another set of arithmetic progression which can be mathematically represented as follows is: a+ (a+d)+(a+2d)+... (a+(n-1)d). The following rule is used for getting the sum of the progression series. (Patiganita 85 b)

श्रेढीक्षेत्रे तु फलं भूमुखयोगार्धलम्बहतिः ।

Sreddeekshethre thu phalam bhoomukhayogaardhalambahathi

The area of a progression (if it can be represented as a geometrical figure) is equal to the product of the half of the sum of the base and the face and the altitude. i.e Sn of a+(a+d) + (a+2d)+..... (a+(n-1)d)= (a+½ (n-1)d)n

Then arithmetic mean of n terms is given as follows by converting the above form into another series, representing the mean (as per rule)

(a+pd)+(a+p+1xd)+.... + (a+p+(n-1))d

Mean of the above series = a + (½(n-1)+p)d

When the above mean value is multiplied with the number of terms n, the sum of the series will be obtained, as follows. Sum of the series = n (a+(½(n-1)+p)d). The same rule appears in Brahmasphuta siddhanta (XII. 42) and Ganitasara sangraha (VII.231.1a)

Aryabhatta I has given a complex equation for finding out the number of terms in a series, if the sum is known, in the above type of progression (Aryabhateeya 2-20)

गच्छो/ष्टोत्तरगुणिताद् द्विगुणाद्युत्तरविशेषवर्गयुतात् ।
मूलं द्विगुणादूनं स्वोत्तरभजितं सरूपार्धम् ।।

Gacchoashtottharagunithaad dvigunaadyutthara visesha
· *vargayuthaath*
moolam dvigunaadoonam svotthara bhajitham saroopaardham

Multiply the number of terms by 8 and by the common difference, increase that by the square of the difference between twice the first term and the common difference, and then take the square root, then subtract twice the first term, then divide by the common difference and add (to the quotient and take half the value, that gives the number of terms in a series of nature.

a+(a+d)+(a+2d)+(a+3d) + ...

$n = \frac{1}{2}a + (\underline{\sqrt{8ds + (2a-d)^2} - 2a}) \div d + 1$

This rule has also appeared in Siddhanta sekhara (XIII. 24), Lilavati (rule 128) and Brahmasphuta siddhanta (XII. 18). In Patiganita (rule 87), Sreedharacharya gives an almost similar equation for calculating the number of terms.

अष्टोत्तरहतफलतो द्विगुणादिप्रचय विवरकृतियुक्तात् ।
मूलं द्विगुणमुखोनं सचयं द्विचयोद्धृतं गच्छः ।।

Ashtottharahataphalatho dvigunaadi prachaya vivara kruthi yukthaath
moolam dvigunamukhonam sachayam dvichayoddhrutham gaccha:

Multiply the sum of the series by 8 times the common difference and to that product add the square of the difference between twice the first term and common difference. Take square root of that. That diminished by twice the first term and increased by the common difference and divided by twice the common difference gives the number of terms in the series i.e $n = (\sqrt{8ds+(2a-d)^2} - 2a+d) \div 2d$

Aryabhatta II in Mahasiddhanta (XV. 50) and Bhaskara II in Lilavati (128) have given another equation for the determination of the number of terms in a series:

$$n=(\sqrt{2ds+(a-d/2)^2} - a+d/2) \div d$$

Sripati in Siddhanta sekhara (XIII. 24) puts the above equation in this form also:

$$n= (\sqrt{s/\tfrac{1}{2} d+(a-\tfrac{1}{2}d/d)^2} - (a-\tfrac{1}{2}d)/d)$$

In Patiganita (rule 14 ii) a method for finding the number of terms in an arithmetic progression when the initial term and the common difference are unity, is given.

द्विगुणीकृत सङ्कलितान्मूलं गच्छोऽवशिष्टसमम् ।।

Dviguneekrutha sankalithaanmoolam gacchovasishtasamam

The number of terms is equal to the square root of twice the sum of the series which must be the same as the residue left (after the extraction of square root). I.e $n = \sqrt{2Sn}$

For finding first term of the series Sreedharacharya has given a rule (Patiganita 86.I)

आदिः पदहृतगणितं निरेकगच्छघ्नचयदलेनोनम् ।।

Aadi:padahruthaganitham nirekagacchaghnachayadalenonam

The sum of a series, as divided by the number of terms of the series, being diminished by half the common difference as multipled by the number of terms minus 1, gives the first term of the series i.e First term $a = S/n - \tfrac{1}{2} d(n-1)$

In Patiganita the equation for finding the first term is given in rule 88

विपदपदवर्गदलाहतमिश्रघनात्फलमपास्य परिशिष्टं ।
व्येकपदार्धेन भजेद् व्येकेन पदाहतेनादिः ।।

Vipadapadavargadalaahathamisraghanaathphalamapaasya parisishtam vyekapadaardhena bhajeth vyekena padaahathenaadi:

Having subtracted the sum of the series from the mixed amount as multiplied by one half of the number of terms squared minus the number of terms, divide the residue by one half of (the difference of) the number of terms minus one, as diminished by one and multiplied by the number of terms.

Thus the first term is $a = (½(n^2-1)(a+d)-S) \div (½(n-1)-1)n$

Patiganita (86 ii) gives the rule for calculating the common difference of a series.

पदहृतफलं मुखोनं निरेकपददलहृतं प्रचयः ।

Padahruthaphalam mughonam nirekapadadalahrutham prachaya:

The sum of a series as divided by the number of terms (of the series) being diminished by the first term and then divided by half of the number of terms minus 1, gives common difference of the series.

I.e. Common difference $d = (S/(n-a)) \div ½ (n-1)$. An applied problem is given in Bhaskarabhashyam (106.4)

द्वयादित्र्युत्तर संख्यं दिने दिने कार्तिके क्रमान्मासे
प्रददाति महीपालः पञ्चदशाहे गतेविप्रः ।
ब्रह्मिष्ठः सम्प्राप्तस्तस्मै दत्ता दशाहधन सख्यां
पञ्चदिनोत्थान्यस्मै कथय धनं किं तयोस्तत्र ।।

Dvyaadithriyutthara sankhyam dine dine kaarthike kramaanmaase pradadaathi maheepaala: panchadasaahe gathervipra:

brahmishta: sampraapthasthasmai datthaa
dasaahadhana sankyaam panchadinotthaanyasmai
kathaya dhanam kim thayosthathra

In the month of karthika, a king gave away some money daily starting with 2 on the first day and increasing that by 3 per day. Fifteen days having passed, there arrived a Brahmana (Vedic scholar). The amount for the next 10 days was given to him and that for the next 5 days (of the month) to another. What do the last two person get?

Here the initial amount on the 15th day is to be calculated as the nth value in the arithmetic progression starting from 2 and incresing by 3. The total sum (S10-sum for 10 days- from 15th to 25 days are to be separately calculated as the first term the amount of 15th day and the common difference as 3 and the number of terms as 10. The same is to be repeated for the 26th to the 30th days (5 days) to calculate sum for 5 days (S5) for second person. Solution for getting the partial sums can thus be calculated.

Problem on the progression of 1 + (1+2)+(1+2+3)+ ... type of series. Bhaskaracharya has given many problems on this type of progression (109.1)

पञ्चानामष्टानां चतुर्दशानां च याः क्रमाच्चितयः ।
गच्छस्तरास्त्रिकोणा (रूपविधानं च) मे वाच्यम् ।।

Panchaanaamashtaanaam chathurdasaanaam cha yaa:
kramaacchithaya:
gacchastharaasthri konaa (roopavidhaanamcha) me vaachyam

There are (three pyramidal) piles (of balls) having respectively 5, 8 and 14 layers which are triangular. Tell me the number of units (balls) in each of them.

This problem is worked out as follows. In the topmost layer of pyramidal piles, there is 1 ball; in the second layer from the top, there are 1+2=3 balls; in the third layer 1+2+3=6; like wise in the 4th layer 10 balls and so on. Number of balls in the first pile having 5 layers, is equal to 1+(1+2)+....... upto fifth layer. The value is (5x6x7) 1/6. This formula is given in Aryabhateeya (2-21)

एकोत्तराद्युपचितेर्गच्छाद्येकोत्तरात्रिसंवर्ग: ।
षड्भक्त: सचितिघन: सैकपदघनो विमूलो वा ।।

Ekottharaadyupachithergacchaadyekottharaathrisamvarga: shadbhaktha: sachiti ghana: saikapada ghano vimoolo vaa

Of the series which has one for the first term and one for the common difference, take three terms in continuation, of which the first is equal to the given number of terms, and find their continues product. That product or the number of terms plus one subtracted from the cube of that divided by 6 gives the chitighana

Chitighana literally means, the solid content of a pile in the shape of a pyramid on a triangular base. The pyramid so constructed as 1 ball in the top and 1+2 balls next and so on. i.e. n(n+1)(n+2)/6 and $(n+1)^3$ - (n+1) divided by 6 = Sn

Problem related with a series: $1^2+2^2+3^2+......n^2$ type. Aryabhateeya (2-22a) gives the rule for getting the sum of the squares of terms in a series.

सैकसगच्छपदानां क्रमात् त्रिसंवर्गितस्य षष्ठोंऽश: ।

Saikasagacchapadaanaam kramaath thri samvargithasya shashtomsa:

Continuous product of three quantities i.e the number of terms, plus one, the same increased by the number of terms and the number of terms when divided by 6 gives the sum of the series of squares of natural numbers (varga chiti ghana)

I.e for $1^2+2^2+3^2+.....n^2$ series. The sum = n(n+1) (2n+1) x 1/6

The sum of the squares of the terms of the given series can be found out using the equation given by Sreedharacharya in Patiganita (rule 105)

Σ (a+(r-1)d)^2 = (a+(a+2d)+a+4d)+....to n terms) x a+ ($1^2+2^2+3^2+$.... $(n-1)^2$. This has been described as follows:

द्विगुणितचयेन गणितं मुखसङ्गुणितं निरेकगच्छस्य ।

कृतिसंकलितेन युतं चयकृतिगुणितेन वर्गयुतिः ।।

Dvigunitha chayena ganitham mukhasangunitham nirekagacchasya

kruthisankalithena yutham chayakruthi gunithena vargayuthi:

The sum of the arithmetic series with twice the common difference when multiplied by the first term and then increased by the sum of the squares of natural numbers ranging from 1 to one less the number of terms, as multiplied by the square of the common difference, gives the sum of the squares of the terms of the given series.

Bhaskaracharya I gives an example for this type of series in Bhaskarabhashyam (111.1)

सप्तानां अष्टानां सप्तदशानां चतुर्भुजाश्चितयः ।

एकविद्यानां वाच्यं पदस्तरास्ता हि वर्गाख्याः ।।

Sapthaanaam ashtaanaam saptadasaanaam chathurbhujaaschithaya:

ekavidyaanaam vaachyam padastharaasthaa hi vargaakhyaa:

There are (three pyramidal) piles on square bases having 7, 8 and 17 layers which are also squares. Say the number of units there in.

There are three pyramids. In the topmost layer there is one brick, in the next layer there are four bricks (2^2), in the third layer

9, (3^2) bricks and so on. Hence the number of bricks used in the three piles separetely are 140, 204 and 1785, respectively. The solution for this problem is possible from the equation given in Aryabhateeya (2-22) described above.

Problem of a series of the type: $1^3 + 2^3 + 3^3 + ... + n^3$.

The square of the sum of the series of natural numbers gives the sum of the series of cubes of natural numbers (Ghana chiti ghana) i.e. Sum of $1^3+2^3+3^3+......+n^3$ series = $(\frac{1}{2} n(n+1))^2 = (1+2+3+......+n)^2$

Bhaskaracharya I gives the problem in which the above formula is applied (111.2)

चतुरश्रघनश्चितय: पञ्चचतुर्नवस्तरा विनिर्देश्या: ।

एकावघटितास्ता: समचतुरश्रेष्टका: क्रमश: ।।

Chathurasraghanaschithaya: panchachathurnavastharaa vinirdesyaa: ekaavaghatithaasthaa: samachathura sreshtakaa: kramasa:

There are three pyramidal piles having 5, 4 and 9 cuboidal layers. They are cuboidal bricks (of unit dimension) with one brick in the topmost layer. Find the number of bricks used in them.

There are 1^3 bricks on top and 2^3 in second layer 3^3 in the third layer and so on. The number of bricks in the three piles are 225, 100 and 2025, respectively. The Equation for solving the problem is given above. n, for three piles are respectively 5, 4 and 9. This rule is also given in the Patiganita (rule 103) for Σn^3.

सपदपदवर्गतोर्धं घनसंकलितं स्वसंगुणं भवति ।

Sapadapadavargathordham ghanasankalitham svasangunam bhavathi

One half of what is obtained by adding the number of terms to the square of the number of terms, when multiplied by itself, gives the sum of the cubes of natural numbers. (from 1 upto given number of terms) i.e $\Sigma n^3 = ((n^2+n)½)^2$

This rule is the same as that given by Aryabhatta and can be used to solve the problems like that given in Patiganita (117)

एकादिचयपदानां घनसंकलितं सखे कियद् भवति ।
आशु दशानां प्रकथय तथैव संकलित संकलितम् ।।

Ekaadichayapadaanaam ghanasankalitham sakhe kiyad bhavathi aasu dasaanaam prakathaya thathaiva sankalitha sankalitham

Friend, quickly say what is the sum of cubes of 10 terms of a series whose 1st term and common difference are each unity. And sum of the successive sums of those terms.

Formula for finding out the value for the series of $\Sigma n+\Sigma n^2+\Sigma n^3$ is given in Patiganita (rule 102 and 104)

द्विगुणितसैकपदघ्नं सैकपदं प(द)दलहृतं भवति ।

Dvigunitha saikapadaghnam saikapadam pa(da)dalahrutham bhavathi

The number of terms plus one, as multiplied by twice the number of terms plus one, being (further) multiplied by half the number of terms.

$\Sigma n+\Sigma n^2+\Sigma n^3... = ½(2n+1)(n+1)n$. The other rule is as follows.

सैकपदवर्गताडितपदं द्विकोपेदगुणं भवति ।
संकलितकृतिघनानां संकलितैक्यं चतुष्कहृतं ।।

Saikapadavargathaadithapadam dvikopedagunam bhavathi sankalithakruthighanaanaam sankalithaikyam chathushkahrutham

The number of terms as multiplied by the square of (the sum of) the number of terms +1, when (further) multiplied by the number of terms plus 2 and divided by 4, gives (i) the sum of successive sums of natural numbers (from 1 upto the given numbers of terms)

(ii) the sum of squares of those natural numbers and (iii) the sum of the cubes of those natural numbers. i.e $n(n+1)^2$ (n+2) divided by 4.

Patiganita also gives another example as follows (118)

संकलितकृतिघनानां संकलितसमासमानां मे कथय।
षण्णां सखे पदानां गणयित्वा यदि विजानासि ।।

Sankalithakruthighanaanaam sankalithasamaasamaanaam me kathaya
shannaam sakhe padaanaam ganayithvaa yadivijaanaasi

Friend, if you know, then say after calculation (i) the sum of successive sum of 6 natural numbers (ii) the sum of the squares of the first 6 natural numbers and (iii) the sum of the cubes of first 6 natural numbers. (these can be calculated by above rules).

In Patiganita the sum of another series of the cubes of numbers is given (rule 107)

श्रेढीफलस्य वर्गे प्रचयहते (चय) विहीन वदनगुणम् ।
मुखफलवधं निदध्यादिष्टादिचयेन घनयोगः ।।

Sreddeephalasya varge prachayahathe (chaya) viheena vadanagunam
mukhaphalavadham nidadhyaadishtaadichayena ghanayoga:

To the square of the sum of the given arithmetic series, as multiplied by the common difference, and the product of the first and the sum of the series, as multiplied by the first term minus the common difference, the result is the sum of cubes of the terms of the series with given terms and common difference.

In a series $a^3+(a+d)^3+(a+2d)^3+....(a+(n-1)d)^3$,

The sum = s^2 x d + a x s x (a-d). Where s = a + (n-1)d

A problem on the application of this rule (Patiganita 121)

पञ्चादिद्विकवृद्धीनां पदानां ये क्रमात् घनाः ।
चतुर्णां तत्समासेन गणयित्वा निगद्यताम् ।।

Panchaadidvika vriddheenaam padaanaam ye kramaath ghanaa:
chathurnaam thathsamaasena ganayithva nigadyathaam

Say result of adding together the cubes of the four terms which begin with 5 and increase successively by 2.

Various types of rules / equations for calculating the sums, number of terms, common difference etc. in mathematical series, are given in many ancient books.

If the discoveries of those equations are traced in the modern mathematics, it would remain as an incomplete task because nothing much is known on the history of this topic. Almost all these equations/ formula in the theoretical and applied field, were known to Indian mathematicians. It is important to remember that many more such rules and applied mathematical problems are available in the books mentioned above. The above set of examples and rules have been quoted to show that this subject was of great interest to our forefathers millennia ago.

Determination of unknown value from sums, products, etc.,

Determination of specific values from sums/products/ ratios/ differences of two or more numbers is common. Simple and well defined procedures were known to ancient Indians for finding the solutions. Neelakanta Somayaji in Tantra samgraham (1500 AD) has given rules under the subtitle, Dasa prasnotharam (Answers for ten questions). He says:

राश्योरयोगोभिघातो वर्गयोगस्तदन्तरम् ।
एषु द्वाभ्याम् दशविधं राश्योरानयनम् भवेत् ।।

Raasyorayogobhighatho vargayogasthadantharam
eshudvaabhayaam dasavidham raasyoraanayanam bhaveth

For finding out the unknown values in a problem, when the sum, difference, square and their differences, etc., are given, there are ten different methods. In Bhaskaracharya II's Lilavati (page 86) an example of this problem is given.

ययोर्योगशतं सैकं, वियोग: पञ्चविंशति: ।
तैराशी वद मे वत्स वेत्सि संक्रमणं यदि ।।

Yayoryogasatham saikam, viyoga: panchavimsathi:
thairasee vada me vatsa retsi sankaramanam yadi

When two numbers are added, it gives 101, and subtracted the result is 25. Tell me boy what are the numbers?

If the numbers are say, x and y. Then, x+y = 101 and x-y = 25. When these two are added, value for 2x will be obtained. Half of that will be, one of the unknown numbers (x), which is 38 and the other (y) can be determined by substituting the value for x. This rule is stated in different way in Vedaganitham [51].

वर्गान्तरात् योगभक्तो भेदस्तेनापि पूर्ववत् ।

Vargaantharaath yogabhaktho bhedasthenaapi poorvavath

Half of the sum of the sum and difference will give one value and half of the difference of sum and difference of the values will give the other

I.e if the sum is A and the difference is B, then ½ (A+B) gives x and ½(A - B) gives y where x and y are two unknown numbers. Similar problem when difference and product of two numbers are known, is given by Bhaskaracharya I in his Bhashya for Aryabhateeya (113.1)

संवर्गोष्टौ दृष्टो व्यक्तं तत्रान्तरं भवेद्द्वितयं ।
अष्टादशके मुनयो गुणकारौ तौ तयोर्वाच्यौ ।।

Samvargoshtow drushto vyaktham thathraantharam bhavedvithayam
ashtaadasake munayo gunakaarow thow thayorvaachyow

The product of two numbers is correctly seen to be 8; their difference is 2. For two other numbers the product being 18 and difference is 7. Tell, the numbers multiplied in the two cases (i.e all the four numbers)

By assuming the numbers, as x, y, ... etc., and following the procedure, answer will be obtained. Product of numbers from the sum and sum of the squares can be determined according to the method given by Aryabhatta I (Aryabhateeya 2-23)

सम्पर्कस्य हि वर्गाद् विशोधयेदेव वर्गसम्पर्कम् ।
यत्तस्य भवत्यर्धम् विद्याद् गुणकारसंवर्गम् ।।

Samparkasya hi vargaath visodhayedeva vargasamparkam
yatthasya bhavathyardham vidyaad gunakaarasamvargam

From the square of the sum of two factors, subtract the sum of squares. One half of that should be known as the product of the two factors.

If a and b are two factors, then $a \times b = \frac{1}{2} (a+b)^2 - (a^2+b^2)$

Mishra and Singh, say that the credit of finding a solution for the first degree indeterminate equation, by a method called Kutta (literally means pulverizer) by breaking into smaller fragments by means of continued division, goes to Aryabhatta I. The method resembles the continuous fraction process developed by Euler in 1764. More than 1250 years before Euler, Aryabhatta I could find out a solution for the indeterminate equations. Higher levels of applications have been achieved by many commentators of Aryabhateeya later. Hardikar[53], has also proved that the solutions of indeterminate and first order equation were discovered by Indians, millennia ago.

A number of applied problems are given on this subject in many Sanskrit books. A few examples are given below from Sreedharacharya's Patiganita (73, 74)

मुद्गानां कुडवा: सप्त लभ्यन्ते नवभि: पणे:
पणेन कुडवस्यार्धं तण्डुलानामवाप्यते ।
तत: पणत्रयं सार्धं गृहीत्वा//शु वणिङ्मम
तण्डुलानां प्रयच्छांश मुद्गानां च द्विसङ्गुणम् ।।

Mudgaanaam kudavaa: saptha labhyanthe navabhi: pane:
panena kudavasyaardham thandulaanaamavaapyathe
thatha: panathrayam saardham gruheethvaafasu vaningmama
thandulaanaam prayacchaamsa mudgaanaam cha dvisangunam

7 kudavas (unit of measurement) of mudga are obtained for 9 panas and ½ kudava of rice is obtained for one pana. Then O! merchant take 3½ panas and quickly give me one part of rice and two parts of mudga.

Finding out the quantity per unit pana is to be followed for the answer. The quantity of mudga is (49/32) and rice (49/64).

Bhaskaracharya II has given a problem of the simple order intermediate equation for finding out the unknown number from a final value when the initial number has undergone a 'series of processsing' in Lilavati (77-2)

अमलकमलाशशेस्त्रयंशपंचाशषष्ठैस्त्रिनयनहरि सूर्या येन तुर्येण चार्या ।
गुरुपदमथषड्भि: पूजितं शेषपञ्चै: सकलकलसंख्यां क्षिप्रमाख्याहि तस्य ।।

Amalakamalaasasesthrayamsa panchaashashtai
sthrinayanahari soorya yena thuryena chaaryaa
gurupada mathashatbhi: poojitham seshapanchai:
sakalakalasankhyaam kshipramaakhyaahi thasya

One third of the total lotus flowers were offered for performing pooja to Sankara, 1/5 to Vishnu, 1/6 to Surya, 1/4 to Devi and remaining 5 to Guru. What was the total number of lotus flowers?

Solution for this problem can be obtained by assuming that the number of total lotus is x and taking the sum of all fractions. I.e $1/3\ x + 1/5x + 1/6x + 1/4x + 5 = x$. From this the value of x can be calculated. Another type of similar problem for determining a number is given by Bhaskaracharya I in Aryabhateeyabhashya (124.1)

द्विगुणं रूपमेतम् पञ्चविभक्तम् त्रिताडितं भूयः ।
द्व्यूनं सप्तविभक्तम् लब्धम् रूपं कियत् भवेत् पूर्वं ।।

Dvigunam roopametham panchavibhaktham thrithaaditham bhooya:
dvyoonam sapthavibhaktham labdham roopam kiyath bhaveth poorvam

A number is multiplied by 2, increased by 1, divided by 5, multiplied by 3, then diminished by 2 and divided by 7, the result is 1. Say what is the number?

Answer can be derived by tracing back method, stepwise assuming the unknown number as x. Yet another example given in Bhaskarabhashya (133.1):

पञ्चभिरेकं रूपं द्वे रूपे चैव सप्तभागेन ।
अवशिष्यते तु राशिविगण्यतां तत्र का संख्या ।।

Panchabhirekam roopam dve roope chaiva sapthabhaagena
avasishyathe thu raasiviganyathaam thathra kaa sankhyaa

A number leaves 1 as the remainder when divided by 5 and, 2 when divided by 7. Calculate the number.

Using intermediate equation method, solution for this problem could also be found out.

The method of equating two parameters of which one is known and the other is unknown, is also adopted in mathematics. Bhaskaracharya I has equated two persons who are equally rich, having two items in different quantities and told to find out the price of the unknown item. (Aryabhateeyabhashyam I, 127 and 128)

कुङ्कुमपलानि चाष्टावेकस्य धनस्य रूपका भवति
द्वादशपलानि विद्यावन्यस्य धनस्य रूपकास्त्रिंशत् ।
तुल्यार्धेण च क्रीतं कुङ्कुमं द्वाभ्यां कियत् पलार्धेण
इच्छानितत्र बोद्धुं मूल्यं वित्तं च तुल्यमेव तयोः ।।

Kunkumapalaani chaashtaavekasya dhanasya roopakaa bhavathi dvaadasapalaani vidyaavanyasya dhanasya roopakaasthrimsath thulyaardhena cha kreetham kunkumam dvaabhyaam kiyath palaardhena icchaanithathra boddhum moolyam vittham cha thulyameva thayo:

A certain person has 8 palas of safforn and money amounting to 90 rupakas, another person possesses 12 palas of saffron and 30 rupakas (and two are equally rich). If two persons have bought the saffron at the same rate per pala, I want to know the price of one pala of saffron and also equal wealth possesed by the two, (equivalent in rupakas)

This problem has to be solved using a simple equation $8x+90=12x+30$: from this x (price of saffron) can be obtained and substituting the value for x, total wealth can be determined.

In yet another problem given in Bhaskarabhashya (128.4) this question is asked.

नव गुलिका सप्त(च) रूपकसमास्त्रयाणां (तु) गुलिकानां ।
त्रयोदशानां च रूपकाणां तदा किं गुलिकामूल्यम् ।।

Nava gulikaa saptha (cha) roopakasamaasthrayaanaam (thu) gulikaanaam thrayodasaanaam cha roopakaanaam thadaa kim gulikaa moolyam

If 9 gulika and 7 rupaka are equal to 3 gulika and 13 rupaka, what is the price of one gulika? (the answer can be determined through the same method followed above)

In Patiganita (52. ii) an important problem on the quality of metallic alloy is given, in respect to the colour of product.

From this the composition should be found out. It is like finding out the average of many averages.

हेमगुणवर्णयोगे हेमैक्यहृते भवेद्वर्णः ।

Hemagunavarnayoge hemaikyahruthe bhaved varna:

The sum of the products of weight and varna of several pieces of gold being divided by the sum of the weight of the pieces of gold, gives the varna of alloy

I.e n pieces of gold of weight wa, wb, wc, wd, ...wn and varnas va, vb, vc, vd.... vn, then varna of the alloy v = (wa va +wb vb+wc vc+....wn vn)/ (wa+wb+wc+....wn)

A problem of significance is given in **Lilavati** by Bhaskaracharya II (p.129.ex.13-1)

ये निर्झरा दिनदिनार्ध तृतीय षष्ठैः संपूरयन्ति हि पृथक् पृथगेव

मुक्ताः । वापीं यदा युगपदेव सखे विमुक्तास्ते

केन वासरलवेन तदा वदाशु ।।

Ye nirjaraa dinadinaardha thrutheeya shashtai:
sampoorayanthi pruthak pruthakeva mukthaa:
vaapeem yadaa yugapadeva sakhe vimukthaasthe
kenavaasaralavena thadaa vadaasu

By opening 4 inlets separately, one pond gets filled respectively within 1, ½, 1/3, and 1/6 days. If all the four inlets are opened together, how much time (in fraction of the day) is required to fill the pond.

Equations of the higher order:

Higher order equations, including quadratic equations were handled by Indian mathematicians. Solution for arriving at the answer of quadratic equation is given in "Vedaganitam" (Page 35, 36).

चतुराहतवर्गसमैः रूपैः पक्षद्वयं गुणयेत् ।
अव्यक्तवर्ग रूपैर्युक्तौ पक्षौ ततो मूलम् ।।

Chathuraahatha vargasamai: roopai pakshadvayam gunayeth avyaktha varga roopairyukthow pakshow thatho moolam

Add on both sides of an equation, 4 times the unknown value. Again add on both sides the square of the unknown value and take the square root. (This summarised procedure can be mathematically presented as follows:)

$ax^2 + bx = c$ Multiply both sides with 4a

$4a^2x^2 + 4\ abx = 4ac$ Add b^2

$4a^2x^2 + 4abx + b^2 = 4ac + b^2$

$(2ax + b)^2 = 4ac + b^2$ I.e $2ax + b = \sqrt{(4ac + b^2)}$

from the above, x can be calculated, says the rule.

Sreedharacharya and Bhaskaracharya I have discussed many examples on the application of the quadratic equations. One each from these two authors are given. (Patiganita example 100)

वानरकुलत्रिभागः स्वत्र्यंशसमन्वितः सरः प्रययौ ।
मूलं च पिपासति द्वौ चूततले स्थितौ शेषौ ।।

Vaanarakulathribhaga: svathryamsa samanvitha: sara: prayayow moolam cha pipaasathi dvow choothathale sthithow seshow

One third of a troop of monkey with one third of itself has gone to the tank; the square root of the whole troop is afflicted with thirst, and the remaining 2 monkeys are sitting under the mango tree. What is the total number of monkeys?

The problem can be written as follows:

$1/3a + 1/9a + \sqrt{a} + 2 = a.$

This equation is quadratic in nature and the solutions can be found out as explained before using the standard method.

Bhaskaracharya's Lilavati (page 95 example 1) gives the following exercises

बाले मरालकुलमूलदलानि सप्त तीरे विलासभरमन्थरगाण्यपश्यम् ।
कुर्वञ्च केलिकलहं कलहँसयुग्मं शेषं जले वद मरालकुलप्रमाणम् ॥

Bale maralakula mooladalaani saptha theere vilaasabhara manthara gaanyapasyam kurvancha keleekalaham kalahamsayugmam sesham jale vada maraalakula pramaanam

I saw that one half of 7 times of the square root of the total number of swans were slowly moving away in the river. Remaining 2 are playing in water. What is the number of total swans? (equation: $7/2 \sqrt{a}+2=a$)

Mahaveeracharya (815 AD) in his Ganita sarasangraha (4-41) has also given an interesting example, the translated version of which is given below.

"Among the total elephants, 1/3rd of them and three times the square root of the remaining are in the valley. One male with three female elephants is in the river. Find out the total number of elephants". (Equation : $1/3a + 3\sqrt{a}+1+3=a$)

All these authors have given in detail the rules and methods through which it should be worked out. It is noteworthy here that the commentators of the original works have gone deep into various aspects of the equations and have expanded the theoretical and practical scope, by incorporating and updating the knowledge.

The credit of discovering binomial theorems and their application should go to Indians. The theorem in its modern and simplest form can be written as $(a+b)^n = a^n+...+b^n$ i.e n is positive integer. This form of writing the equation was done by an Indian scholar, Halayudha of 10th century AD. Higher level binomial equation was arrived at by him based on the principles of syllables, given in Chanda sutra of Pingalacharya (200 BC). He used the examples to show not only the theoretical capability of the ancient mathematicians but also the correct method of its application in distinguishing the sound intensity. Derivation was arrived at by him as follows: If a three syllablic Madhya Chanda based on guru and lakhu sounds were followed, then variation of guru and lakhu sound will be on the following pattern: 3 guru sound occur once, 2 guru and 1 lakhu occur thrice, 1 guru and 2 lakhu sounds occur thrice, 3 lakhu occur once. The equation can be derived easily. If guru is g and lakhu is 1 then,

$(g+1)^3 = g^3+3g^2 1+3g1^2+1^3$. This equation is the same as $(x+y)^3$. Similarly for finding the pratishta Chanda, in the Chanda sastra of Pingalacharya, the following equation can be indirectly applied in this form: $(g+1)^4$ which is expanded as $g^4+4g^3 1+4g^2 1^2+4g1^3+1^4$ I.e 4 guru sound occur once, 3 guru and 1 lakhu occur four times, 2 guru and 2 lakhu occur four times, 1 guru and 3 lakhu occur four times and 4 lakhu occur once.

More information was also known on the higher levels of equations, as stated by Madhavacharya in Kriyakramakari. Here, Madhavacharya quoted another famous mathematician Chitrabhanu who has given the following equations.

$(a^3 - b^3 - (a - b)^3) / 3(a - b) = ab$

$(a^3 - b^3 - (a - b)^3/3 ab = a - b$

$(2 (a - b) + (a - b) /3 (a^2 + b^2)= a - b$

Summing up the descriptions on these mathematical equations and rules one can understand that the algebra of even higher order were used in India centuries before the present state of development in the subject. The credit of discovery of the quadratic equation and all other binomial equations have been given to the Europeans Apianus (1527 AD), Stifel (1544 AD), Acheubel (1545 AD), Tartagalia (1556 AD) and Bombelli (1572 AD). But is was the Indian scholars who gave the foundations for the binomial theorems with theoretical and applied back ground and common applications.

INTRODUCTION

Swami Vivekananda on Indian heritage!

"By the Vedas no books are meant; they mean the accumulated treasury of spiritual laws discovered by different persons in different times. Just as the law of gravitation existed before its discovery, and would exist if all humanity forgot it. So as is it with the laws that govern the spiritual world. The moral, ethical, and spiritual relations between soul and soul, and between individual spirit and the father of all spirits, were there before their discovery, and would remain even if we forgot them. The discoverers of these laws are called Rishies and we honour them as perfected beings. I am glad to tell this audience that some of the very greatest them were women.

Romain Rolland

The true Vedantic spirit does not start out with a system of preconceived ideas. It possesses absolute liberty and unrivalled courage among religions with regard to the facts to be observed and the diverse hypothesis it has laid down for their coordination. Never having been hampered by a priestly order, each man has been entirely free to search wherever he pleasured for the spiritual explanation of the spectacle of the universe.

Astro physicist R. A. Millikan

It seems to me that the two great pillars upon which all human well being and human progress rest are first the spirit of religion and second the spirit of science or knowledge, neither can attain its largest effectiveness without support from the other. To promote the latter we have universities and research institutions, but the supreme opportunity for every one with no exception lies in the first that is spirituality!

Dr. Fritjof Capra of Berkeley University, Department of Physics on the integration of the ancient Indian spirituality and modern western science!

The basic elements of the Eastern world view are also those of the world view emerging from the modern physics.

Eastern thought and more general mystical thoughts provide a consistent and relevant philosophical background to the theories of contemporary science.

This book (Tao of Physics) aims at improving the image of the science by showing that there is an essential harmony between the spirit of Eastern wisdom and Western science. It attempts to suggest that modern physics goes far beyond technology- that the way or Tao - of physics can be a path with a heart, a way to spiritual knowledge a self realisation.

Geometrical Studies

A well defined chronlogical pattern can be seen in the development of Geometry in India. Mohanjo daro, Harappa, Lothal and dozens of other archeologically important sites (and some more are being excavated) have shown that the constructions have definite geometrical patterns. They are triangular, quadrilateral, circular or imposed structures and all of them were constructed by following refined geometrical applications. Some of these structures have a mathematical history, of not less than 5000 to 6000 years.

However, from the available literature in Sanskrit, the Sulba sutras stand first. These books deal with the ritual in the Srouths sutra and at the end of the Srouths sutra comes the Sulba sutras. But the Sulba sutras have a definite Sanskrit style and compositional structure of the Vedic Sanskrit language. The prominent among the Sulba sutras are Bhoudhayana, Apastamba, Katyana and Manava Sulba sutras. Even though the periods of these books are generally fixed at the beginning of first millennia BC, some historians estimate the age to be in 2 and /or even 3rd millennia BC. Even if we fix the period to 1000 BC. It is far older than the defined period of Greek mathematicians, which falls between 500 - 200 BC. The selected quotations from Sulba sutras can give the modern geometrical concepts developed in ancient India.

Bhoudhayana Sulbasutra : This is chronologically estimated to be the first among all the Sulbasutras. It may be around 1000BC and this Sulbasutras got the present form.

Quotations, without detailed explanations are given below, as they are self-explanatory, geometrical informations.

व्यायाममात्री भवतीति गार्हपत्यचितेर्विज्ञायते ।
चतुरश्रेत्येकेषां परिमण्डलेत्येकेषां ।

Vyaayaamamaathree bhavatheethi gaarhapathyachither vijnayathe chathurasrethyekaashaam parimandaleethyekeshaam

According to tradition the Garhapatyaygi fire altar has the measure of area one vyayama. It is a square according to one tradition and a circle by another (II.61-63). This is an example to show the emergence of the geometrical background in Sulba sutras.

प्रौगचितिम् चिन्वितेति । *Prowgachithim chinvithethi*

For prouga chiti a fire altar in the form of isosceeles triangle is to be constructed (III. 161): shows how theorems and equations related to triangles emerged in the ritual books.

उभयत: प्रौगम् चिन्वितेति । *Ubhayatha: prougam chinvithethi*

Ubhaya prouga chiti, a rhombus with two isoceeles triangles is to be constructed (III. 172) (relation between the rhombus and isosceles triangles are dealt with)

यावानग्नि: सारत्निपादेशाष्टवदुभयत: प्रौगम् कृत्वा ......

Yavaanagni: saarathni paadesaashta vadubhayatha: prowgam kruthva....

(For yavanaagni a rhombus equal in area to two aratnis and one pradesa (1½ purusha in area) is laid (III. 173) (method to generate a geometrical figure from its area).

Different types of bricks are described as follows. Bricks with one side equal to one fifth of purusha (Purusha is a measurement equal to the height of the yajamana). Quarter bricks bounded four sides ½ pradesa (6 angulas: average diameter of the finger is one angula), 1½ pradesa (18 angulas), 1 pradesa and √2 pradesa. Adhyardha bricks bounded four sides ½ vyayama (48 angulas) 1 aratni (24 angulas) and √2 aratni. These make 6 types of bricks (12.7)

This example gives a lot of information on the use of square root, relation among various measurements, manufacture of bricks, having different shapes and sizes, etc. and also the approach to arrive at various structures by including many parameters, as a branch of ganita.

समचतुरश्रस्यक्ष्णयारज्जु द्विष्टावतिं भूमिं करोति ।

Samachathurasrasyakshnayaa rajju dvishtavathim bhoomim karothi

The diagonal of a square produces double the area of the square (1.45) a rule to be followed in the construction of chiti. It is a geometrical theorem.

दीर्घचतुरश्रस्यक्ष्णयारज्जुः पार्श्वमानि तिर्यन्मानि
च यत्पृथग्भूते कुरुतस्तदुभयं करोति ।

Deerghachathurasrasyakshnayaarajju: paarsvamaani thiryanmaani cha yatpruthakbhoothe kuruthasthadubhayam karoti

Areas produced separately by the length and breadth of rectangle together equal to the area of the (square) produced by the diagonal (1.48) (gives the directions, to make the geometrical figures under specified rules:)

तासां त्रिकचतुष्कयोर्द्वादशिकपञ्चिकयोः पञ्चदशिकाष्टिकयोः
साप्तिकचतुर्विंशतिकयोः द्वादशिकपञ्चत्रिंशिकयोः
पञ्चदशिकषट्त्रिंशिकयोः इत्येतासूपलब्धिः ।।

thaasaam trika chathushkayordvaadasikapanchikayo: panchadasikaashti kayo: saaptikachathurimsathikayo: dvaadasika panchathrimsathikayo: panchadasikashad-thrimsikayo: ithyethaasoopalabdhi:

This is observed in rectangles having sides 3 and 4 (=5), 12 and 5 (=13), 15 and 8 (=17), 7 and 24 (=25), 12 and 35 (=37) and 15 and 36 (=39) (1.49).

Here, with examples Pythagorus theorem (Bhoudhayana theorem) is explained as: when two squares are constructed on the length and breadth of a rectangle, the square obtained from the diagonal of the rectangle will have sides without fractions- as whole numbers given in parenthesis-for the rectangles having above set of lengths and breadths.

Apastamba Sulbasutra:

This appears to be of later origin than Bhoudhayana Sulbasutra, according to some historians. More descriptions and significant information on the structure of altars and geometrical figures are given in Apastamba Sulba sutra. Some methods and geometrical structures given in the Boudhayana sutras are also repeated here with minor variation in language style. A few important and relevant quotations are given below. Some of them are improvements and addenda to earlier writings.

For different agni (yajna ritual) altars with varying areas are to be constructed. (8.3)

एकविधः प्रथमोऽग्निर्द्विविधः द्वितीयास्त्रिविधस्तृतीयः
त एवमेवाद्यन्त्यैकशतविद्यात् ।।

Ekavidha: prathamognirdvividha: dvitheeyarthrividha sthrutheeya: tha evamevadyanthyai sathavidyaath

The first agni is one fold (1½sq. purusha) in area, the second 2 fold (2½ sq. purusha) and third 3 fold (3½sq. purusha) and so on. In this way one has to go upto hundred and one fold agni (with the sacrifical altar having area 101½ sq. purusha) - (Even for the same type of yajna, when repeated year after year by same yajamana, the size of the altars is to be increased. This increase in size has a mathematical relation. Hence this also emphasises application of geometry at various instances)

पुरुषमात्रेण विमिमीते इति विज्ञायते ।

Purushamaathrena vimimeethe ithi vijnayathe

According to tradition all the measurements are to be made with purusha unit i.e to be measured with bamboo (8.7).

This shows that there was a definite mechanism for the standrad of measurements and the measuring scale.

पादमात्र्यो भवन्ति आरत्निमात्र्यो भवन्त्युर्वस्थिमात्र्यो
भवन्त्यणूकमात्र्यो भवन्तीति विज्ञायते ।।

Paadamaathryo bhavanthi aarathnimaathryo bhavanthyoo rvasthimaathryo bhavantheethi vijnayathe

Other measurement units to find out the area are pada, aratni,urvasti, anuka, etc. (11.2) They are related as follows.

चतुर्भागीयमणूकं पञ्चमभगीयारत्नी ततोर्वस्थि ।।

Chathurbhaageeyamanookam panchabbhaageeyaaratnee thathorvasthee

Anuka is 1/4 of a purusha, aratni is 1/5 of a purusha, and so is urvasti which is 1/6 of purusha.(11.3) (It gives a clear picture of the relationship between the different units of measurements).

Using the areas, diagonals, or one of the side, the geometrical structures were constructed. When area of the circle is given, it has to be constructed after finding out the diameter. Hence the method of finding out the diameter was expected to be known. The same is the case for all other structures. Constructing square from its diagonal, rhombus from diagonal, square inscribed circle and so on... Rules are clearly given for this purpose. Similarly from the mesurements of the altars and the number of bricks required to construct the same, bricks are to be designed and made based on the geometrical knowledge. No broken or smaller bricks can be used for final adjustment in construction purposes. Hence the brickmaker is expected to have a perfected geometrical back ground of high standard, so that the person involved in the construction of altars can extrapolate and intrapolate the measurements of bricks and their sizes. Some examples on this line are also given below.

यावानग्निः सारत्नीप्रदेशाष्टावर्तिं भूमिं परिमण्डलं
कृत्वा तस्मिंश्चतुरश्रम् अवदध्यावत् सम्भवेत् ।।

Yaavaanagni: saarathnee pradesaashtasathim bhooomim parimandalam kruthvaa thasmimschathu rasram avadadhyaavath sambhaveth

A circle of area equal to that of the fire altar with 2 aratnis and one paradesa is made and the largest possible square is inscribed in it (12.10) (The knowledge on the construction of a circle, after calculating the diameter from the area is the first step. Then a square, suitably fixing in the circle, is to be inscribed after calculations). Here measurements of the structures are not crude or simple. It is complex too and an approximation will not work.

द्विपुरुषं पश्चादर्धपुरुषं पुरस्ताच्चतुर्भागोनः पुरुष आयामोऽ
ष्टादशकरण्यौ पार्श्वयोष्टः पञ्चदश परिगृह्णाति तत्पुच्छम् ।।

Dvipurusham paschaadardhapurusham purasthaachhathur bhagona: purusha: aayamofshtadasa karanyow paarsvayoshta: panchadasa parigruhnaathi thatpucham

(An area bounded by length of) 2 purushas on the western side, 1½ purusha on eastern side, √18 on each of other two sides and having height 3/4 purusha can accommodate 15 bricks (19.1).

Hence size of the bricks has to be calculated from the given data of the number of bricks permitted in building this altar.

षोडशीम् चतुर्भिः परिगृह्णीयात् अष्टमेन त्रिभिरष्टमैश्चतुर्थेन चतुर्थ सविशेषेणेति ।।

Shodaseem chathurbhi: parigruhneeyaath ashtamena thribhirashtamaischathurthena chathurtha savisesbenethi

The bricks to be made by four sides having the measurements. 1/8, 3/8, 1/4 and $\sqrt{2}/4$ purusha (19.2)

This obviously stands out as an example on the complicated bricks making with the background of geometry. Sutra (1.5) says:

चतुरश्रस्यक्ष्णयारज्जुर्द्विष्टावर्तिं भूमिं करोति समस्य द्विकरणि ।।

Chathurasra syakshnayaarajjurdvishtavathim bhoomim karothi samasya dvikarani

The diagonal of the square produces double the area (of the square) and it is $\sqrt{2}$ of the side of the square.

It is important to note here that the method for the determination of $\sqrt{2}$ is given in Boudhayana sulba sutra, for the first time in mathematics (I.61, 62) This is also explained under Katyayana sulbasutra.

प्रमाणं तृतीयेन वर्धयेत्तच्च चतुर्थेनात्सचतुस्त्रिंशोनेन सविशेषः ।।

Pramaana thrutheeyena vardhayethaccha chathurthenaathsachathusthrimsonena savisesha:

Increase the measurement (side of a square) by third and this third by it's own fourth less the thirty fourth part of that fourth. This is the value with special quantity in excess for $\sqrt{2}$.

Apastamba sutra (5.3 to 5.6) lines give measurements of sides of rectangles which can give diagonal measurements in whole numbers. This is an example of applied knowledge.

त्रिकचतुष्कयोः पञ्चिकक्ष्णया रज्जुः। द्वादशिकपञ्चिकयोः त्रयोदशिक क्ष्णयारज्जुः । पञ्चदशिकष्टिकयोः सप्तदशिकक्ष्णयारज्जुः ।।

Thrika chathushkayo: panchikakshnayaa rajju:
dvaadasika panchikayo: thrayo datika kshnayaarajju:
panchadasikaashtikayo: sapthadasikakshnayaarajju:

The diagonals of rectangle with sides 3 and 4 is 5, 12 and 5 is 13, 15 and 8 is 17.

Transforming one figure into another is frequently discussed in the book.

मण्डलम् चतुरश्रम् चिकीर्षन् विष्कम्भं पंचदशभागान्
कृत्वा द्वावुद्धरेत् त्रयोदशावशिष्यन्ते सानित्याचतुरश्रम् ।।

Mandalam chathurasram chikeershan vishkambham
panchadasa bhaagaan kruthvaa dvaavuddhareth
thrayodasaavasishyanthe saanithyaachathurasram

To transform a circle into a square, diameter is divided into 15 parts, 2 of them are removed, leaving 13 parts. This gives the approximate side of the square (3.3)

This however cannot give a good result. In Manava Sulba sutra better approximations are given for this calculation, which is referred elsewhere also.

The following important rules are also given in Apastamba Sulbasutra: Rule 3.2 is for transforming a square into a circle, 3.1 is for transforming a square into reactangle, 12.1 which has been stated earlier is inscribing a square in a circle of known area. Similarly inscribing different figures in other geometrical figures is given for the construction of required/suggested types of altars.

Katyayana sulba sutra: This book gives a variety of structures for altars: Katyayana Sulba sutra (4-I) says:

द्रोणचिद्रथचक्रचित्कंकचित्प्रौगचिदुभयतः प्रौगः समूह्यापुरीषा इत्यग्नयः ।।

Dronachidrathachakrachithkankachith prougachidubhayatha; prowga: samoohyaapureeshaa ithyagnaya:

The altar is of the shape of a trough, chariot wheel, falcon, triangle, rhombus and a kind of pot in the shape of wheel.

In all these structures the size, sides, shape of the bricks and their numbers are specified. Hence the construction procedure have to be based on the geometrical information. Methods of finding the square roots are also given. Stanza 2.9 describes the method for finding the diagonal of a square.

करणीं तृतीयेन वर्धयेत्तच्च स्वचतुर्थेनात्मचतुस्त्रिंशोनेन स विशेष इति विशेषः ।।

Karaneem thrutheeyena varddhayethachha svachatheerthenaa thmachathu sthrimsonena sa visesha ithi visesha:

The measure of the side is to be increased by one third of its value, again its own one fourth, less the 34th part (of that of fourth).

This is the diagonal of a square, whose side is the measure and this is approximate, which is also explained in other suthras. I.e if the side is a the diagonal a+a/3+(1/3x4) - 1/3x4x34. This can give a fine approximation for the value of $\sqrt{2}$ a = 1.4142156a..... The value for $\sqrt{2}$ = 1.414212. The modern and ancient values are almost the same. This value and method are reproduced here from Bhoudhayana Sulbam. Thibaut says about this in his observation ".... thus it is clear that the ancient Hindus have attained remarkable degree of accuracy in calculation of the approximate value for $\sqrt{2}$"

A detailed description on the construction of yajnasala (ritual hall) is given in the Manava sulbasutra (3.1-3.3). Infact this description gives the beauty and rule of the geometrical contents in the sulba sutra.

प्रग्वंशम् दशकम् कुर्यात्पत्नीशालां चतुःशयम् । प्राग्वंशतृषु वेद्यन्तो वेद्यन्तात् प्रक्रमे सदः । नवकम् तु षडो विद्याच्चत्वारः षडशोन्तरम् । चत्वारिस्त्रिक हविर्धानामर्धदशाष्टादन्तरम् । पादम् यूपवतेमित्वा शेषमौत्तरवेदिकम्' अग्नीद्रम् षडारत्न्येव षड्त्रिंशत्प्रक्रमा रज्जुः ।

Pragvamsam dasakam kuryaathpathneesaalaam chathu: sayam praagvam sathrushu vedyantho vedyanthaath prakrame sada: navakam thu shaddo vidyaachhathvara: shodasontharam chathvaaristhrika havirdhanaamardhadasaashtaadantharam paadam yoopavathemithvaa seshamowttharavedikam agneedram shadaarathnyeva shadthrivimsathprakramaa rajju:

The sacrifical hut pragvamsa occupies (on the ground a square area of) side 10 aratni, the hut for the wife (patnee sala) (a square) of 4 aratnis, end of the (maha) vedi is at a distance of 3 prakrama from the pragvamsa, and the hall sadas is one prakrama away from the western end of the mahavedi. The (praci) sadas is 9 prakramas. The havirdana (a square) of 12 prakramas is 4 prakramas from the sadas and 10½ prakrama from the yupavata. One pada is allowed for the yupavata and the remaining belongs to the uttaravedi. The agnidra hut (a square) of 6 aratnis, the chord measures 36 prakramas.......

Manava Sulba sutras: Almost all the mathematical descriptions given in above three can be seen in this Sulbasutra also. It is very important to note that many fundamental knowledge which are not described in other Sulbasutras have found their place in Manava sulbasutra. An example is the method for finding the volume of a structure given in the line 10.9

आयम बाहुम् निक्षिप्य विस्तारस्तु तथापृथक्
सोऽध्यर्धम् गुणयेद्राशिम् ससर्वगुणितो घनः ।।

Aayama baabum nikshipya visthaarasthu thathaapruthak soadhyardham gunayedraasim sasarvagunitho ghana:

Multiply the length with the breadth separately and that again by the height. This always gives the result in cubic measure.

This appears to be the first ever seen in a literature for finding out the volume of a structure. These lines are written centuries before Archimedes, who found out the volume of an object by dipping it in a water tank. A few more quotation from the Manava sulba sutra on the circular structure are given, (11.13)

विष्कम्भ: पञ्चभागश्च विष्कम्भस्त्रिगुणश्च य:
सा मण्डल परिक्षेपो न वालमतिरिच्यते ।।

Vishkambha: panchabhaagascha vishkambhasthrigunascha ya: saa mandala pari kshepo na vaalamathirichyathe

The fifth part of the diameter added to three times the diameter gives the circumference of the circle. Not a hair is left over.

For circle 3.14 x diameter is the circumference. As per Manva sulbasutra it is 3.2 x diameter, which is perhaps the first approximation for Π. The descriptions given in Apastamba sulba sutra (3.3-given earlier) to convert a circle into a square was only an approximation. More refined rule is given here (11.14)

दशधा चिन्द्य विष्कम्भम् त्रिभागानुद्धरेत्तत:
तेन यच्चतुरश्रम् स्यान्मण्डले तदपप्रथि: ।।

Dasadhaa chindya vishkambham thribhaagaanuddharethbatha: thena yachhathurasram syaanmandale thadapaprathi:

Divide the diameter of a circle into ten parts and leave out three parts. The square drawn with this (as side) and placed within the circle just projects.

The measurements of the bricks given in rule 14.21 can throw light, not only on the geometrical accuracy followed in brick manufacturing but also the ceramics techniques followed during the period.

चत्वारि कराण्यन्येषां त्रिचतुर्थेन कारयेत् नवभागा
अक्ष्णार्धक्ष्णा: पञ्चकोणा: च भागश:

Chathvaari karaanyanyeshaam thrichathurthena kaarayeth navabhaagaa akshnaardhakshnaa: panchakonaa: cha bhaagasa:

Four kinds of bricks were used with one third and one fourth (of a purusha) measurement. These are one ninth of the original (40, 40), triangular (30, 30, 30 √2), half triangular (15 √2, 30, 15 √2) and five cornered bricks (15 √2, 15 √2,15, 30, 15).

From the above selected quotations, a lot more information can be elucidated. It is obvious that the geometrical background was excellent, in sulbasutras They acted as the strong foundation for building up the modern mathematics.

Even the actual proof of Pythagorous theorem was given in Greek only in 300 BC by Euclid, says Burk. Proclus (460 BC) has made it clear that it is not an original contribution of the Greeks, as also proved by Mishra and Singh. Hence why not the mathematician think of renaming the theorem as Bhoudhayana theorem!

Little more information if added to the above, the exact practical form of Bhoudhayana theorem is stated to have been known in Brahmanas (explanation of ritual part of the Vedas-also known as karmakanda) during 2500 BC. Satapata Brahmana, Taitireeya Brahmana and Garga samhita also contain this information, Seidenberg has shown that Taitireeya samhita describes not only the algebraic or Computational aspects of this theorem but also geometric or sonstructive aspects which was not known to any others. (This is quoted from Seidenberg 1983, the geometry of Vedic rituals, in Agani, Vol. II. Frits Staal, Asian Humanities press, Berkeley, page 125-126). Burk has concluded that the theorem was known to Indians with all its proofs, in the far past Pythagorus.

Many studies have thrown light on the geometrical contents in these Sulbasutras on which modern mathematicians could arrive at important conclusions, thousands of years later. Construction of a line perpendicular to other is derived from Katyayana Sulbasutra (rule 1.4 and 1.5). Similarly construction[62] of squares having areas either sum of two other squares, or differences of areas of two squares are also given in Apastamba sulbam 2.4, Boudhayana sulbam 2.1 and 2.2 and Katyayana sulbam 2.13 and 3.1

Triangles : Details on the triangles are given in the Sulbasutras. They contain measurements of sides, hypotenuse, areas and also the relations among these parameters. Inscribing other geometrical structures in the triangle and vice versa are also explained, including the methodology for doing so. Bhakshali manuscripts contain a lot of information on the geometrical figures including the triangles. Bhaskarachraya I has given the examples of application of the rules (given by Aryabhatta I) on the lamp and shadow calculations. Bhaskarabhashyam to Aryabhateeyam gives the examples with details on triangles (92.3).

यष्टिप्रदीपमूलात् पञ्चाशद्विवरसंस्थितः शङ्कुः ।
तस्यच्छाया पङ्क्तिर्वाच्यस्तस्मिन् कियान् दीपः ।।

Yashti pradeepamooolaath panchaasadvivarasamsthitha sanku:
thasya cchayaa pangthirvaachyasthasmin kiyaandeepa:

The shadow of the gnomon situated at a distance of 50 angulas from the foot of the lamp post is 10 angula. Say what is the height of the lamp.

Height of the lamp post can be found out from length of the shadow and the length of the tip of shadow and lamp using Bhoudhayana theorem. Another interesting problem on the same line is about the bamboo triangle, given by the author in the same book (99.4)

अष्टादशकोच्छ्रयो वंशो वातेन पातितोमूलात् ।
षडगत्वा/सौ पतितास्त्रिभुजं कृत्वा क्व भग्नः स्यात् ।।

Ashtaadasakocchrayovamso vaathena paathithomoolaath
shadgathvaavasow pathithaasthribhujam kruthvaa kva bhaghna: syaath

A bamboo of beight 18 cubits fell by the wind, it falls at a distance of 6 cubits from the root, thus forming a right triangle, where is the break?

This problem can be worked out as follows: The sum of lengths of hypotenuse and height is 18 cubit and the base of the triangle formed is 6 cubits. From this the height (x) can be calculated as $(18\text{-}x)^2\text{-}x^2=6^2$.

Same problem is repeated by Pruthudaka in his commentary on the Brahmasphuta siddhanta (XII. 41) and also in Mahavira in Ganitasara sangraha (VII.191, 192) Sreedharacharya, Bhaskaracharya I and II and many others have given problems of this type related to bamboo pole, lamp post-gnomon and lotus - river depth, etc. Through this problem one can get the required exposure on the sound background of the ancient knowledge in the geometry of triangles.

Quadrilaterals: Brahmasphuta siddhanta (XII, 28) gives the equation for the diagonal quadrilaterals in relation with its sides:

"The sum of the products of two pairs of sides about any diagonal divided by the sum of the products of pairs of the sides about other diagonal and the result multiplied by the sum of prducts of opposite sides give the square of the first diagonal". Similarly the second diagonal can also be calculated. First diagonal is x, (for sides a, b) and second y (for sides c, d) adjacent to them,

Then, $x^2 = (ab+cd)(ac+bd)/(ad+bc)$ And

$g^2 = (ad+bc)(ac+bd)/(ab+cd)$

Patiganita (110-111) gives a rule for finding out the area of various types of natural and aritifical geometrical figures in the following way:

आयतसमचतुरश्रे द्वित्रिसमभुजे विषमचतुरश्रम्
समविषमद्विसमभुजत्र्यश्रण्यथवृत्त चापे च ।
क्षेत्राणिदशैतानि हि फलमेषां साधयेत् स्वकरणे(न)
एतत् परिकल्प्यान्येषां गजदन्तनेमिपूर्वाणाम् ।।

Aayatha samachathurasre dvithri samabhuje vishamachathurasram
samavishamadvisamabhuja thrya sranyathavruthha chaape cha
kshethraani dasaithaani hi phalameshaam saadhayeth svakarane(na)
ethath parikalpyaanyeshaam gajadanthanemi poorvaanaam

The rectangular quadrilateral, the equilateral quadrilateral, equibilateral quadrilateral equitrilateral quadrilateral....., the inequilateral quadrilateral, the equilateral triangle, the scalene triangle, isosceles triangle, circle and the segment of the circle are the ten primary plane figures; the area of these figures should be determined by applying their own rules. And by considering these, one should obtain the areas of other figures, such as an elephant, a buffalo, etc.

This is the method adopted in modern mathematics too, i.e dividing the total figure into geometrical sub figures/structures whose areas can be determined using standard equations. Then partial areas are added to get the actual area of the figure. Narayana, in Ganita kaumudi, has given the same 10 figures as mentioned in Patiganita. Mahavira in Ganitasara sangraha gives three varieties of triangles, five varieties of quadrilaterals and eight varieties of curvilinear figures including ellipse, conical, concave and convex and has given the detailed explanations.

Aryabhatta I (Aryabhateeya 2.6) gives the rule for finding out the area of any triangle as the product of the height and half of the base which is the same as that known now.

त्रिभुजस्य फलशरीरं समदलकोटि भुजार्धसंवर्गः।

Thribhujasya phalasareeram samadalakoti bhujaardha samvarga:

The area of a triangle is the product of the prependicular and half the base.

Bhaskara in his bhashya for Aryabhateeya gives problems for the calculation of areas of different types of triangles (55.1)

सप्ताष्टनवभुजानां क्षेत्राणां यत्फलं समानां तु ।
पञ्चश्रवणस्य सखे षड्भूसंख्याद्वितुल्यस्य ।।

Saptaashta navabhujaanaam kshethraanaam yathphalam samaanaam thu panchasravanasya sakhe shad bhoosankhyaadvithulyasya

Tell me the area of the equilateral triangle whose sides are 7,8 and 9 units respectively and also the area of isosceles triangle whose base is 6 units and lateral sides each 5 units. The above problems deal with equilateral and isosceles triangles. Similar problems on scalene triangles are also given by Bhaskara in his bhashyam (56.5)

कर्णस्त्रयोदश स्यात् पञ्चदशान्यो मही द्विसप्तैव ।
विषमस्त्रिभुजस्य सखे फलसंख्या का भवेदस्य ।।

Karnasthrayodasa syaath panchadasaanyo mahee drisapthaiva vishamasthri bhujasya sakhe phalasankhyaa kaa bhavedasya

What is the area of a scalene traingle in which one lateral side is 13 units, other 15 unit and the base is 14 units.

This problem is also given in Brahmasphuta siddhanta (XII 21. ii), and was solved using the equation which is followed by the modern Mathematicians. I.e. finding out the semi perimeter of the triangle, and following the equation $\sqrt{S \times (S - A)(S - B)(S - C)}$, where S is the semi perimeter i.e half of the sum of all the three sides and A, B and C are the sides of the triangle. However Bhaskara I even though was a contemporary to Brahmagupta, has not adopted this equation in Bhaskarabhashya. He followed a different way which gives only an approximate value for the area of scalene triangles.

Polygonals: Bhaskaracharya II has given a very interesting information on the cyclic equilateral triangle and for Polygonals, in Lilavati (294-45,46 and 47)

त्रिब्ध्यङ्काग्निनभश्चन्द्रैस्त्रिबाणाष्टयुगाष्टभिः वेदाग्निबाणखाश्चैच
खखाभ्राभ्ररसैः क्रमात् बाणेषुनखबाणैश्चद्विद्विनन्देषुसागरैः
कुरामदशवेदैश्च वृत्तव्यासे समाहते खखखाभ्रार्क संभक्ते लभ्यन्ते
क्रमशोभुजाः वृत्तान्तस्त्र्यपूर्वाणां नवास्रान्तं पृथक् पृथक्

Thribdhyankaagninabha schandraisthri bhaanaa shtayugaashtabhi: vedaagni baanakhaaschaicha khakhaabhraabhrarasai: kramaath baaneshu nakha baanai schadvidvi nandeshu saagarai: kuraamadasavedaischa vruthbavyaase samaahathe khakhakhaabhraarka sambhakthe labhyanthe kramasobhujaa: vrutthaantha sthraya poorvaanaam navaasraantham pruthak pruthak

For cyclic equilateral triangle, cyclic square, cyclic equilateral pentagon,.... to cyclic equilateral nonagon, (cyclic figures having 3 to 9 sides with equal side measurements) their sides can be calculated respectively when diameter is multiplied separetely with 103923 (triangle) 84854 (quadrilateral) 70534 (pentagon), 60000 (hexagon) 52055 (septagon) 45922 (octagon) and 41031 (nonagon) and divided by 120000, the value will be the measurements of the sides of cyclic equilateral triangles to cyclic equilateral nonagon. Bhaskaracharya has given the example: If 2000 is the diameter of circle, equilateral geometrical figures inscribed inside that circle will have sides as follows:

Geometrical figure	Bhaskara's value	Modern value
Triangle	1732 + 1/20	1732.043
Square	1414 + 13/60	1414.211
Pentagon	1175 + 17/30	1175.5619
Hexagon	1000 + 00	999.996
Septagon	867 + 7/12	867.5799
Octagon	765 + 11/30	765.3636
Nonagon	683 + 17/20	683.85

Obviously the level of accuracy can be seen. No further discussion need be given for providing the capability of the ancient Indians on the structural knowledge of Polygonals. This information on the relationship between the diameter and the side of the cyclic figures is based on the application of the mathematical equations; a very important contribution, made nine centuries ago.

Rectangles: Finding out the area of the rectangles was well known. An excercise is given by Bhaskaracharya I in Bhaskarabhashya (67.1).

अष्टौपञ्च च पङ्क्तिर्विस्तारे दैर्घ्यमप्यमीषां यत् ।
अष्टिर्द्वादश मनवो गणितं कियदायतानां तु ।।

Ashtowpancha cha pangthirvisthaare dairghyamapyameeshaam yath.
ashtirdvaadasa manavo ganitham kiyadaayathaanaam thu

Breadths of three rectangles are 8, 5 and 10 units and their lengths are 16, 12 and 14 units respectively. What are the areas of rectangles?

Bhaskaracharya has explained the area of rectangle as the product of adjacent sides, while commenting the same rule given by Aryabhatta, in his book. Aryabhatta I has said that the product of length and breadth of a rectangle is its area.

Trapezium : Determination of area of a trapezium is more complicated than the above two classes of geometrical figures. Aryabhatta I has given the equation for the area of trapezium. Using this descriptions one can easily derive the modern equation. Aryabhatta has gone through a different route, which is intersting for a student of the subject (Aryabhateeya 2-8)

आयामगुणे पार्श्वे तद्योग हृते स्वपातरेखे ते ।
विस्तारयोगार्धगुणे ज्ञेयं क्षेत्रफलमायामे ।।

Aayaamagune paarsve thadyoga bruthe svapaatharekhe the
visthaara yogaardhagune jneyam kshethraphalamaayaame

Multiply the base and face of height, and divide (each product) by the sum of the base and the face. Results are prependiculars on the base and face. The result obtained by multiplying with half the sum of base and face by the height is to be known as the area (of trapezium).

The modern equation for finding out the area of the trapezium is the same. i.e ½(a+b)p where a and b are the opposite parallel sides and p the distance between them.

Bhaskaracharya gives an exercise for finding out the area using the equation in Bhaskarabhashyam (63.1)

भूमिश्चतुर्दशस्यात् वदनं चैवरूपाणि ।
कर्णौ त्रयोदशाग्रौ संपाताग्रं फलं च वद ।।

Bhoomischathurdasasyath vadanam chaivaroopaani
karno thrayodasaagrow sampathaagram phalam cha vada

If the base of trapezium is 14 units, the face 4 units and the lateral sides 13 units each give out the junction line and area.

Quadrilateral: In Patiganita (rule 117) a method for determining the area of a quadrilateral is given exactly in the same way as that given by Brahmagupta for traingle. In fact, it is said that Brahmagupta has given the equation for quadrilateral which can also be applied to scalene traingles, when one side is considered equal to 0.

भुजयुतिदलं चतुर्थभुजहीनं तद्वधात्पदं गणितम् ।
सदशासमलम्बानामसदृशलम्बे विषमबाहौ ।।

Bhujayuthidalam chathurtha bhujabeenam thadvadhaatlpadam ganitham
sadasaasamalambaanaama sadrusalambe vishamaabhow

(For finding out the area of a quadrilateral), set down half the sum of the (four sides of the quadrilateral) in four places, diminish them (respectively) by the sides, multiply, and take the square root. This gives the area of a quadrilateral.

This rule can be mathematically summarised as follows: √S x (S-A) (S-B) (S-C) (S-D); where A,B,C and D are sides and S half of the sum of sides (semi perimeter). This equation is also given in Brahmasputa siddanta XII 21(ii) and by Mahaviracharya in Ganitasara sangraha (VII 50 (ii) and also in Siddhanta sekhara (XII.28). In the above equation, if one side (Say D) is 0 then the quadrilateral becomes a triangle and the area will be √S x (S-A)(S-B)(S-C). This equation is also given, in another ancient mathematical book known as Yuktibhasha, for the explanation connected with cyclic figures.

When semi perimeter obtained by taking half of the sum of the sides of quadrilateral and the product obtained after each side is subtracted from the semi perimeter taking the square root of the product gives the area of triangle or quadrilateral. All the above equations/theorems in modern mathematics are known as Herons' equation/ theorem, even though they have been mentioned by Brahmagupta, Sreedharacharya, Mahaveera and Aryabhatta II. Brahmagupta had a sound knowledge in the cyclic figures too. He has given an equation for the diagonals of cyclic quadrilateral as follows "If one of the diagonals in a qudrilateral can be written as $\sqrt{(ab+cd)(ac-bd)/(ad+bc)}$ the other diagonal is $\sqrt{(ad+bc)\ (ac+bd)/(ab-cd)}$."

This is said to be one of the important contributions of Brahmagupta. Moreover, these two equations are meant for cyclic quadrilaterals, which are given with the equation for area. Hence there is another view among mathematicians that the equation for finding out the area of the quadrilateral given above by Brahmagupta; $\sqrt{S\ (S-A)(S-B)(S-C)\ (S-D)}$ is also for the cyclic quadrilateral and not for the non cyclic types. If so, Brahmagupta was correct too. The equation for the diagonals of the cyclic quadrilaterals is claimed to be the discovery by W. Snell in 1919 AD, in Europe. Snell's claim was for an equation discovered more than one thousand years ago, by Brahmagupta, with all clarity of modern mathematics.

Cyclic quadrilaterals: The mathematical equation for the radius of a circumcircle of a quadrilateral is given by Aryabhatta school as shown:

$R = 1/4\ (ab+cd)\ (ac+bd)(ad+bd) \div (a+b+c-d)\ (b+c+d-a)\ (c+d+a-b)\ (d+a+b-c)$. No mathematican has given anything equivalent to this during that period. In the above equation the denominator (a+b+c-d)(b+c+d-a)(c+d+a-b)(d+a+b-c) is written as (s-d) (s-b) (s-c) (s-a) where s is the perimeter equal to a+b+c+d. In this form the radius of the cyclic quadrilateral was rediscovered by Lhuiler in 1782 AD, centuries later then Indian discovery, put forward by the Aryabhatta school of mathematics.

In the 14th century, Narayana Bhatta developed two theorems connected with the cyclic quadrilaterals, which were not recorded earlier by any other foreign scientist. They are: 1. Three and only three diagonals are possible for four sides of cyclic quadrilaterals. 2. The area of cyclic quadrilateral is the product of these three diagonals divided by twice the circum-diameter.

The first theorem given above is also said to be Brahmagupta's contribution.

Different methods existed for finding out the area of quadrilaterals. Aryabhatta II has commented on determining different parameters of quadrilaterals without using the diagonals, thus given in Mahasiddhanta (XV. 70):

कर्णज्ञानेन विना चतुरश्रे लम्बकं फलं यद्वा वक्तुम्
वाञ्छति गणको यो/सौ मूर्खः पिशाचो वा ।।

Karnajnanena vina chathurasre lambakam phalam yaddhaa vakthum vaancchathi ganako yoasow moorkha: pisaacho vaa

A Mathematician who wished to tell the area or the altitude of a quadrilateral without knowing a diagonal is either a fool or an insensible person.

In Brahmasphuta siddhanta (Ganitadhyaya XII.21) the method for the determination of the gross area of a triangle or a quadrilateral is given.

स्थूलफलं त्रिचतुर्भुज बाहुप्रति बाहुयोगदलघात: ।
भुजयोगार्धम् चतुष्टयोभुजोनघाताल्पदं सूक्ष्मम् ।।

Sthoolaphalam thrichathurbhuja baahuprathi baahuyogadalaghaatha: bhujayogaardham chathushtayo bhujonaghaathaalpadam sookshmam

The product of half of the sums of the opposite sides is the area of tringle or quadrilateral. I.e area = ½ (a+c) x ½ (b+d).

"Whenever a diagonal separates a quadrilateral into two halves, as triangles, the sum of the areas of the triangles will be the area of the quadrilateral": says Bhaskaracharya II in Lilavati.

Circles: Among many geometrical figures, circles and spheres have attracted the attention of both ancient and modern mathematicians, much more than any other figures. In circles the relations among the radius/diameter, circumference and the area are the three focussing points around which the studies were conducted. A series of theorems have also been developed during the 15th and the 16th centuries in Europe on the relations among these three parameters of circles. A detailed study on the subject could throw light on the ancient Indian contribution on this subject. It is a matter of fact that many of those theorems attributed to European mathematicians are really the contributions of Indians.

Circles have been dealt in detail in the Sulba sutras. To an extent of resonable accuracy, the relations among diameter, area and circumference have been given in these books. More interesting is the principle adopted for inscribing other geometrical figures in the circles. The growth of knowledge of ancient Indians on circles steadily increased to great depths and resulted in formulating theorems many of which are now known in the names of Newton, Kelvin, Gregory, Euler and others.

The Indian contributions on these theorems/rules/equations took place centuries before the period of the Western Scientists.

Sulbasutras brought in application, the methods for constructing circular structures for the ritual altars. Aryabhatta (Aryabhateeya 2-13) defines drawing circles:

वृत्तम् भ्रमेण साध्यम् वृत्तक्षेत्रं भ्रमेण साध्यते ।

Vruttham bhramena saadhyam vratthakshethram bhramena saadhyathe

Circles can be drawn by rotation-using a compass - This rule is also given in Brahmasphuta siddanta (XXII.7) and in Sishyadhivruddhi Tantra (2.VIII.2) by Lallacharya. Aryabhatta I in Aryabhateeya (2.10) gives the relation between the diameter and circumference of a circle accurately:

चतुरधिकं शतमष्टगुणं द्वाषष्टिस्तथा सहस्राणां
अयुतद्वय विष्कम्भस्यासन्नो वृत्तपरिणाहः ।।

Chathuradhikam sathatmashtagunam dvaashashtisthathaa sahasraanaam ayuthadvya vishkambasyaasannoo vruthbhaparinaaha:

When 100 increased by 4 multiplied by 8 and added to 62,000 gives an approximate value for the circumference of a circle having diameter 20,000 units.

This gives a value of 62832/20,000 for ∏ and it is equal to 3.1416. This is the most accurate explanation for defining the value of ∏. Actual value of ∏ is 3.14285..... However Aryabhatta has approached this problem perfectly by saying that the value is near approximation using the Sanskrit word *asanno*. The accuracy level, indirectly followed till the first century BC and also in Sulbasutras was only upto 3.2. Bhaskaracharya I, while giving his commentary to Aryabhateeya (60.1) gives a problem for determining the circumference of a circle from its diameter. This shows that there were methods for calculating the unknown parameters using the known parameters of circles during the first half of the first millennia AD.

अष्टद्वादशषड्का: विष्कम्भस्तत्वतो मया दृष्टा: ।।
तेषां समवृत्तानां परिधिफलं मे पृथक् ब्रूहि ।।

Ashtadvaadasa shadkaa: vishkambhasthathvatho mayaa drushtaa: theshaam samavrutthaanaam parithiphalam me pruthak broohi

Diameter of 3 circles are correctly seen by me to be 8, 12 and 6 units respectively. Tell me separately the circumference and areas of the circles.

This exercise, since written in the commentary of Aryabhateeya, is the application of the formula of 2∏r for determining the circumference of a circle having radius r. In the use of this formula, Aryabhatta's value of ∏ has been taken as 3.1416. A reverse method is also applied by Bhaskaracharya I in his usual style of giving the mathematical excercises, using fractions for finding out diameter from circumference. (Bhaskarabhashya 76.2)

नवनवयमरामाणामष्टाभि: शरयमांशहीनानां खखरसवृन्दस्य च मे
व्यासावचक्ष्व विगणस्य ।।

Navanavayamaraamaanaamashtaabhi: sarayamaamsaheenaanaam khakharasavrundasya cha me vyaasaavachakshva viganasya

Calculate the diameter of a circle whose peripheries (circumference) are 3299 minus 8/25 units and 21600 units. methodology is application of reverse of the above formula. Another equation given by Virasena in his commentary called Dhavalateeka, written in 816 AD, on a mathematical work which was written much earlier by Pushpadatta namely Sakhandagama[65] says thus:

व्यासम् षोडशगुणितम् षोडशसहितम् त्रिरूपरूपैर्भक्तम् ।
व्यास त्रिगुणित सहितम् सूक्ष्मात् अपि तद् भवेत् सूक्ष्मम् ।।

Vyaasam shodasagunitham shodasasahitham thriroparoopairbhaktham vyasa thrigunitha sahitham sookshmaath api thath bhaveth sookshmam

The diameter multiplied by 16 increased by 16 divided by 113 and again combined with thrice the diameter is the circumference the more accurate than the accurate one. The above statement can be mathematically summarised as:

Circumference = 3d + (16d+16)/113

It is equivalent to Π =355/113+16/113d. The value is not correct (3.28328). But Hayashi, T. Kusubha, T and Yano, M. have published the interesting information under the title: Indian value for Π derived from Aryabhatta in the Historica Scientifica[66]. The significance of this work is not perhaps in its approximate value but in the application of round the way method, adopted. Bhaskaracharya II in Lilavati (Kshetravyavahara-rule 40) gives relation between the circumference and the diameter, to arrive at the formula, as given by modern mathematicians.

व्यासे भनन्दाग्नि हते विभक्ते खबाणसूर्यैः परिधिः ससूक्ष्मः ।
द्वाविंशतिघ्ने विहृतेऽथ शैलैः स्थूलोऽथवा स्याद् व्यवहारयोग्यः ।।

Vyaase bhava nandaagni hathe vibhakthe
khabbaana sooryai: paridhi: susookshma:
dvaavimsathighne vihruthefha sailai:
sthoolofthavaa syaath vyavahaarayogya:

Multiply the diameter with 3927 and divide with 1250. It will give the correct circumference. Or multiply diameter with 22 and divide by 7, then the circumference for common purpose will be available.

First of the above answers is correct value for ∏ 3.1416.

Charles M. Wish of the British East India Company has made this comment in the Royal Asiatic Society on some of the Ancient Indian achievements. "The approximation to the true value of the circumference with given diameters exhibited in these three works of Tanthrasangraha, Karanapaddhati and Sadratnamala, are so wonderfully correct, that European mathematicians who seek for such proportion in the doctrine of fluxions or in the more tedious continual bisection of an arc, will wonder by what means the Hindu has been able to extend the proportion to so great a length". These comments are on the three books written in the first half of this millennia whereas Aryabhatta I and Bhaskaracharya I have written in the middle of the first millennia i.e 499 AD and 628 AD. These two scholars have given accurately the values as 22/7 for arriving at the modern mathematical answer for ∏. It is sure that scholars like Charles M. Wish would have got wonder struk if they had heard about these two mathematics/astronomy giants of the world who lived in India, 1500 years ago.

VEDIC GEOMETRY
APASTHAMBA SULBASUTRA- 600 BC

Vedas are the basis of all the Indian customs and rituals. The most important and directly connected Vedic related literature are the Brahmanas, Aranyakas and Upanishads. There is another set of literature known as vedangas. The literal meaning of vedanga is ' the organs of the vedas'. There are six vedangas which are Sikha, Niruktha, Vyakarana, Chandasaastra, Kalpasastra and Jyothisha. Among these six classes the Kalpasastra i.e. the fifth vedanga is connected with many rituals which are linked to the social life of Indians directly or indirectly.

Kalpasastra consists of four major parts known as Sroutha sutra, Gruhya sutra, Dharma sutra and Pitrumedha sutra. There may be some minor parts also in some Kalpasastra. In the Sroutha sutra part of Kalpasastra, a detailed description of the sacrifical rites - yajnas - is given. These yagas or yajnas are performed by the kings, rushies, or great men of honour. Gruhya sutras consists of variety of rituals to be performed by the family head with his wife, children and their relatives. It can thus be explained from the meaning of Gruhyasutra that the rites performed by the gruhastha are given in gruhya sutra. In dharma sutra (Or Dharma sastra) all the rules and regulations of the society including the king are specifically mentioned. In fact the nation was ruled based on dharmasastra rules. In the Pithrumedha sutra, rituals connected with the death of a family member are given. Infact pitrumedha sutra describes the pithru karma.

In the first part of the Kalpasastra, (in the Sroutha sutra the sulba sutra) is included. The subject matter of Sulba sutra is geometry or Vedic geometry. The description given in Sulba sutra is directly linked with the dimensions of sacrificial altars of various shapes and sizes. The geometry of the halls, minor and major rooms meant for sitting the priests, master/ yajamana, general public, kitchen, etc is given. These rooms and halls are to be constructed under specific dimensions, size and shape. The most common geometrical shapes are square, rectangle, isoscless - equilateral- Rt angled - squalene triangles, circles, semi circles, other shapes inscribed circles and circle inscribed in other shapes, etc.

Infact all the geometrical explanations given in sulbasutras are directly connected for the explanation of the construction of the above Yaga saala or sacrificial halls and rooms.

There are four major sulbasutras which are Boudhayana, Apasthamba, Katyayana and Manava sulbasutras. Among these, the oldest one is Boudhayana. This gives detailed explanation of Pythagorous theorem with examples. More refined version of the Sulbasutra is Apastamba. And the latest one appears to be Manava sulbasutra.

Many types of fire alters are made for performing different kind of sacrifical yajna. Example; rathachakra chiti, the altar having the shape of a chariot wheel. Syena chithi, the altar having the shape of falcon, kanka chithi the altar having the shape of a bird,... these shapes and sizes are selected based on the type and aim of the yajna/yaga.

Explained here is the Apasthamba sulba sutra, which might have composed two, three or four thousand years ago. On the period of these sulbasutras, there are a variety of opinions.

APASTAMBA-SULBASUTRA

In beginning of the text itself, Maharshi Apasthamba starts the narration of the sacrificial altar construction

1. We shall explain the methods of constructing (different) figures (on the ground for building sacrificial altars).

2. A cord of length equal to a given measure is increased by its half so that the whole length is divided into three parts of half the measure each. In the third part on the western side, a mark is given at a point shorter by one-sixth (of the third part). With the two ties fastened to the two ends of the east-west line (prsthya) the cord is stretched towards the south by the mark and a pole is fixed on it. The same is done towards the north. The same is repeated on the other side (eastern) after interchanging the ties. Thus are determined (the four corners of the right rectilinear figure). Thereby the sides are shortened or lengthened.

3. Alternatively, a cord of a given measure is increased by its length; the original length plus its fourth part will constitute the diagonal and the remaining (three-fourth part of the length) the breadth (of the rectangle). Thereby, the construction of a (right rectilinear) figure is explained.

4. The area (of the squares) produced separately by the length and the breadth of a rectangle together equal the area (of the square) produced by the diagonal. By the understanding of these (methods) the construction of the figures as stated (is to be accomplished).

(This line infact is the Pythagorous theorem. As a part of the theorem, the next important geometrical point is also mentioned)

5. The diagonal of a square produces double the area (of the square). It is root of 2 (dvikarani) of the side of the square (of which it is the diagonal).

(This is also the explanation given in the modern geometry. Here in ancient geometry, square root of any number is explained as karani, whereas in the modern Sanskrit it is called as varga moolam.)

6. The measure is to be increased by its third and this (third) again by its own fourth less the thirty fourth part (of the fourth); this is (the value of) the diagonal of a square (whose side is the measure).

(This explanation gives the oldest method for finding out the square root of the number two. The length of the side of the square + one third of that value +(1/4 of the length - 1/34 of the length of the side of the square) gives the diagonal of the square)

7. Here is another method (of construction of square). Ties are made at both ends of a cord of length equal to the given measure. Marks are given at its middle and at mid points of its two halves. After stretching the cord along the east-west line poles are fixed at the ties and the marks. With the two ties fixed at the two poles at the two outer marks (mid-points of two halves), the cord is stretched towards the south by the middle mark and a sign is given (at the point reached). With the two ties fixed at the middle pole, the cord is (again) stretched by its middle mark towards the south over the sign (previously made) and a pole is fixed (at the point reached). With one tie (of the cord) fixed at this pole and the other tie at the eastern pole, the south-eastern corner is (now) obtained by (stretching the cord with) its middle mark. By removing the tie from the eastern pole and fixing it to the western pole, the south-western corner is likewise obtained by (stretching the cord with) the middle mark. In the same manner, the north-western and the north-eastern corners (are obtained).

8. Now another method of construction (of a square). Poles are fixed at both ends and in the middle of the east-west line. A cord measuring half of the east-west line is taken and increased by its visesa (the difference between its length and the diagonal of

the square produced by it). After giving a mark at this point, the cord is (further) increased by half of the east-west line. Ties are made at both ends of the cord. Fixing the tie at the saviseya end at the middle pole and the other tie at the eastern pole, the cord is stretched by the mark so as to obtain the south-eastern pole, the cord is stretched by the mark so as to obtain the south-eastern corner. By removing the tie from the eastern pole and fixing it to the western pole, the south-western corner is likewise obtained by (stretching the cord with) the mark. In the same way, the north-western and the north-western corners (are obtained).

9. The breadth (of a rectangle) being the side of a given square (pramana) and the length the side of a square twice as large (dvikarani), the diagonal equals the side of a square thrice as large (trkarani).

(divkarani and trikarna are respectively square root of two and three)

10. Thereby is explained the side of a square one-third the area of a given square (trtiyakarani). It is the side of a square one-ninth the area of the square (explained in the preceding rule, that is, of the square on the trkarani).

11. The combination of two equal squares has been described. The combination of two squares of unequal measures (sides) (now) follows. A (rectangular) part is cut off from the larger (square) with the side of the smaller; the diagonal of the cut-off (rectangular) part (produces the square which) combines both the squares. This has been stated. (here the combining two and or more geometrical structures are also explained)

12. If it is desired to remove a square from another, a (rectangular) part is cut off from the larger (square) with the side of the smaller one to be removed; the (longer) side of the cut-off (rectangular) part is placed across so as to touch the opposite side;

by this contact (the side) is cut off. With the cut-off (part) the difference (of the two squares) is obtained.

13. That (the longer side of the cut-off rectangle in the above rule) which is placed across is the diagonal equal to the side of a square four times as large (as the given square). The area (of the squares) produced separately by the cut-off side and the other (the breadth of the rectangle) together equal the the area (of the square) produced by this diagonal. If the breadth produces one square purusa, the other side produces three square purusas. This has been stated/

(The unit of measurement purusha is equal to the average height of a man. This according to Aryabhatta I is equal to 96 times the average diameter of the finger or four cubits. Eight thousand times the purpusha unit is one yojana which according to INSA publication is 12.11 kms)

14. If it is desired to transform a rectangle into a square, a (square) part is cut off (from the rectangle) by the breadth. The remainder (of the rectangle) is divided (into two equal parts) and placed on two sides. the empty space (in the corner) is filled up with a (square) piece. The removal of it (of the square piece from the square thus formed to get the required square) has been stated.

(Like combining two or more geometrical figures, transforming one figure into an other is also described systematically in all the sulbasutras)

15. If it is desired to transform a square into a rectangle, the side is made as long as desired; (after diagonal intersection), what remains as excess portion is to be placed where it fits. (Like Bsl.2.4, the rule is defective and does not lead to proper geometrical operation.

16. If it is desired to transform a square into a circle, a cord is stretched from the center (of the square) upto its corner (so as to

measure out a length equal to half the diagonal). It is (then) stretched (from the center) towards the (eastern) side. with one-third of the excess part (lying outside the eastern side) added (to the portion of the cord between the center and the side), the (required) circle is drawn. This is the (approximate) circle, for (almost) as much is added as is cut off (from the corners of the square).

(Transforming a circle into a square using geometrical principle is given above. This is well in agreement with the modern concepts also. Similarly given below is transforming a circle into a square using the diagonal of the circle)

17. To transform a circle into a square, the diameter is divided into fifteen parts and two of them are removed, leaving thirteen part. This gives the approximate (side of the) square (desired).

18. The (square) measure is to be done by means of the (linear) measure.

19. A square (of unit area) is to be understood in the absence of anything to the contrary.

20. (A cord of length) twice the measure produces four (square measures); thrice the measure nine (square measures).

21. The number of units of measure in a cord is to be squared (to get the area of the square in that measure). (Alternatively, as many units of measure there are in a cord so many rows of squares on each side will be in a square of side equal to the measuring cord.) This is the meaning.

22. A cord 1 ½ purusa long makes 2¼ (square purusas); a cord of 2½ purusas makes 6¼ (square purusas).

23. Now follows the method (of finding the area of a square) when the side is increased. With the side (of the given square) and the length by which the side is increased is drawn (a rectangular area) which is placed on either side (of the square). A square is formed with the length by which the side is increased and placed in

the corner (to produce the enlarged square whose area is the sum of the given square, the two rectangles and the corner square piece).

24. With half the side of a square, a square one-fourth in area is produced, because four such squares to complete the area (of the original square) are produced with twice the half side. with one-third the side of a square is produced its ninth part.

(After giving geometrical explanations for the basic principles of various figures, Apasthamba starts narrating the Fire altars and their constructions)

25. The distance between the garhapatya and the ahavaniya in the arrangement for the laying of sacrificial fires is known from the tradition. The Brahmana has to place it (the ahavaniya) (at a distance of) 8 prakramas, the price 11 prakramas and the merchant 12 prakramas (from the garhapatya towards east).

(In the Yagasaala, there are a number of fire altars, where the sacrifices are made for different vedic devathas. The size, shape and distance of these fire altars are followed by traditions but with specific measurements. Thus mentions the Maharshi. Garhapathyagni is the name of a fire altar and then ahavaneeyagni. In the above statement prakrama is another unit of measurement which is commonly used in the altar construction)

26. For general use and not for any particular class, this distance is indefinite, (above) 24 prakramas to be ascertained by eye estimation and should not deviate from it much.

(Here it is mentioned that an aproximate mesurement by eye estimation is enough in the above case)

27. According to tradition, the (place of the) daksinagni is near the south-east corner of the third part of the distance of the garhapatya (from the ahavaniya).

(Menioned above is the position of the dakshinagni in the yagasaala)

28. The distance between the garhapatya and the ahavaniya is divided into five or six (equal) parts, a sixth or a seventh part is added, the whole (of the cord measuring the original distance plus the added part) is divided into three (equal) parts, and a mark is given at the end of the third part from the western end. (With two ties) fastened to (poles at) the two ends of (the distance between the garhapatya and the ahavaniya, the cord is stretched to the south by the mark and a pole fixed (at the point reached by the mark). This is the place of the daksinagni. This is according to sruti. (According to sruthi means Vedas)

29. The east-west line (praci) has the measure of the sacrificer (96 angulas) or of indefinite measure like that of clarified butter in relation to fire. Such is the case with the breadth. The two amsas (shoulders) of the fire-altar) is shorter on the eastern side, broader on the western side and curved in the middle. It is like a wooden doll. Such is the tradition of the darsikya fire-altar.

(The measure purusha, angula etc are taken directly from the height of the yajamana - master - who performs the sacrifice)

30. To the west of the ahavaniya is constructed the four-sided elongated figure of which the length has the measure of the sacrificer (96 angulas). A cord equal to this measure is increased by itself and a mark given at the middle. With the two ties (of the cord) fastened to the (poles at the) south-western and south-eastern corners, it is stretched towards the south by the mark and a pole fixed (at the spot reached by the mark). Fixing both ends of the cord at this pole, an are of a circle is drawn from the south-western to the south-eastern corner (with the middle mark of the cord). the same is done on the northern side (of the fire-altar). The western and the eastern sides are to be similarly circumscribed by means of a cord double the (respective) side.

31. According to tradition, the saumikya vedi measures 30 padas or prakramas on its western side, 36 (padas or prakaramas) along

the east-west line and 24 (padas or prakramas) on its eastern side.

(Saumukhya vedi is another fire altar connected in yaaga saala. Paada is a measurement approximately equal to the length of the foot, we now call as one foot - approximately 12 inches)

32. To a cord of 36 (padas or prakramas) another piece of 18 (padas or prakramas) is added and a mark is given at a distance of 12 and another mark at a distance of 15 from the western end (of the cord which is added.). With ties at both ends (of the cord) fastened to (poles fixed at) two ends of the east-west line, the cord is stretched towards the south by the mark at 15 and a pole fixed (at the point reached by the mark). The same is done towards the north. These (two points thus obtained) are the two western corners (sronis) (of the altar). After interchanging the ties at two ends, the cord is stretched (towards the south) by the mark at 15 and a pole is fixed at the mark at 12. The same is done towards the north. These are the two eastern corners (amsas) (of the altar). This is the method of construction with one cord.

(Given below are a series of measurements for the right angle triangle where, diagonal/hypotenuse having whole number measurement is obtained i.e without fraction . If 3 is the base and 4 is the height of the right angle triangle the hypotenuse will be 5)

33. The diagonal of a rectangle of sides 3 and 4 is 5. With these (sides) increased by three times themselves, the two eastern corners (of the altar), and with these (sides) increased by four times themselves, the two western corners (are determined).

34. The diagonal of a rectangle of sides 12 and 5 is 13. With these (sides), the two eastern corners (of the altar) and with these (sides) increased by twice themselves, the two western corners (are determined).

35. The diagonal of a rectangle of sides 15 and 8 is 17. With these (sides), the two western corners (of the altar) (are determined).

The diagonal of a rectangle of sides 12 and 35 is 37; with these (are fixed) the two eastern corners.

36. The knowledge of these (squared numbers) makes possible the construction of figures of the sacrificial altars.

(Mahavedi is the most important part of the yagasaala. Its measurement is given below and also the position)

37. The (area of the) mahavedi is 1000 minus 28 (square) padas. From the south-east corner (a perpendicular) is dropped (on the western side) at a point 12 padas towards the south-western corner (from the east-west line). The (triangular) portion cut-off is placed invertedly on the other side. That makes a rectangle. By this addition (the area) is enumerated.

38. According to tradition, the sautramaniki sacrificial altar is one-third of the saumikya vedi. (To find its dimensions), 1/3 of a prakrama is to be substituted for prakrama (in the values given for the saumikya). Alternatively, the transverse sides will be 3 times 8 and 10 and the east-west line (prsthya) 3 times 12. The (area of the) sautramaniki sacrificial altar is 324 (square) padas.

(Sautramani vedi is the altar for performing the yaga ritual part known as sautramani)

39. According to tradition, the (area of the) altar for the asvamedha sacrifice is double (the area of the saumiki vedi). (Here) 2 of a prakrama takes the place of a prakrama.

40. One prakrama equals 2 padas or 3 padas; on account of uncertainty in the meaning of the term (prakrama) one may take such value of prakrama as one may wish. The measure (of pada) may be that of the sacrificer or of the adhvaryu, because one directs the effects of the other.

(Given above is an explanation for the priest, to follow the same unit measurement, if there is any confusion in the correct dimension of the measurment)

41. According to tradition, the altar for the conventional

animal sacrifice (nirudhapasubandha vedi) has the measures of a chariot. There it is said that the western side (of the altar) measures 1 aksa (104 angulas), the east-west line 1 isa (188 angulas) and the eastern side 1 yuga (86 angulas) or the distance between the two outside holes.

(The above given is the explanation and name of the altar known as Ratha chakra chiti - means the altar having the shape of the chariot wheel - constructed for performing the pasubandha yaaga)

42. This (is to be constructed) by the methods of one cord already mentioned. Having stretched the cord by the mark at fifteen, the western corners are to be fixed by ½ aksa (52 angulas) and the eastern corners by ½ yuga (43 angulas).

(more measurement units are being added when the explanation progresses. Given here is another measurement aksha and yuga and their equivalent in angula is given. One angula is average diameter of the male finger which may be approximately 1.6 cms. In some cases for the same measurement the dimension appears to vary)

43. Now, these (units of chariot measure) are explained. I aksa equals 188 angulas, I aksa 104 angulas and 1 yuga 86 angulas. These are according to the (Vedic) Carana school and are known as chariot measures.

44. The western side is 4 aratnis or other measures, the east-west line 6 and the eastern side 3. This (is to be constructed) by the method of one cord already mentioned. Having stretched the cord by the mark at fifteen, the western and the eastern corners are to be fixed by 2 and 1 ½ (aratnis) (respectively).

45. According to tradition, the paitrki vedi is a square, and has the measure of a sacrificer. The (is to be constructed) by the method of one cord already mentioned. Having stretched the cord by the

mark at fifteen, the western and the eastern corners are fixed by half the measure.

(Measurement of the sacrificer means, the height of the master of yaaga)

46. According to the tradition of the soma sacrifice, the (side of the) uttara vedi measures 10 padas. This (is to be constructed) by the method of one cord already mentioned. Having stretched the cord by the mark at fifteen, the western and the eastern corners are to be fixed by half the measure.

47. These are measured by the yuga, pada or saya measures of the sacrificer.

48. One may take such value of pada, yuga, aratni and samya as one may wish when these (words) are used as units of measure, on account of uncertainty in the meaning of these terms.

("On account of the uncertainty of the measure" this statement is given because the dimension of the above units vary because it is connected with the master of the sacrifice. The height of the master vary, hence the dimension of purusha unit vary....)

49. In the measurement, the two sides should lie along the east according to tradition

50. According to tradition, the sadas (shed) is 9 aratnis wide and 27 aratnis long in the south-north diretion; according to some, its length is 18 aratnis. This (is to be constructed) by the method of one cord already mentioned. Having stretched the cord by the mark at 15, the western and the eastern corners are to be fixed by 4 ½ (aratnis).

51. According to tradition, the uparavas are each I pradesa along, separated from one another by 1 pradesa. A square of side equal to 1 aratni is made, poles fixed at the (four) corners, and a circle of radius equal to half pradesa is drawn (with each pole at the

corner as center) as per tradition.

52. According to tradition, the garhapatya fire has the measure of 1 vyayama. It is a square by one tradition and a circle by another.

(Pradesa and vyayaama are two measurements used in the explanations. Till now the explanations were focussed on the measurement of the yaaga saala. Now the explanations of bricks for making the fire altars are given)

The bricks (to be used for the garhapatya fire) is to measure 1/3 vyayama (32 angulas) long by 1/7 vyayama (13 angulas 24 tilas) wide. There are 21 such bricks (required for each layer). In the first layer, the length (of the brick) is tu;rned towards east, and in the second layer towards north.

53. For the circular (garhapatya fire), a circular mound of earth is made and a pole fixed at the middle. (With this pole as center) a circle is drawn with (a radius equal to) ½ vyayama plus the extra (as per rule 3.2 for transforming a square into a circle). Within it (the circle) a square of the maximum size possible is drawn and divided into 9 parts (squares); each segment of the circle (between the circumference and the square) is to be divided into 3 parts.

54. In the placement (of bricks), the corners of square (in the first layer) point towards intermediate directions; in the other layer, these corners lie at the centers (of the segments of the first layer). (With these two layers) alternating with each other, as many layers as desired are to be constructed.

55. The dhisnya fire, according to tradition, has the measure of the wooden vessel (pisilamatra); it is a square by one tradition and a circle by another.

56. Having made a circle mound of earth, the agnidhriya fire is divided into 9 parts and a stone is to be placed. The other (dhisnya fire) is divided into as many parts as prescribed and covered with

bricks as they fit.

57. The tradition has it that he who constructs the fire-altar is certain to be (rich). It is constructed in the likeness of the birds, that is, after their shape, in pursuance of express direction (in the matter).

58. With the help of a bamboo rod of length (equal to a purusa) as mentioned, 4 (square) purusas are measured out for the body (of the fire-altar) and 1 (square) purusa is measured out for each of the southern wing, the northern wing and the tail. The southern wing is lengthened towards south by 1 aratni and likewise the northern wing towards north. The tail is lengthened towards west by 1 pradesa or 1 vitasti.

59. The first agni is one-fold (1 ½ sq, purusa); the second two-fold (2 ½ sq. purusa); the third three-fold (3 ½ sq. purusa) and so on; in this way one continues upto hundred-and-one-fold agni (101 ½ sq. purusa).

(A very important point is mentioned in the above stanza. In Ekaaha yaaga. Ie. the yaaga performed in one day the altar should have the area 1½ sq, purusa, if it is performed in two days, it is known as dwayaaha the area of altar should be (2½ sq, purusa... like this for sataaha (100 days yaaga) the measurement should be 101½ sq, purusa.

60. But indeed the seven-fold (agni) only is to be contsructed (first); (for) the seven-fold is the proper fire-altar. Thereafter, higher altars (are obtained) by increasing the area by one (sq.purusa) successively; this is the tradition.

61. The one-fold and the following (fire-altars up to the six-fold) do not have wings and tails, but the seven-fold does (have them) according to the injunctions of the Sruti.(vedas)

62. In the case of eight-fold and higher order fire altars, their differences in area from the seven-fold should be divided in seven and half equal parts and each part added to each purusa (of the

original seven-fold altar). This is because the deformation (of the fire-altar) is disallowed in the Sruti.

63. According to tradition, (the term) to be measured with a purusa means 'to be measured with a bamboo rod'.

64. Two holes are made (at the ends of) a bamboo rod at a distance equal to the height of the sacrificer with uplifted hands, and a third hole is made at the middle. Having placed the bamboo rod along the east-west line, poles are fixed in the holes from the western extremity) with the other end (from the west) towards south-east. The pole is then removed from (the hold in the eastern extremity and fixed at the western extremity, and a circle is drawn (about the pole in the western extremity) with the other end (from the east) towards south-west. The bamboo rod is taken off and one end of it is fixed to the middle (of the east-west line) with a pole; it is then placed towards the south so as to pass over the point of intesections of the two circles and a pole is fixed in the hole at the other extremity. The bamboo rod is fixed to this (last) pole by its middle hole and laid (east-west) touching the outer edges of the two circles; two poles are fixed through the two extreme holes. This is a square of (side equal to) one purusa.

65. Going about in this way, four squares each of one (sq.) purusa in the body (atman) are measured out. One (sq) purusa (is then measured out) for each of the southern wing, the tail and the northern wing. As stated, the south wing is to be increased towards south by one aratni and so on.

66. A bamboo rod equal to the diagonal of a square of side one purusa is placed across from (the western end of) the east-west line and another (rod of one purusa) is placed on the east side (from the eastern end). By them (that is, by their meeting point) the south-east corner is fixed. By reversing (the placement of the two rods), the south-west corner is fixed. Proceeding as before, the north-east corner is fixed.

67. As in the case of the uttara vedi, it is measured out with the help of a cord or a bamboo rod.

68. When the fire-altar having wings and tail is increased to higher folds or reduced, the saptamakarani of the fold (vidha) is to be substituted by the purusa and the area (of the fire-altar) drawn.

69. Of the bricks, the side (of the first type) should measure one-fifth of a purursa; the second type has one of its sides longer by half; the third type is one-fifth of a purusa long and one pradesa broad; bricks with each side equal to one pradesa form the fourth type; square bricks of side equal to one-fifteenth (if a purusa) constitute the fifth type.

70. The height of the brick is to be made a fifth of the janu and that of the nakasat and pancacosa half of that measure.

71. What is lost by burning (and drying) is to be made good by loose earth because of the flexibility of its quality.

72. In the placement (of bricks), 10 bricks longer by half (that is, 36 angula X 24 angula) and turned towards west are placed on the east side of the body (atman); 10 (of them) turned towards east on the west side (of the body); (of the body); 5 (of them) at each end of the two wings; 5 (of them) at both junctions of the wings (with the body) such that half of the bricks (that is, the added half 12 angula of the adhyardha) lie in the wings; and 5 bricks turned towards north and south on both sides of the tail.

73. After placing bricks of side equal to 1 pradesa in the tail, all the (remaining) space of the fire-altar is to be covered with bricks of side equal to one-fifth (of a purusa).

74. The number (of 200bricks) is to be completed with bricks of side equal to one-fifteenth (of a purusa).

75. In the other layer, 10 bricks longer by half and turned towards north are placed on the south side of the body and 10 (of

them) turned towards south on the north side (of the body). (The placement of bricks) in the tail will be the same as in the wings for the first layer and that in the wings the same as in the tail (for the first layer). In the junction (between the tail and the body), (the placement of bricks should be) in the reverse order.

76. The whole (of the remaining) space of the fire-altar is to be covered with bricks of side equal to one-fifth (of a purusa).

77. The number (of 200 bricks) is to be completed with bricks of side equal to one-fifteenth (of a purusa). (With the two layers) alternating with each other as many layers as desired are to be constructed.

78. There are five layers; these are covered with five (layers of) loose earth, ending up with a layer of earth; (this is done) for various purposes (served by) loose earth.

79. The construction of the fire-altar for the first time should be it 1000 bricks upto (the height of) the knee; for the second time with 2000 bricks upto (the height of) the navel; for the third time with 3000 bricks upto (a height of) the mouth; and so on upto higher and height. Those who desire heaven should construct by increasing the height measure with innumerable bricks; this is tradition.

80. In the case of (fire-altars employing) 2000 bricks, the piles will be two layered; in the case of 3000 bricks, three layered; in the case of 4000 and larger number of bricks, the number of bricks (for the layer) remains the same (as that for the 3000).

81. According to tradition, a smaller fire-altar should not be laid after a larger one has been constructed.

82. According to tradition, the fire-altar is to be constructed with four sided (bricks); in the absence of anything mentioned in particular, a square is to be understood.

83. (The bricks should be) of the measure of pada, aratni,

urvasthi and anuka; this is the tradition.

(Given above are different measurement of the bricks used for various yaagas. Their relations are given below)

84. Anuka is one-fourth (of a purusa), aratni one-fifth (of a purusa), and so is urvasthi (one -sixth of a purusa).

85. The quarter bricks have the measure of a pada; there one is free to choose owing to the wide range of the meaning of the word (pada).

86. In the placement (of bricks), 8 bricks of size quarter of the one-fourth (that is, 15 x 15 sq. angula.) are to be placed at each end of the two wings and B similar bricks at the (two) junctures (between the wing and the body) such that 6 angulas (of the bricks) lie within the body, 8 bricks (of the same type) are placed on the western corners (of the body, 4 on each, lined) towards cast and 8 bricks on the eastern corner towards west.

87. In the space (of the body) between the two junctures (with the wings), bricks of six one-fifth (of a purusa) and their quarters (are placed).

88. After placing bricks of size equal to 1 pradesa in the tail, the whole of the (remaining space of the) fire-altar is to be covered with one-fourth bricks.

89. The number (of 200 bricks) is to be completed with quarter bricks

90. In the other layer, one-fifth bricks are placed in the middle of the juncture of the tail (with the body), 14 bricks of six quarter of them (of one-fifth, that is, 12 x 12 sq. ang.) are placed around in the body as they fit.

91. The whole of the (remaining) fire-altar is to be covered with one-fifth bricks.

92. The number (of 200 bricks) is to be completed with

quarter bricks. (With the two layers) alternating with each other as many layers as desired are to be constructed.

93. For one-fold etc. (upto the six-fold five-altar), square bricks of side equal to one-twelfth and one-thirteeneth of the side (of the fire-altar) are to be made as also their quarters. (With the two layers) alternating with each other as many layers as desired are to be consutructed.

94. From one-fold etc., (upto the six-fold), bricks are used in the first, second and third construction; in all cases and also for higher consutrction, their number is according to the prescription of the Sruti (that is, 1000 bricks for all constructions).

95. The kamya (fire-alters) are (endowed with) different merits and (are prescribed for the fulfilment of specical desires) according to the science of merits (gunasastra).

(There are two types of yaagas performed. One is kaamyeshti, the yaagas performed for wealth and physical benefits and blessings of the gods. The other yaagas known as moksheshti are performed for attaining the immortality and getting heaven - swarga prapti)

96. Those who have many foes should construct a (fire-altar in the form of an isosceles) triangle: this is the tradition.

(Here many foes means, the king who has many enemies wishes to conquer them should perform a yaaga for victory)

97. A square twice as large as the area of the (seven-fold) fire-altar with (two) aratnis and (one) pradesa is laid; the mid-point of the eastern side (of the square) is joined to the two western corners (of the square, and the area lying outside these lines is cut off); this is the exact triangle (equal in area to the seven-fold fire-altars of 7 ½ sq. purusa).

98. Bricks are to be made as in the case of one-fold etc.

fire-altars (that is, of side equal to one-twelfth and one-thirteen of side of the altar); these should have the shape of an isosceles triangle.

99. According to tradition, those who wish to destroy existing and future enemies should construct a fire-altar in the form of a rhombus (made of two isosceles triangles, ubhayata prauga).

(The size and shape of the fire altar varies in yaagas performed for different purposes, that is the tradition)

100. This (rhombus) looks like two inverted (fore parts) of a cart (joined together). As in the case (of the isosceles triangle), a rectangle (twice the area of 7 ½ s1. Purusa) is constructed and the mid – points of the eastern and western sides are joined to the mid-points of the southern and northern sides (of the rectangle, and the area lying outside these lines is cut off); this is the exact rhombus. (Bricks for this fire-altars are to be made in the same manner) as described in the case of the isosceles fire-altars.

101. According to tradition, a fire-altar in the form of a chariot wheel is to be constructed (when it is desired) to destroy enemies.

102. A circle of area equal to that of the (seven-fold) fire-altar with (two) aratnis and (one) pradesa is made and the largest possible square is inscribed in it.

103. Bricks (for the construction of the chariot wheel fire-altar) are made with the twelfth part of the side (of the inscribed square).

104. Of these (bricks) are placed in each circular segment and the remaining space (of the segment) is divided into 8 parts.

105. In the placement (of the first layer), the corners of the square should lie in the intermediate directions and in the other layer in the centres of the segments (of the first layer). (With these two layers) alternating with each other, as many layers as desired are to be constructed.

106. According to tradition, those who desired food should construct a fire-altar in the form of a trough.

107. The trough are indeed of two types, e.g. the square-shaped and the circular.

108. One can construct the fire-altar of any one of these (two) types as one may wish.

109. Rather from considerations of quality a square (dronacit) should be constructed.

(dronachit means the fire altar having the shape of a trough)

110. According to tradition, the handle (of the trough) should lie on (its) western side.

111. The area of the handle is one-tenth of the total area (of the fire-altar). This being placed in the form of the tail (separate from the body), the area (of the square body) is found by the difference (of two squares) as already stated.

(From hereafter, the description on the bricks is given, their shape , size and other parameters are given)

112. The (square) bricks are to be made with the twelfth part of the side (of the square body). Bricks longer by half (adhyardha) and quarter bricks are also made.

113. In the placement (of the bricks in the first layer), to bricks longer by half are arranged on the eastern side of the body, turned towards west, at the (west) end of the handle and at the two western corners (of the body).

(the construction of the fire altar by arranging the bricks is given below. The details on how the gaps are to be filled by partial bricks or sand etc is given. According to tradition. In the fire altar construction, nothing should work on the so called 'common sense' instead, the procedure folowed is according to tradition.)

114. The remaining space of the fire-altar is covered with square bricks.

115. The number (of 200 bricks) is to be completed with quarter bricks.

116. In the other layer, the bricks longer by half are placed along the southern side of the body, turned towards north and along the northern side, turned towards south; the same is done along the southern and the northern side of the handle.

117. The remaining space of the fire-altar is covered with square bricks.

118. The number (of 200 bricks) is to be completed with quarter bricks. (With these two layers) alternating with each other, as many layers as desired are to be constructed.

119. Those who desire beasts should construct the samuhya, according to tradition.

(in the Vedic hymns, prayer for wealth and comfort are very common. Those who are desirous of these wealth, have to perform specific yagas. That is mentioned above)

120. Bricks are to be placed all around the samuha (fire-altar).

121. The catvala pits (in the ground) are to be placed in every direction and levelled with clay with water (purisa); this is the tradition.

122. Those who desire villages should construct the paricayya (fire-altar); this is the tradition.

123. The paricaya is that (fire-altar) in which bricks are placed around the central svayamatrnna (bricks).

124. According to tradition, the upacayya is to be constructed by those who desire villages. It is prepared in a manner opposite to that of the paricayya (that is, the construction proceeds from outside to the center).

125. Those who desire prosperity in the abode of the Fathers should construct the fire-altar in the form of a pyre (smasanacit); this is the tradition.

126. The pyres are indeed of two types, e.g. the square-shaped and the circular.

127. One can construct the fire-altar of any one of these (two) types as one may wish.

128. Rather from considerations of quality a square (smasanacit) should be constructed. In the square type, it should be like the trough without the handle, as already stated.

129. According to tradition, those who desire beasts should construct the fire-altar with the meters (in place of bricks).

130. According to one opinion, the entire (sacrificial ceremony) should be performed by means of meters, according to another, by the usual sacrificial fires.

131. Those who desire heaven should construct a fire-altar in the shape of a falcon; this is the tradition.

132. This (fire-altar) has curved wings and extended tail. The west side (of first half of the wing) is pushed upwards towards east and the east side (of the wing from the middle to the end) is pushed downwards towards west. In this way the wings of birds are said to be bent at their middle (part).

133. The (area of the) fire-altar is to be made seven-fold with) two) aratnis and (one) pradesa. (Of the rectilinear syenacit), the pradesa (portion of the tail) and the fourth part of the body (atman) together with 8 caturbhagtyas (also from the body) (are to be taken out). Out of these (areas), three (caturbhagiyas)form the head (of the falcon) and the remaining (area) is to be distributed between the two wings.

134. 5 aratnis make 1 purusa, 4 aratnis 1 vyayama, 24 angulas

1 aratni and half (of 24 angulas, that is, 12 angulas) 1 pradesa. That is the definition.

135. The length of the wing is 9 ½ aratnis and ¾ angulas.

135. A tie is made at either end of a cord 2 purusas long and a mark given at the middle (of the cord). Having fastened the ties at the two western corners of the (southern) wing, the cord is stretched towards east by the mark; the same is done on the eastern side (of the wing). This makes the bending (of the wing). Thereby is explained (the bending of) the northern wing.

136. The body is 2 purusas long and 1 ½ purusas broad.

137. At the place of the tail, a rectangle ½ purusas broad and 1 purusa long towards west is constructed; a similar rectangle is constructed on its southern and northern side. These (latter, i.e. the southern and the northern) two (rectangles) are diagonally cut off such that the length (of the tail) at its juncture (with the body) is ½ purusa.

138. At the place of the head, a square of side ½ purusa is drawn; the mid-point of its eastern side is joined to the mid-poiints of the southern and the northern sides (and the parts lying outside these lines are cut off).

139. The western and the eastern corners (of the body) are cut off (by lines) in the direction of the junctures (of the body with the tail and the head). This is the (form) of the falcon.

140. Bricks are made with length equal to one-fifth purusa (24 ang) and breadth one-sixth purusa (20 ang.), the two sides being inclined (with each other) in such a way that these fit (with the shape of the wing). This is the first type.

141. Two of these (first type) bricks are joined along the east line (that is, the length). This is the second type.

142. That side of the first type, which is one-sixth purusa long is

extended by one-eighth of a purusa (15 ang), and (the extended part) is bent so as to fit (with the shpae of the fire-altar). This is the third type.

143. A (square) brick of side one-fourth of a purusa (30 ang) is lengthened by half; the (square) portion of side one-fourth purusa is diagonally cut off. This is the fourth type.

144. The fifth type of bricks is half of the (square) brick of side one-fourth of a purusa.

144. By dividing it (the fifth type) by the diagonal, the sixth type (is obtained).

145. A rectangle of breadth one-tenth of a purusa (12 ang) and length one-fifth of a purusa (24 ang) in the direction from east to west is drawn. One each such rectangle is placed on its southern and its northern side. These two (latter, that is, the southern and the northern, rectangles) are cut off by diagonals passing through their south-west corners. This is the seventh type.

146. Like-wise another type is formed in which the northern rectangle is cut off by the diagonal passing through the north (western) corner (the cutting off of the southern rectangle being as before). This is the eighth type.

147. The ninth type is obtained by dividing by both diagonals a (square) brick of side one-fourth of a purusa.

148. In the placement (of bricks in the first layer) 60 bricks of the first type turned towards north, are placed in each wing.

149. Along each side of the tail, 8 bricks of the sixth type are placed (in this way); three of them at the end (of the tail) and one above them and again three and one (above them).

150. At the juncture of the tail (with the body), 2 bricks of the fourth type partly covering both (the tail and the body) are placed. West of them (are placed) 2 bricks of the fifth class touching each other edge to edge.

151. The remaining space (of the tail) is covered by 10 bricks of the fourth type. 8 bricks of this type, turned towards east and west, are placed in the four corners (of the body).

152. The remaining space (of the body) is covered by 26 bricks of the fourth, 8 of the sixth and 4 of the fifth type.

153. In the head 2 bricks of the fourth type partly covering both (the head and the body) are placed and 2 of the same type, turned towards east, above them.

(The most important part of the above explanation is that the fire altar is made out of 200 bricks. The area of the bricks is clearly mentioned, their shapes are also mentioned. The total area of the fire altar and each part of the altar has been clearly explained. There is no provision for using broken bricks, nor provision for breaking the bricks. Hence the geometrical knowledge imparting starts from the designing of the bricks onwards and also the size reduction during burning (baking the bricks) are to be accounted).

154. Thus is formed the (first) layer of 200 bricks.

155. In the other layer, 5 bricks of the second type are to be placed at each of the two bendings (of the two wings). At either juncture (of the wing with the body) (5) bricks of the third type are placed in such a way that the portion of each brick extended by one-eighth purusa lies within the body. The remaining space (of each wing) is covered by 45 bricks of the first type, turned towards east.

(The above fire altar is either having the shape of a falcon or that of kite. Hence their body, head, tail, wings etc are made using specific type and shape of bricks . the shape has specific relation with geometrical figure.)

156. 5 bricks of the seventh type are placed along each of the two sides of the tail. Next to such brick in the second row on one

side and in the fourth row on the other side, one brick each of the seventh type is to be placed. The remaining space (of the tail) is to be covered by 13 bricks of the eighth type.

157. 8 bricks of the fourth type, turned towards south and north, are placed in the western and the eastern corners (of the body). The remaining space (of the body) is covered by 20 bricks of the fourth type, 30 bricks of the sixth and 1 bricks of the fifth type.

158. In the head are placed 2 bricks of the fourth type and east of them 4 bricks of the ninth type.

159. Thus is formed the (second) layer of 200 bricks.

160. (With these two layers) alternating with each other as many layers as desired are to be constructed.

161. Those who desire heaven should construct a fire-altar in the shape of a falcon; this is the tradition.

162. This (fire-altar) has curved wings and extended tail. The west isde (of first half of the wing) is pushed upwards towards east and the east side (of the wing from the middle to the end) is pushed downwards towards west. In this way the wings birds are said to be bent at their middle (part).

163. 120 (square) bricks each 1/16 of a (square) purusa (said, ¼ pu.or 30 ang) give the area of the seven-fold (fire-altar of 7 ½ sq. pu) with (two) aratnis and (one) pradess. Of them, 40 (can be accomodated) in the body (atman), 3 in the head, 15 in the tail, 31 in the southern wing and the same (number) in the northern (wing).

164. A rectangle, 1 ½ purusa broad and 2 purusas long, is constructed. (an area equal to) 2 bricks of 1/16 (square purusa) is discarded from each of the two western and the eastern corners, leaving (an area equaivalent of) 40 (sodasi) bricks. This is the body.

165. At the place of the head, a square of side ½ purusa is drawn; the mid-point of its eastern side is joined to the mid-points of the

southern and the northern sides (and the parts lying outside these lines are cut off). (An area equaivalent of) 3 (sodasi) bricks is left. This is the head.

166. A rectangle of breadth 1 purusa and length 2 purusas, further extended by an area of 1/16 square purusa makes the southern wing. Likewise (is made) the northern wing.

167. At the end of (each) wing, 4 squares of side equal to ¼ of a purusa are made, diagonally divided, and their halves discarded. An area (equaivalent of) 31 (sodasi) bricks is left.

168. In the middle of the wing less the end portion (that is ¼ purusa or 30 ang with which the 4 squares were made), an east-west line is drawn. From (the western point of) the juncture of the wing (with the body) a cord of lemgth 1 purusa is stretched and a point at the end of 1 purusa is given (where the end of the cord meets the east-west line). At a distance of 1 purusa from this point towards east another point is given. From these two poits lines are to be drawn to the different end points (of the wing at the junction with the body and at the end less ¼ purusa where the 4 squares were made). This is the curving of the (southern) wing. Thus is explained (the curving of) the northern wing.

169. (An area bounded by a length of) 2 purusas on the western side, ½ purusa on the eastern side, 18 (times ¼ purusa or 30 ang) on each of the two (southern and northern) sides and having a height of ¾ purusa can accommodate 15 (sodasi) bricks. This is the tail.

170. The one-sixteenth (sodasi) brick is to be bounded by four sides (whose measures are) : 1/8 purusa 3/8 purusa, 1/4 purusa and 2/4 purusa.

171. A half brick is bounded by three sides, two sides by ¼ purusa each and the other by 2/4 purusa.

172. A quarter brick is bounded by three sides, - one side by ¼

purusa and the other two by 2/8 purusa each.

173. A brick for use in the wing (paksestaka) is bounded by four sides,-two sides by ¼ purusa each and the other two by 1/7 purusa each.

174. A brick for use in the middle of the wing (paksamadhyiya) is bounded by four sides, - two sides by ¼ purusa each and the othe two by 2/7 purusa each.

175. A bricks for use at the end of the wing (paksagriya) is bounded by three sides, - one side by ¼ purusa, one side by (1/4+1/7) purusa, and the remaining side by (2/4+1/7).

176. For making the brick for use in the wing (paksestaka) a rectangle of breadth 1/7 purusa and length ¼ purusa is made and then lengthened by a diagonal (so that the other diagonal is shortened and the figure assumes the form of a parallelogram). The slabs are bent by the seventh of the distance between the root (apyaya) and the bending point of the wing (paksanamam).

177. In the placement (of bricks), 4 quarter bricks are placed in the east of the head, 5 on the western side of the juncture of the head (with the body), 11 on the eastern side of the (eastern) juncture of the wings (with the body), 11 on the western side of the (western) juncture of the wings (with the body), 5 on the eastern side of the juncture of the tail (with the body) and 5 on the west of it, and 15 at the end of the tail.

178. 4 bricks for use at the end of the wing (paksagriya) are each placed at the end of the two wings and 4 at the juncture of the wing (with the body) each lying partly in both (the wing and body).

179. Around these latter (paksagriya bricks partly lying) in the body, 4 one-sixteenth bricks are placed on either side as these fit.

180. 4 bricks for use in the middle of the wing (paksamadhyiya) are each placed in the middle of the two wings. The two wings are

(then) to be covered by bricks for use in the wings (paksestaka), (the longe sides of the bricks being) turned towards east.

181. The remaining space (of the fire-altar) is to be covered with one-sixteenth bricks; at the (inclined) edges (of the fire-altar) the diagonal sides (of these bricks) are to face outwards; elsewhere (their placement should be) as in the head.

182. In the other layer, 2 one-sixteenth bricks with their diagonal sides facing outwards are to be placed in the east of the head; west of them two of these with their diagonal sides facing inwards (are to be placed) partly covering the head and the body.

183. 2 half bricks are to be placed as these fit, and these are to be enclosed by 2 half bricks with their diagonal sides outwards.

184. One-sixteenth bricks with their diagonal sides facing outwards are to be placed where the sides of the body meet (that is, at the western and eastern corners).

185. 4 half bricks (are to be placed) at each end of the two wings. Two wings are (then) to be covered with bricks for use in the wings (paksestaka), (their longer sides) turned towards north.

186. 3 half bricks (are to be placed) at either side of the tail.

187. The remaining space (of the fire-altar) is to be covered with one-sixteenth bricks; at the (inclined) edges (of the fire-altar) the diagonal sides are to face outwards, elsewhere (their placement should be) as in the tail.

188. If square or triangular areas arise (due to the removal of sodasi bricks for completing the number 200), These are to be covered by half or quarter bricks. Anukas in the place of pancadasabhagtyas (are to be placed).

(With these two layers) alternating with each other as many layers as desired are to be constructed.

The kite-shaped fire-altar (kankacit) the fire-altar in the form of an alaja bird are explained after the falcon-shaped (fire-altar).

189. Like the falcon their two wings are larger than the tail and more curved (at the middle); the inclined tail is long (at the end and short where it joins with the body); neither the body nor the head is circular. This is according to the scriptures. Or, in pursuance of the sacred tradition, (the fire-altar may be) without the head.

190. And it is taught thus: 'One who wishes to live with the head on in this world should provide the kankacit with the head'. Why is it said when one (always) has (the head)?

191. Naturally the two wings are curved and the tail is narrowed because such modifications are so heard. Where no (such) modification is heard, the body retains its natural form.

192. Thus it is constructed in the form of the falcon, and the shape has been explained after the sacred tradition.

193. According to tradition, the fire-altar for the asvamedha (sacrifice) is three times as large (as the seven_fold with aratni and pradesa).

194. All (sorts of) enlargements are possible in this case as nothing particular is mentioned.

195. The enlargement of the wings and the tail is stated to be brought about by the addition of rectangles.

196. It is (further) taught that, for the asvamedha sacrifice, the fire-altar is of twenty-one.

Thus one can see that the Vedic geometry has its base from the Vedic rituals like havanas and yajnas. The basic principle of all the yajna saalas lies in the sulba sutra. Thus in India, the geometry has its beginning in the applied field itself. In designing and construction of the yajna based houses and altars.

Challenge of solving ancient IndianMathematics Problems

Dear Students, Teachers, and Parents!

Bharatheeya Vidya Vihar is a school started (in 2005) by the Indian Institute of Scientific Heritage, at Mazhuvanchery, Eranellur, in Trissur Guruvayoor Highway. The aim of establishing this academic institute is for demonstrating the method of incorporating the value based education for the students, teachers and parents. The school has undertaken the mission of spreading the values, heritage knowledge on all aspects of human life, spirituality, modern knowledge, etc.... not only within our campus, but also in every other schools with the support and cooperation of the management.

1. Development the future of a student is like constructing a building. The bricks used for the building are like the modern knowledge, the cement used is like the values taught. The cement gives strength, durability and beauty for the building. Similarly the values contribute to the beauty, strength and durability of the life.

2. One should not give values and modern knowledge in separate periods in the class rooms. They are to be given together. The bricks and cement are placed simultaneously during the construction. Cement connects the brick layers. The values give connection of the modern knowledge with the life. Hence the cry for separate periods/hours for teaching values in the class rooms cannot be logically accepted. The values are to be discussed in the class rooms for few minutes when physics/chemistry/English/history/biology/..... etc are taught, whenever and wherever possible.

Generally everyone asks the question; what is meant by values and value based education. We definite it like this: All the information and knowldege disseminated fort he (1) psychological and (2) physiological benefits, (3) strengthening family bondage and relations, (4) improvin social bondage and (5) national integration are the values in the life. In one line the values are those knoledge given for the psychological, physiological, family relation based, social bondage based and national integration based benefits.

For the psychological benefit, proper utilization ears, eyes tongue and mind for learning and teaching, is advised.

For physiological benefit taking good food (ahaara). following valuable acharas (customs and good rituals) and alos the karma (dharma)

For strengtheining the family relation, one should teach teh message of mathru/pitru/putra/putree/bharatru/patnee/.... dharma and also the concept of maathru devo bhava, pitru devo bhava. The students should be informed about their dharma towars their parents particularly when they (parents) become old.

Every student should be informed about his/her dharma towards the society, particularly towards the sick, old aged peope, orphans, poor, ... A message should always be given to them that they should spent atleast 3% each of their time, energy and monthly income in the course of time for serving the society. They should also learn the culture of India from the heritage knowledge given in epics, poems, stories, and from the great messages given by our ancestors.

Every student should know the past, present and future glory of our motherland and feel proud of being an Indian. The scientific, technological, spiritual, social literature based....... heritage of India should be taught with example like Ajanta, Ellora, Huge temples, Delhi Iron Pillar, etc. The achievements of modern Inida in the field of atomic energy, green-white-blue revolution and revolutionary imporvements taking place in India in the field of the electronics/ communication/ transportation/academic / health / IT fields etc should be taught for making every student proud of our country. How the world is estimating and assessing India as the great furture economic / scientific / technological power also should be taught so that every Indian should get thrilled with our capability and should feel proud with self respect and self confidence.

Every students, teacher, and parents should collect as many information bits on the above five subjects as possible and learn, teach and practice in their life, which are the values in the value based education. We are sure every school authority will definitely use the above messages for teaching their students.

1. What are the respective remainders obtained when the sums of 1 to 10, each multiplied by 10, terms of the series whose first term and common difference are unity are severally subtracted from the sum of 100 terms of the same series ?

2. Multiply 1296 by 21, 896 by 37, 8065 by 60

3. Tell me the squares of 1 to 9 , 25, 36, 63, 432 and 7802

4. Quickly say what are the cubes of 1 to 9, 15, 256 and 203

5. Say the sum of ½, 1/3, 1/6 and 1/12 and of 2 + ½, 3 – ¼, and 6

6. Friend, if you know the method of calculation , quickly say the sum of 1 ½ terms, ½ term, and of 1/3 term of the series whose first term (aadi) and common difference (chaya) are each unity

7. Subtract ¼, 1/3 and 1/6 from 1 and say what remains. Also subtract 3 - ½ and 2 + 1/3 from 5 and say the remainder

8. Say what remains as the remainder when the sum of 2 plus ½ terms is subtracted from the sum of 5 + ½ terms of the series whose first term and common differnce are unity

9. 2 plus ½ is multiplied by 1 plus ½ and 60 plus 1/3 is multiplied by 5/2: what are the products say separately.

10. 6 + ¼ is divided by 2 + ½ and 60 + ¼ is divided by 3 + ½ ; say the quotients separately

11. Say, friend if you know , the square of 2 + ½ , of 15 + ¼ , of ½ and of 1/3.

12. Say, if you know, the cube of 7 + ½ of 17 + ¼ , of ¼, and of 1/3.

13. What sum is obtained by adding together the fractions having the integers 2 to 6 for their denominators, and 1 for their numerators, and by adding together the fractions having the integers 3 to 9 for their denominators and the integers 2 etc., for their respective numerators.

14. Tell me the sum of ¼ of ½ of ½, 1/10 of 1/6 of 1/5 of 1/3 , and 1/7 of 1/6 of (2 +1/2)

15. It has been severally divided by fractions having the integers 3 to 6 for their denominators and the integers 2 etc for their respective numerators. Say what sum will be obtained when they are added together.

16. Say the amount, when 1 – ½, 5 – ¼ and 8 –1/3 are added together

17. What is obtained by adding (3-1/2)-1/4 of (3 –1/2) – 1/6 ((3 – ½) – 1/4of (3-1/2)) and ½ - ½ of ½-1/4 of (1/2 –1/3 of ½) ?

18. What amount is obtained by reducing 5 puranas, 3 panas, 1 kakini, - 1 varataka, - 1/5 of that of a varataka to puranas ? This is a problem connected with ancient Indian coins/ currencies

19. What amount is obtained by adding together ½. ¼ of ¼, 1 divided by 1/3, ½ + ½ of ½, and 1/3 - ½ of 1/3 ?

20. If 1 pala and 1 karsha of sandal wood are obtained for ten and a half panas, then for how much will 9 palas and 1 karshas of sandal wood of the same quality be obtained ?

21. If I 1/3 palas of black pepper are obtained for 1 ¼ panas, then how much of that will be obtained for (10 – 1/3) panas ?

22. If one and a half dronas and three Kudavas of grain is obtained for 8, say, if you know, for how much will one khari and one drona of that grain be obtained

23. If 60 +1/2 kharis of grain is obtained for 100 + 1/3 rupas, how much of that grain will be obtained for a quarter of a rupa ?

24. Where one suvarana gets 70 +1/3 rupas, say, friend, what will 1 masha as lessened by 1/10 of a masha get there.

25. A certain lame person goes to distance of 1/8 of a yojana in 1/ 3 of a day, say in how much time will he go to a distance of 100 yojanas

26. An insect goes to a distance of 1/6 of an angula in ¼ of a day, in how much time will it go to a distance of 10 and half a yojanas

27. The best amongst the elephants goes forward at the rate of ½ (1 +1/4)(1 – 1/3) (1 +1/2) of a yojana in 6 x 1/5 x 1/9 x 1/3 (1 +1/4) of a day and comes back at the rate of 2(1 –1/3) yojanas in (1 +1/2) days. Say, friend, in how much time will he go to a distance of 100 yojanas

28. In how much time will a man, earning at the rate of (8 –1/2) rupas in (1 + 1/3) days and spending on his food at the rate of ½ per day, be a lord of 100 rupas ?

29. When a given quantity of pearls is measured at 8 pearls a necklace, the number of necklaces is twenty; say, mathematician, what the number of necklaces would be (when the same quantity of pearls is measured) at 6 pearls a necklace.

30. Being measured by the masha of 5 raktikas, a quantity of gold amounts to 300 suvarnas, say how much would that quantity of gold, amount to, when measured by the masha of 6 raktikas

31. How much gold of 11 varnas can be had in exchange for 168 suvarnas of 16 varnas ?

32. Quickly say how many blankets of length 6 hastas and breadth 2 hastas can be made of the yarn which yields 200 blankets of length 9 cubits and breadth 3 cubits.

33. How much gold of 10 ¼ varnas will be obtained in exchange for 100 suvarnas and 8 mashas of gold of 12 ½ varnas ?

34. If the interest on 100 for a month be 5, what is the interest on 60 for a year ? from the interest say the time, and from them both the unknown principal.

35. If 1 ½ be the interest on 100 ½ for one third of a month, what will be the interest on 60 ¼ for (8 – ½) months.

36. When the price of a suvarna of gold of 16 varnas is 60 , then say the price of 63 suvarnas of gold of 10 varnas

37. If 8 dronas of rice are carried to a distance of one yojana for 6 panas, say for how much will a khari together with a drona of rice be carried to a distance of 3 yojanas

38. If 3 laborers earn 5 rupas in 2 days , say what will 8 laborers earn in 9 days ?

39. If a blanket, whose breadth is 2 cubits and length 8 cubits, gets 10, what will 2 other similar blankets of breadth 3 cubits and length 9 cubits get ?

40. If a rectangular piece of stone with length, breadth and thickness equal to 9, 5 and 1 hastas respectively costs 8, what will two other rectangular pieces of stone of dimensions 10, 7 and 2 hastas cost ?

41. If the diet of an elephant of diamter 2 hastas , height 6 hastas and length 7 hastas is one drona, what should be diet of an elephant of diameter 3 hastas , height 9 hastas and length 10 hastas

42. If 2 palas of dry ginger are obtained for 6 and one pala of long pepper for 9, how much of long pepper will then be obtained for 6 palas of dry ginger ?

43. If 16 mangoes are obtained for 2 panas and 100 wood apples for 3 panas, say then how many wood apples will be obtained for 6 mangoes.

44. If 16 workers of age 16 get 200, say then, o mathematician , how much will 2 workers of 20 years of age get ?

45. If 3 camels of 10 years of age get 108 puranas, say then what will 5 camels of 9 years of age get.

46. The rate of interest being 5 % per month, a certain sum is seen to amount to 96 in a year. Say, friend what is the capital and what the interest ?

47. The interest on 100 ½ for one month and a quarter being 1 ½, a certain sum amounts to 36 ½ in a period of 7 ½ months. Find the sum and the interest accrued thereon.

48. The rate of interest being 5% per month, the commission of the surety (bhavyaka) 1 % per month, the fee of the calculator (vrutti) ½ % per month and the charges for the scribe ¼ % per month, a certain sum amounts to 905 a year. Find the capital the interest and the shares of the surety, fee for accountant and the scribe.

49. A rich man lent to somebody a sum of 100 rupas at 5% per month and from him took a house bearing a rent of 40 rupas per month. Say learned man, after how much time is the debtor relived of his debt, and what does the rich man get by the gain of bare accommodation.

50. There are four bonds capitals amounting to 100, 200, 300, and 400 are given to someone on intrest at the rates of 2,3,4 and 5 % per month in the respective order; and months amounts to 2,3,5 and 4 multiplied by 2, have passed. Say, how would a single bond (eka patra) be now made out of these.

51. O' Learned man how a single bond be made of the above 4 bonds with the same capitals as previously stated and with rates per cent per month of interest augmented by ½ in each case and months elapsed increased by ¼ in each case .

52. A sum of money is put to interest at 5% per month. When will it become twice of itself ?

53. And when will another sum of money put to interest at 3 ½ percent per month become 1 ¼ of itself ?

54. Two, three, five and four prastas of seeds are contributed by four partners and 210 prasthas of grain is the produce; what are the shares of the partners separately ?

55. ½ prasthas is the contribution of one, 1/3 prastha of another, 1/9 prastha of still another, and 1700 prasthas is the produce. Say what are their shares in the produce separately.

56. Seven kudava of mudga (green gram) are obtained for 9 panas, and ½ kudava of rice is obtained for one pana, then O' merchant, take 3 panas and a half and quickly give me one part of rice and two parts of mudga.

57. ½ pala of asafetida, 2 palas of long pepper, and 7 palas of dry ginger are each obtained for one rupa. Give me equal quantities of each of them for one rupa.

58. The capitals of three men are 1, 3 and 5 rupas or 1/3, ¼ and ½ rupas respectively. By purchasing and selling certain articles at the same rates and by selling the remnant articles at the rate of 1 for 3 rupas, they become possessed of equal riches. Find the rates of purchase and sale.

59. The capitals of four men are 1 ½, 2, 3 and 5 rupas. By purchasing and selling certain articles at the same rates and by selling the remnant articles at the rate of 1 for ½ of a rupa, they become possessed of equal riches. Find the rates of purchase and sale.

60. Pigeons are sold at the rate of 5 for 3 rupas, cranes at the rate of 7 for 5 rupas, swans at the rate of 9 for 7 rupas and the peacocks at the rate of 3 for 9 rupas. Knowing the rates as stated above bring 100 birds for 100 rupas for the amusement of the princess.

61. The rate of sales of pomegranates, mangoes and wood apples are respectively 1 for 2 rupas, 5 for 3 rupas and 2 for 1 rupas. Bring 100 fruits for 80 rupas.

62. When a person, traveling at the speed of 8 yojanas per 5 minus ½ days, has alreddy traveled for 6 minus ¼ days, another person, who travels at the speed of 3 yojanas a day starts traveling from the same place along the same track. Say after calculating, when the latter traveler would overtake the former.

63. One man travels at the speed of 8 yojanas a day and another at 2 yojanas a day from the same place and after reaching the destination come back by the same track. The length of the track is 100 yojana. Say where is the meeting of the two. One going ahead and the other coming back

64. While a leathern oil bottle (kutapa) filled with 200 palas of oil, was being carried by a porter to a distance of 8 yojanas for 5 panas as wages, a hole happned to occur in the bottom of it through which the oil leaked out on the way continuously. If 20 palas of oil be left in the bottle what wages should be paid to the porter ?

65. One man saw a dance for one quarter of the day, another for two quarters of the day, another for three quarters of the day and yet another till the end of the day. The dancing party has to be paid by them a sum of ninety six rupas in all. If payment is to be made in proportion to the time of seeing the dance, how much of that sum should be paid by each of them separately ?

66. A palanquin is to be carried to a distance of 3 krosas by 10 men for 100 rupas. Of those men, 2,3 and 5 stop away after going over 1,2, and 3 krosas respectively. Calculate the wages of each of them separately.

67. Five scholars, enchanters of Vedas were invited by a person to take part in the worship of the five faces of the five faced god Siva on a dakshina of 300 rupas. And they,. On the completion of worship of, two, three, four and five faces respectively, went away one by one. Say what are their dakshinas.

68. In what time will the four drains, which severally fill up a cistern in ½, 1/4., 1/5 and 1/6 of a day , fill up that cistern if they are opened simultaneously , to flow into it ?

69. If for carrying 24 jack fruits over a distance of 5 krosas a porter is to get 9 of those jack fruits, what will he get if he carried them over a distance of 2 krosas ?

70.* Twenty four jackfruits were carried to a distance by a man for 4 out of those 24 jack fruits as wages; the remaining jack fruits were carried over the remaining distance by another person for 5 of them as wages. The load was thus carried by the two persons over a distance of 5 krosas. Say how much of that distance was gone over by each of them ?

71. O' friend, a cook prepares varieties of food with the six savours , pungent, bitter, astringent, acid, saline and sweet. Say what is the possible number of varieties ?

72. One fourth, one third, and one sixth of a pillar are respectively buried under the water, mud and sand of a river and three cubits are visible. Give out the measure of the length of that pillar

73. After giving away one half of a quantity, then 2/3 of what remains, then ¾ of what remains there after and then 4/5 of what remains there after, the residue left is 3. What was the quantity ?

74. Of a herd of cows, one half went away towards the east and one fourth towards the west, the difference of the - two as multiplied by 2 and divided by 5 went away towards the north and three cows are left. What is the numerical strength of the herd ?

75. A number is diminished by its square root, what remains is diminished by its one- sixth, what remains after that is diminished by its square root, what remains after that is diminished by is one fifth and what remains after that is diminished by twice the square root of itself; the residue now left is 8 . Find out the number .

76. One third of a troop of monkeys together with one third of itself has gone to the tank; the square root of the whole troop is afflicted with thirst; and the remaining two monkeys are sitting under the mango tree . what is the number of the monkeys in the troop ?

77. After giving away one, then one sixth of what remains, then one fourth of what remains after that, then one third of what remains after that, and then the square root of the original number, the residue left is 5. what is the original number?

78. Say what is that number which being multiplied by 5/2, then divided by 3, then squared, then increased by 9, then reduced to its square root, and then diminished by 1, becomes 4.

79. What is the sum of 5 terms of the series whose first term is 2 and common difference 3? and what of one half of a term? Say the sum of one fifth of a term of the series whose common difference is 5 and first term 2?

80. In a leathern oil bottle full of oil there occurs a minute hole, and the oils leaks through it. The bottle has to be carried to a distance of 3 yojanas. If the wages for the first yojana be 10 panas and those for the subsequent yojanas successively less by 2 panas, what are the wages for a krosa?

81. One man gets 3 rupas and the other men get 2 rupas more in succession; say, what do the first 4 ½ men get.

82. If a laborer gets 1 ½ in the first month and 1/3 more in succession in the following months, what will he get in the first 3 ½ months.

83. A man taking 3 rupas with him, went out to make profit. If his capital becomes double after every month, what will it become after 3 years?

84. The first bangle is obtained for 8 panas, and the last bangle for 13 panas. If the total number of the bangles be 24, say what is the price of all of them.

85. One man goes with initial speed 3 yojanas per day and acceleration 1 yojana per day per day and another man goes with the constant speed of 10 yojanas per day. In what time will they cover the same distance?

86. After one man had traveled for 6 days with some initial speed (adi) and acceleration (uttara) another man went by the same track with an unknown initial speed and acceleration of 2 units per day per day. Say how will they meet each other two times .

87. In a gamble two persons alternately won 30, 10, 100 and 8 casts of dice beginning with 9 and increasing successively by 6. say who is the winner.

88. If the casts of dice (alternatively won by the two persons) be 7, 3, 9, and 12 and the first term and common difference of the series formed by the take money, as stated before, then say after calculation who wins.

89. Say what is the sum of i. The sum of the first five natural numbers, ii. The square of 5 and iii. The cube of 5.

90. Friend, quickly say what is the sum of the cubes of 10 terms of the series whose first term and common difference are each unity; and also the sum of the successive sums of those terms.

91. O' Friend, if you know then say after calculation the sum of i. The sum of successive sums of the first 6 natural numbers, ii. The sum of the squares of the first 6 natural numbers and iii. The sum of the cubes of the first 6 natural numbers.

92. Tell me the sum of the squares of the first six terms of the arithmetic series whose first term is two and the common difference three.

93. Say after adding together the cubes of the four terms which begin with 5 and increase successively by 2.

94. In an equilateral quadrilateral, the face, the base and the altitude are all equal to the flank sides, each being 1 ½ hastas in length. Say, friend what is the area of the quadrilateral.

95. Give out the area of that rectangular quadrilateral in which the base and face are each 5 ½ hastas, and the flank sides and altitude each 3 hastas.

96. In a triangle the flank sides are 4 – ¼ and 3 ¼ hastas, the base is 3 ½ hastas and the altitude is 3 hastas, what is the area of that triangle ?

97. In an equilateral triangle the base is 8 ½ hastas and the altitude is 7 hastas and 8 2/3 angulas. What is the area thereof ?

98. If you know the method of finding the area of place figures, say the area of the isosceles triangle, whose flank sides are each 5 hastas, altitude 3 hastas and the base 8 hastas.

99. In a quadrilateral the face is 1 1/3 hastas, the base is 9 1/3 hastas, the flank sides are each 5 hastas and the altitude is 3 hastas what is the area ?

100. In an in-equilateral quadrilateral with equal altitudes, the base is 10 hastas, the face is 4 1/6, the flank sides are 9 – 1/3 and 6 + ½ hastas and the altitude is 6 ½ hastas minus 1/60 of an angula. what is the area of the figure ?

101. What is the area of an elephant's tusk whose base is 2 cubits and altitude cubits; and also of the figure of the shape of a felloe whose base and face are 3 cubits and altitude is 10 cubits ?

102. The central length of a crescent moon is 8 cubits and the central width 3 cubits. Treating it as made up of a pair of triangle, quickly say what its arc is ?

103. In a thunderbolt, the central length is 10 hastas, the faces are each 5 hastas, and the central width is 2 hastas. What is its area, if it be regarded as made up of two quadrilaterals ?

PART II

The mathematical problems given by Bhaskaracharya I in 629 AD in Aryabhateeya Bhashya (Commentary for Aryabhateeya)

104. Tell me separately the cubes of integral numbers beginning with 1 and ending in 9, and also of $(8x8)^2$ and (25^2)[23]

105. If you have clear understanding of cubing a number, say correctly the cubes of 6 , 5, 10, and 8 as respectively diminished by 1/6, 1/5, 1/10 and 1/8

106. Calculate in accordance with the ganitha of Aryabhatta, the square root of 6 plus ¼ and of 13 plus 4/9 and state the two results.

107. Correctly state in accordance with the ganitha of Aryabhatta the fractional cube root of 13 plus 103/125

108. Correctly state, in accordance with the rules prescribed in bhatasastra (Aryabhateeya) the cube root of 8291469824.

109. Tell me O' friend , the area of the (three) equilateral triangles whose sides are 7, 8 and 9 units respectively, and also the area of the isosceles triangle whose base is 6 units and the lateral sides each 5 units

110. Say what is the area of the scalene triangle in which the base is 51 units and one lateral side is 37 units and the other lateral side is stated to be 20 units.

111. Quickly tell me the more accurate volume and also the measure of the altitude of the solid of the shape of a trapa (triangular pyramid) in which each edge is 12 units

112. The diameter of three circles are correctly seen by me to be 8, 12 and 6 units, respectively. Tell me separately the circumference and area of these circles

113. Calculate and tell me the diameters of the circles whose peripheries are 3299 minus 8/25 and 21600, respectively.

114. When at an equinox, the Sun is on the meridian, the shadow of a gnomon, divided into 12 units, on level ground is seen to be 5, 9, and 3 ½ units at three different places. Find the latitudes of these places.

115. Tell me the length of the shadow of the gnomon situated at a distance of 80 angulas from the foot of a lamp post of height 72 angulas; and also that of another gnomon situated at a distance of 20 angulas from a lamp post of height 30 angulas.

116. Give out the hypotenuses of three right angled triangle where the base and uprights are respectively 3 &4, 6 & 8, and 12 & 9, respectively.

117. In a circle of diameter 10 units, the arrows are seen by me to be 2 and 8 units, in the same circle another set of arrows is 9 and 1 units. Tell me the corresponding R Sines.

118. A hawk is sitting on the top of a rampart whose height is 12 cubits. The hawk sees a rat at a distance of 24 cubits away from the foot of the rampart, the rat , too sees the hawk. Thereupon the rat, out of fear for the hawk, hastens to its own dwelling situated at the foot of the rampart but is killed in between by the hawk which flew along a hypotenuse. I want to know the distance traversed by the rat and also the horizontal motion of the hawk.

119. A bamboo of height 18 units is felled by the wind. It falls at a distance of 6 cubits from the root, thus forming a right angled triangle. Where is the break. ?

120. A full blown lotus of 8 angulas is seen just above the water. Being carried away by the wind it just submerges at a distance of cubit (24 angulas) . Quickly say the height of the lotus plant and the depth of the water.

121. There is a reservoir of water of dimensions 6 x 12. At the east north corner thereof there is a fish; and at the west north corner there is a crane. For th fear of the crane, the fish crossing the reservoir, hurriedly went towards the south in an oblique direction but was killed by the crane who came along the sides of the reservoir. Give out the distances traveled by them assuming that their speeds are the same.

122. When the 8 out of 32 of the diameter of the Moon are eclipsed by the shadow of diameter 80, I want to know then what are the arrows of the intercepted arcs of the shadow and the full Moon.

123. In a series of Arithmetic Progression the first term is 8, the common difference is stated to be 5 and the numbers of terms is seen to be 18. Give out the middle term and the sum of the series.

124. In the month of Karkitaka a certain king daily gives away some money in charity starting with 2 on the first day of the month and increasing that by 3 per day. Fifteen days having passed, there arrived a scholar well versed in Vedas. The amount for the next ten days was given to him; that for the remaining five days of the month to someone else. Say, what do the last two persons get.

125. Of the 11 Conch shells which are arranged in the increasing order of their prices which are in AP, the first conch shell is acquired for 5 and the last for 95. Say what is the price of all the 11 conch shells.

126. There are three pyramidal piles of balls having respectively 5, 8 and 14 layers which are triangular. Tell me the number of units of balls in each of them (application of the progression 1 + (1+2) + (1+2 +3) +........

127. There are three pyramidal pile on square bases having 7, 8 and 17 layers which are also squares. Say the number of units therein (number of bricks, of unit size used in each of them) (application of $1^2 + 2^2 + 3^2 + \dots + n^2$

128. There are three pyramidal pile having 5, 4 and 9 cuboidal layers. They are constructed of cuboidal bricks of unit dimensions with one brick in the topmost layer. Find the number of bricks used in each of them (application of $1^3 + 2^3 + 3^3 + \dots + n^3$

129. The products of two numbers is correctly seen to be as 8 and their difference is 2. Of two other numbers, the product being 18, the difference is 7. Tell me the numbers multiplied in the two cases.

130. I do not know the monthly interest on 100, but I do know that the monthly interest on 100 + interest on that interest accruing in 4 months is 6. Give out the monthly interest on 100.

131. If one bhara (2000 palas) of ginger is sold for 10 + 1/5 rupakas, tell me quickly the price of 100 + ½ palas of ginger.

132. A serpent of 20 cubits in length enters into a hole, moving forward at the rate of ½ of an angula per muhurta (one muhurta = 48 minutes) and backward at the rate of 1/5 of an angula per muhurta. In how many days does the snake get into the hole completely

133. Out of 11 cattle 8 are tamed and 3 to be tamed so are the cattle described. Out of 1001 cattle, then, how many are tamed and how many to be tamed ?

134. Five merchants collaborate in a business; the capitals invested by them are in the ratio of 1 and the same number increasing by one. The profit that accrued on the whole capital amounts to 1000. Say what should be given to whom.

135. The combined profit of three merchants whose investments are in the ratio of ½. 1/3. and 1/8 respectively, amount to 70 minus 1. What is whose profit ?

136. Given the 100 increases by 5 in a month, say if you are well versed in Aryabhatta's Ganitha . by how much will 20 increase in 6 months .

137. A sum of 20 plus ½ rupakas increased by 1 plus 1/3 rupakas in 1 plus 1/5 months. Say after carefully understating the method of elimination of divisors from the aphorism of the Bhatta tantra (Aryabhateeya) what will be the increase of 7 minus ¼ rupakas in 6 plus 1/10 months.

138. If 9 kudavas of pure parched and flattened rice are obtained daily for an elephant whose height is 7 cubits, periphery 30 cubits and length 9 cubits say how much of parched and flattened rice will have to be obtained for an elephant whose height is 5 cubits, length and periphery 28 cubits.

139. A number is multiplied by 2; then increased by 1; then divided by 5; then multiplied by 3; then diminished by 2 and then divided by 7: the result thus obtained is 1. Say what is the initial number

140. What is that number which when multiplied by 3, then diminished by 1, then halved, then increased by 2, then divided by 3 and finally diminished by 2 yields 1 ?

141. A certain person has 8 palas of saffron and money amounting to 90 rupakas. Another person possesses 12 palas of saffron and 30 rupakas and the two persons are equally rich by these items. If the two persons have bought the saffron at the same rate per pala, I want to know the price of one pala of saffron and also the equal wealth with the two.

142. One man goes from Vallabhi at the speed of 1 ½ yojanas a day; another man comes along the same route from Harukaccha at

the speed of 1 ¼ yojana a day. The distance between the two places is known to be 18 yojanas. Say, o mathematician, after how much time will they meet each other.

143. One man goes from Vallabhi to Ganga at the speed of 1 ½ yojanas a day and at the same time another man proceeds from Sivabhagapura at the speed of 2/3 yojanas a day. The distance between the two places has been stated by the learned to be 24 yojanas. If they travel along the same route, after how much time will they meet each other ?

144. Calculate what is that number which is said to yield 5 as remainder when divided by 8, 4 when divided by 9 and 1 when divided by 7.

145. Quickly say, O mathematician, what is that number which when divided by the numbers beginning with 2 and ending in 6 in each case leaves 1 as the remainder and is exactly divisible by 7.

146. 8 is multiplied by some number and the product is increased by 6 and that sum is then divided by 13. If the division be exact, what is the unknown multiplier and what is the resulting quotient ?

147. By opening four inlets separately, one pond gets filled respectively with 1, ½, 1/3 and 1/6 days. If all the four inlets are opened together, how much time in fraction of the day is required to fill the pond.

148. One third of a troop of monkeys with one third of itself has gone to the tank, the square root of the whole troop is afflicted with thirst, and the remaining 2 monkeys are sitting under the mango tree. What is the total number of monkeys

149. I saw that one half of 7 times the square root of the total number of swans were slowly moving away in the river. Remaining 2 are playing in water. What is the number of the total swans ?

ASTRONOMICAL PROBLEMS GIVEN BY BHASKARACHARYA I (AD 629)

150. The mean position of the Sun has been observed by me at sunrise to be in the sign Leo in the middle of navamamsa Sagitarius. Calculate the number of days elapsed since the beginning of Kalaiyuga (ahargana) when the longitude of the planets was zéro according to the Aryabhateeya

151. The mean longitude of the Sun at sunrise on a Wednesday is stated to be 8 signs, 25 degrees, 36 minutes, and 10 seconds. State correctly after how much time will the sun again assume the same position at sunrise on a Thursday, Friday and Wednesday.

152. The Sun and Moon on a Sunday at sunrise are carefully seen by me to be in the sign Libra. The degrees of their longitudes are 12 and 2, respectively. The minutes are 1 and 40, respectively. After how many days will they assume the same longitudes again at sunrise on a Thursday, Friday and Saturday, respectively ?

153. The mean longitude of the Sun for midnight is found to be 9 signs, 32 minutes, and 40 seconds. Quickly say the number of days elapsed since Kaliyuga and the number of revolutions performed by the Earth (Sun)

154. Quickly calculate how many years and how many days of the current yuga had elapsed when the traversed part of the Moon's apogee amounted to three signs.

Sanskrit terms and the measurements used in mathematical books

Sankalitha - addition, vyavakalith - subtraction, pratyutpanna - multiplication. bhagahara - division, varga – square, varga mula – square root. ghana – cube, ghana mula – cube root.

Table of money measurements: one purana = 16 panas; one pana = four kakinis. one kakini =20 varatakas/ coudies, 12 panas = one dramma. 36 dramma = nishka.

Table of weights: one masha=5 gunjas, 16 masha= one karsha, one karsha of gold = suvarna , 4 karshas = one pala.

Table of the measurements of capacity: One khari = 16 dronas, one drona = 4 adhakas, one adhaka = 4 prasthas, one prastha = 4 kudavas

Table of linear measurements

24 angula = one hasta, 4 hasta = one danda, 2000 danda = one krosa, 4 krosa = one yojana.

Table of time measurements

60 ghatis = ahoratra (a day and night), 30 ahoratra = one masa, 12 masa = one varsha

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How the period of an ancient Indian scientist has been calculated? Calculation of the date of Bhaskaracharya I and his book Aryabhateeya Bhashya written in AD 629 is given below

In the above book 9th stanza of first chapter, 3rd narration gives the period of writing this book in Bhootha Sankhya (number) system as....kha (0) agni(3) adri(7) rama(3) arka(12) rasa(6) vasu(8) randhra(9) indava:(1) - (kha agnyadri rama arka rasa vasu randrendava:) which is 1986123730*. (write the above numbers in the reverse order.

"... Since the beginning of the current kalpa, the number of years elapsed is 1986123730 years (when the book was written).

The number of years elapsed since the beginning of the current kalpa at the commencement of kaliyuga (according to Aryabhata I)

= 6 manwantharaas + 27 ¾ of mahayugas (one manwanthara = 72 mahayugas and one mahayuga = 4320000 years)

= (6 x 72 + 27 ¾) x 4320000 years

= 1866240000 + 119880000 = 1986120000 years

Hence the number of years elapsed since the beginning of the kaliyuga at the time of writing this book

= 1986123730* - 1986120000 = 3730 years

The kali era starts from BC 3102 Feb 17th. When year and month corrections are given we get 3730 – 3101 = 629 AD. Our ancestors could calculate the year so accurately (which has been written in their books.)

CONTENTS

Use of average, ratio and proportions, permutation and combinations, percentage, interest calculations, partnerships, shares, profit sharing, loans and interest, rules of calculation interest, compound interest, bodies in motion, progression, determinatin of unknown values from known values, equation of higher order. Geometry in Boudhayana - Katyayana Apasthamba - Manava sulbasutras, area - perimeter - diagonals - altitudes, etc of triangles, quardrilaterals, polygonals, rectangles, trapeziums, cyclic quadrilaterals, circles, etc. Every ruler of any nation is keen on teaching the heritage of that nation to all citizens. Unfortunately in India, for the last 70 years, we have not learned or taught our heritage. Instead we denigrated our valuable and scientific traditions and heritage. It has become a political tool for the so called secular politicians and policymakers for achieving their goals. Indian Instituteof Scientific Heritage is in the mission of learningand teaching the heritage of our motherland inthe true spirit of science